

STUDY OF DYNAMICAL APERTURE OF NICA COLLIDER WITH ACCOUNT OF MAGNETIC FIELD ERRORS AND COULOMB EFFECTS

A.Ye. Bolshakov, P. R. Zenkevich, ITEP, Moscow, Russia

Abstract

By use of MADX code beam dynamics in NICA collider has been studied. NICA collider has comparatively small kinetic ion energies (1.5-4.5 GeV/u) that results in essential one beam Coulomb effects. These effects are simulated by set of “BEAM-BEAM” elements with appropriate chosen strength and location. Moreover it was taken into account beam-beam interaction, system of chromaticity correction and influence of systematic and random errors of the magnetic field. The simulation results are given and discussed.

Coulomb Forces

In collider Coulomb forces result in effect of “beam-beam” interaction. In linear approximation shift of the betatron tune because of the effect is defined of so named “beam-beam parameter”, which for symmetrical Gaussian beams is defined by

$$\xi = -r_i \frac{N_b}{4\pi\beta^2\gamma\epsilon} \frac{1+\beta^2}{2} \quad (1)$$

Here the classical ion radius $r_i = \frac{Z^2 r_p}{A}$ (r_p - classical proton radius, A and Z are atomic and charge ion numbers), β, γ – relativistic, N_b is the number of ions per bunch, ϵ is r.m.s beam emittance. Non-linear kick in the interaction point results in appearance of a set of non-linear resonances (see, for example, [1]). Let us use model of “frozen beam”, which is assumed that this effect does not influence on a particle distribution in a phase space. In MADX code [2] this effect is described by special element (Beam-Beam element) located in the interaction point. NICA collider has comparatively small kinetic energy (1.5-4.5 GeV) and therefore there are essential “one beam” Coulomb forces, which result in shift of the betatron tune (Laslett tune shift)

$$\Delta Q = -\frac{r_i N_b}{4\pi\beta^2\gamma^3\epsilon} F_b \quad (2)$$

Here we assume that the beams have Gaussian distribution. The bunching factor $F_b = \frac{C_{ring}}{\sqrt{2\pi}\sigma_s}$, where C_{ring} is the ring circumference, σ_s is r.m.s. longitudinal size of a bunch. Dependence of the tune shift with longitudinal coordinate z tune shift

$$\Delta Q(z) = \Delta Q \cdot \exp\left(-\frac{z^2}{2\sigma_s^2}\right) \quad (3)$$

Periodical oscillations of the tune shift because of the synchrotron oscillations result in crossing of high order betatron resonances. Coulomb shift due to simultaneous action of both effects (for two interaction points) $\Delta Q_c = \Delta Q + 2\xi = -\frac{N_b r_i}{4\pi\beta^2\gamma\epsilon} \left[\frac{F_b}{\gamma^2} + (1 + \beta^2)\right]$. In choice of machine parameters we assumed that $|\Delta Q_c| \leq 0.05$. Then maximal beam intensity

$$N_b^{max} = 4\pi\beta^2\gamma \frac{|\Delta Q_c|}{\gamma^2 + (1 + \beta^2)} \frac{\epsilon}{r_i} \quad (4)$$

In high energy region (3-4.5 GeV) in NICA collider the intensity is limited by IBS and $N_b^{max} = 2.4 \cdot 10^9$ ions [3]. Let us mark that for equal tune shifts Coulomb effect due to the beam-beam interaction is more dangerous than one-beam one. Thus for given intensity the most dangerous point in high energy region corresponds to $E = 3$ GeV. In low energy region (1.5-3 GeV) the intensity is defined by Eq. (3) and the most dangerous point corresponds to the lowest energy ($E = 1.5$ GeV). In this report we choose for simulations point $E = 3$ GeV.

