TRANSIENT BEAM RESPONSE IN SYNCHROTRONS WITH A DIGITAL TRANSVERSE FEEDBACK SYSTEM

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Abstract

The transient beam response to an externally applied impulse force in synchrotrons with a digital transverse feedback system is studied. Experimental data from the LHC on damping of coherent transverse oscillations excited by the discrete-time unit impulse are analysed. Good agreement on the measured and theoretically predicted decrements has been obtained. A method of feedback fine tuning, based on measurements of the bunch response to the harmonic excitation impulse, is discussed.

INTRODUCTION

Transverse feedback systems (TFS) in synchrotrons (see Fig. 1) are used for damping coherent transverse oscillations caused by injection errors and for suppression of coherent transverse instabilities [1]. The transverse momentum of a bunch is corrected by the damper kicker (DK) in proportion to the bunch displacement from the reference orbit at the location of the beam position monitor (BPM). The digital signal processor (DSP) module allows to obtain optimum damping by adjustment of the TFS parameters in agreement with the beam time of flight from BPM to DK and the corresponding betatron phase advance.

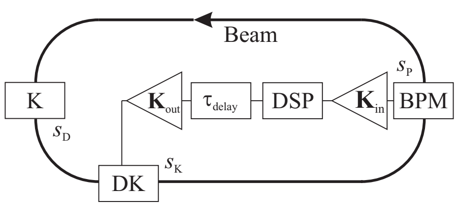

Figure 1: TFS Layout

Coherent oscillations can be originated by DK or a specialised driving kicker K (see Fig. 1). For example, the kicker can be fed with the discrete-time unit impulse or the harmonic excitation impulse. The beam response observed by BPM in this case can be used for tuning the TFS.

The transient beam response to a driving force is analyzed below in framework of the discrete transformation approach developed in [2, 3] for describing transverse beam dynamics in synchrotrons with a digital TFS.

BASIC NOTIONS

Let the column matrix $\widehat{X}[n,s]$ describe the bunch state at the n–th turn and in point s on the reference orbit (see Fig. 1). Its first element $x[n,s]$ is the bunch deviation from the orbit and the second one $x'[n,s]$ is the angle of the bunch trajectory. Let the driving kicker K located in point s_D change the angle of the bunch trajectory on $\Delta x'_D[n]$. For the bunch states at two consecutive turns after passing the damper kicker DK located in point s_K one can write [2, 3]:

$$\widehat{X}[n+1,s] \equiv \widehat{X}[n,s+C] = \widehat{M}(s)\widehat{X}[n,s] + \Delta x'[n,s_K]\,\widehat{M}_K\widehat{E} + \Delta x'_D[n]\,\widehat{M}_D\widehat{E}\,, \qquad (1)$$

where C is the circumference of the reference orbit; elements $E_1 = 0$ and $E_2 = 1$ in the column matrix $\widehat{E}$. Here 2×2 matrix $\widehat{M}(s_2|s_1)$ for passage from s_1 to s_2 is used [4] so that: $\widehat{M}(s) \equiv \widehat{M}(s+C|s)$, $\widehat{M}_K \equiv \widehat{M}(s+C|s_K)$, $\widehat{M}_D \equiv \widehat{M}(s+C|s_D)$.

$\Delta x'[n,s_K]$ is proportional to output voltage V_{out} on the amplifier in a feedback chain and linearly depends on input voltage V_{in}:

$$\Delta x'[n,s_K] = S_K\,V_{out}[n] = S_K\,\mathrm{K}_{out}\,\mathrm{K}_{in}\times \times\sum_{m=0}^{N_F} h[m]\,V_{in}[n-\hat{q}-m]\,u[n-\hat{q}-m]\,, \qquad (2)$$

where S_K is the DK transfer characteristic; K_{in} and K_{out} are voltage gains of the output and input amplifiers (see Fig. 1); $u[n]$ is the Heaviside step function; $h[m]$ are coefficients of a finite impulse response (FIR) digital filter in DSP; N_F is the FIR filter order. The total delay τ_{delay} in the signal processing of the feedback path from BPM to DK adjusts the timing of the signal to match the bunch arrival time. If τ_{PK} is the time of flight of the particle from BPM to DK and $T_{rev} = 1/f_{rev}$ is the particle revolution period, then $\tau_{delay} = \tau_{PK} + \hat{q}\,T_{rev}$.

V_{in} voltage linearly depends on $x[n,s_P]$ displacement:

$$V_{in}[n] = (x[n,s_P] + \delta x_P)\,S_P\,u[n]\,, \qquad (3)$$

where δx_P is a deviation of the BPM electrical centre from the reference orbit, S_P is the BPM transfer characteristic.

Let the driving force be the discrete-time unit impulse with amplitude a_D at the n_D turn so that

$$\Delta x'_D[n]\sqrt{\hat{\beta}_D\hat{\beta}_P} \equiv V_D = a_D\,\delta[n-n_D]\,, \qquad (4)$$

where $\hat{\beta}_i \equiv \hat{\beta}(s_i)$ is the Twiss beta function [4]. In the case of harmonic excitation impulse of $Q_D f_{rev}$ frequency, $N_D T_{rev}$ duration and ϕ_D phase one can write: $V_D = a_D \sin\big(2\pi(n-n_D)Q_D + \phi_D\big)\,\big(u[n-n_D] - u[n-n_D-N_D]\big)$.

The system of linear difference equations (1), (2), (3) and (4) can be solved using unilateral $\mathcal{Z}$–transform [6]:

$$\mathbf{y}(z,s) = \mathcal{Z}\{y[n,s]\} \equiv \sum_{n=0}^{\infty} y[n,s]\,z^{-n}\,, \qquad (5)$$

$$y[n,s] = \mathcal{Z}^{-1}\{\mathbf{y}(z,s)\} = \sum_k \mathrm{Res}\left[\mathbf{y}(z,s)\,z^{n-1};\, z_k\right].$$

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For $\widehat{\mathbf{X}}(z,s)=\mathcal{Z}\{\widehat{X}[n,s]\}$ one can obtain the following:

$$\widehat{\mathbf{X}}(z,s)=\frac{z\widehat{I}-\widehat{\mathbf{M}}^{-1}\det\widehat{\mathbf{M}}}{\det\left(z\widehat{I}-\widehat{\mathbf{M}}\right)}\Bigg(\mathcal{Z}\{\Delta x'_{\text{D}}[n]\}\,\widehat{M}_{\text{D}}\widehat{E}+$$

$$+z\widehat{X}[0,s]+\frac{g\,z^{-\