Code for numerical simulations

Simulations are made using MAD-X code with account of the following factors:

- 1) Chromaticity correction system, which is corrected machine chromaticity and chromaticity due to sextupole errors in the collider magnets.
- 2) Systematic errors and random errors in the collider magnets. Systematic errors are accepted same as in paper [4] (these errors are given at Table 1). Random errors correspond r.m.s. value equal to 1/3 from systematic errors. For simplification we take into account only one set of random errors.
- 3) Beam-beam Coulomb forces with two similar interaction points.
- 4) One-beam Coulomb forces.

Table 1: Systematic errors in the magnets

$\frac{1}{BR} \left(\frac{d^2 B_y}{dx^2} \right) m^{-3}$	$\frac{1}{BR} \left(\frac{d^4 B_y}{dx^4} \right) m^{-5}$	$\frac{1}{BR} \left(\frac{d^6 B_y}{dx^6} \right) m^{-7}$
-0.027	76.312	-1.489E5
$\frac{1}{BR} \left(\frac{d^8 B_y}{dx^8} \right) m^{-9}$	$\frac{1}{BR} \left(\frac{d^{10} B_y}{dx^{10}} \right) m^{-11}$	$\frac{1}{BR} \left(\frac{d^{12} B_y}{dx^{12}} \right) m^{-13}$
-2.669E10	-7.507E14	-6.616E18

Calculations are made in “thin lens approximation” in order to provide a simplicity [5]. The synchrotron motion is simulated by inclusion in lattice the cavity with voltage 1 MV. Initial distribution in phase space is assumed to be Gaussian one, number of macro particles is 20000-300000, number of turns 1000-5000. Influence of “one beam” Coulomb forces is taken into account by use of “beam-beam” non-linear lenses located in the centers of all lattice elements (such method was used earlier in [6]). Beams in the “beam-beam” elements are assumed to be in one direction, which allows us to describe correctly Lorentz force. Dependence of space charge force on

longitudinal distance z is reached by variation of strength this “beam-beam” lenses with z according to Eq. 3.

Dynamical aperture (DA) analysis is made in space of transverse invariants E_x, E_y , where $E_x = \gamma_x x^2 + 2\alpha_x \dot{x}x + \beta_j \dot{x}^2$, $E_y = \gamma_y y^2 + 2\alpha_y \dot{y}y + \beta_j \dot{y}^2$ (here x, y are transverse coordinates, \dot{x}, \dot{y} are corresponding derivatives on variable s , $\alpha_j, \beta_j, \gamma_j$ are Twiss parameters). The beam has the following parameters: 1) number of particles per bunch $N_b = 2.4 \cdot 10^9$; 2) ion energy $E = 3 \text{ GeV}$; 3) r.m.s. beam sizes in longitudinal space $0.6m \times 1.2 \cdot 10^{-3} \frac{\Delta p}{p}$; 4) r.m.s. beam emittances in the transverse space $\varepsilon_x = \varepsilon_y = 10^{-6} \text{ m}$.

Numerical results

Results of numerical simulations of particle trajectories are shown at Fig. 1. Calculations are made for the following bunch parameters: $\sigma_s = 0.6 \text{ m}$, $\sigma_p = 0.00125$, harmonic number $q = 72$, $V = 1 \text{ MV}$, synchrotron period $\tau_{\text{sinchr}} \approx 170 \cdot \tau_{\text{rev}}$. Number of macroparticles $N_{\text{part}} = 300000$, number of turns $N_{\text{turn}} = 1000$. All effects are taken into account (factors 1+2+3+4). The lost particles are marked by black points. We see from the plot that picture is very complicated.

As the first step of treatment of Fig.1 we have found for given horizontal invariant E_x value of vertical invariant E_y corresponding to the lowest value for lost particles. Typical results of such treatment are plotted at Fig.2.

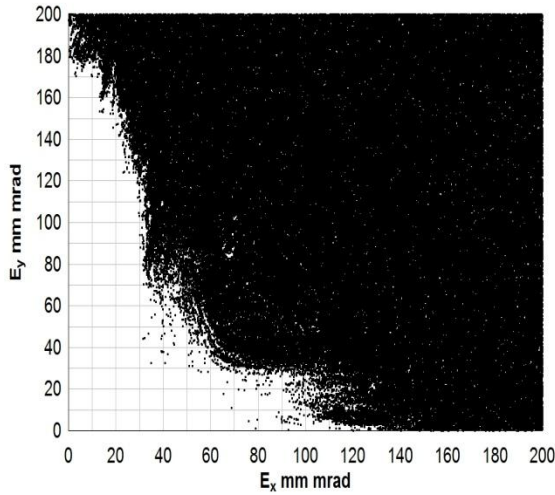


Figure 1: Results of numerical simulations of particle trajectories. Number of macroparticles $N_{\text{part}} = 300000$, number of turns $N_{\text{turn}} = 1000$.

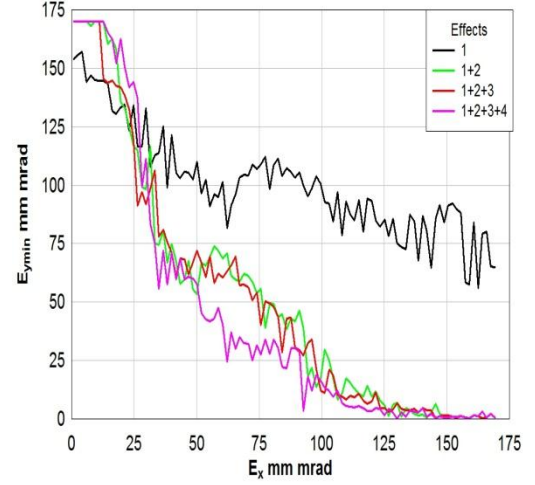


Figure 2: Dependence of minimal value of E_y on E_x for lost particles. Number of macroparticles $N_{\text{part}} = 20000$, number of turns $N_{\text{turn}} = 1000$.

In NICA the narrowest point is located in final focus lenses where chamber is circular that corresponds to equation $r^2 = x^2 + y^2 \leq a^2$ (a is the chamber radius). Let us introduce parameter $E = E_x + E_y$; then avoiding by difference between beta functions we obtain the following condition of particle loss: $E = E_x + E_y \geq \frac{a^2}{\beta_{\text{max}}}$. We see that minimal value E_{min} of parameter E determines particle survival. Dependence of E_{min} on parameter $\mu = (\frac{\Delta p}{p})^2 / (\frac{\Delta p}{p})_{\text{max}}^2$ (here $(\frac{\Delta p}{p})_{\text{max}}$ is the separatrix half-width on momentum deviation) is given at Fig. 3 (parameter μ with good accuracy is proportional to longitudinal invariant).

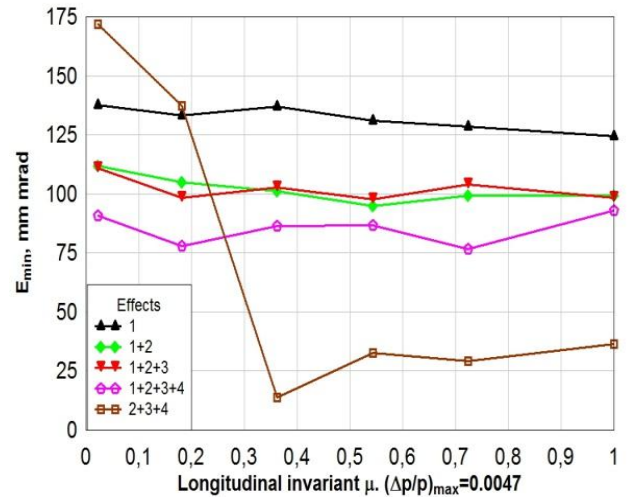


Figure 3: Dependence of E_{min} on parameter $\mu = (\frac{\Delta p}{p})^2 / (\frac{\Delta p}{p})_{\text{max}}^2$. Number of macroparticles $N_{\text{part}} = 20000$, number of turns $N_{\text{turn}} = 1000$.

We see from the plot that beam-beam interaction only weakly affects on DA; the more important effect is one-beam Coulomb interaction. Another important result is the sharp diminishment of DA with increase of μ in absence of the chromaticity correction system. Moreover we have studied dependence of E_{min} on number of turns N_{turn} ; results are presented at Fig. 4.

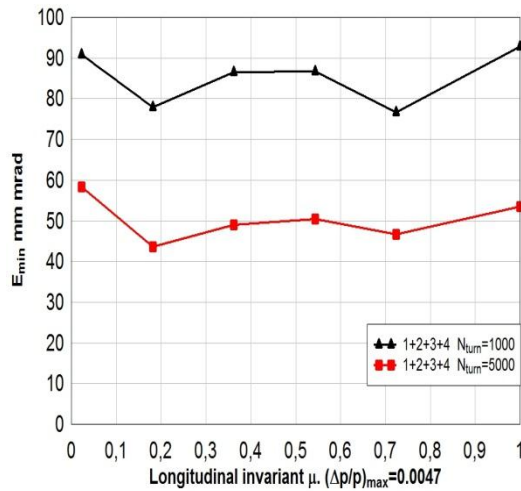


Figure 4: Dependence of E_{min} on parameter $\mu = \frac{(\Delta p/p)^2}{(\Delta p/p)_{max}^2}$. Number of macroparticles $N_{part}=20000$, number of turns is marked at the plot.

Discussion

We see from Fig.4 that dependence on particle number is strong. Significant diminishment of DA with number of turns shows that accepted method of results treatment has some indefinity. In fact we observe that there are particles separated from basic massive of lost particles [Fig.4]. Number of these particles increase with number of turns. This effect is connected with very complicated mechanism of particle losses in for bunched beams, which includes a lot of processes: 1) adiabatic particle capture during resonance crossing [7,8]; 2) modulational diffusion, i.e. diffusion due to random jumps of invariant because fast resonance crossing (see for example [9]); 3) some “rare” processes (for example, Arnold diffusion [9]). It is possible we should take into account only “basic massive” neglecting these particles. Nevertheless in a frame of this method we could make the following conclusions:

- In absence of the chromaticity correction the dynamical aperture is sharply decreased with increasing of longitudinal invariant.
- Main sources of DA reduction in presence of the chromaticity correction circuit are magnetic field errors and one-beam Coulomb forces; beam-beam interaction influence is smaller.

REFERENCES

- [1] E. Keil, “Beam-beam dynamics”, Report CERN/ISR-TH/72-7, (1972).
- [2] H. Grote and F. Schmidt, MAD-X, An Upgrade from MAD8, CERN-AB-2003-024 ABP.
- [3] I. N. Meshkov, “NICA Project at JINR”, 11 International Meeting on Beam Cooling and Related Topics, Ukraine, Alushta, 2011.
- [4] O Kozlov, private communication, JINR, 26.11.2011.
- [5] W. Herr, “Particle tracking with MAD-X including LHC beam-beam interactions” LHC Project Note 344, CERN, May 25, 2004.
- [6] V.V. Kapin, A.Ye. Boshakov, P.R. Zenkevich, “Influence of Beam Space Charge on Dynamical Aperture of TWAC Storage Ring”, XXI Russian Particle Accelerator Conf., Sep. 28 - Oct.3, 2008, Zvenigorod, Moscow region, pp.399-401; <http://accelconf.web.cern.ch/accelconf/r08/papers/FRC AU03.pdf>
- [7] A. W. Chao, M. Month, Particle Trapping during Passage through a High-Order Nonlinear resonance
- [8] G. Franchetti and I. Hofmann, Particle Trapping by Nonlinear Resonances and Space Charge, NIM, A-261, p. 195-202, 2006.
- [9] J. Lichtenberg, A. Lieberman, Regular and Stochastic Motion, Springer, N.Y., 1983.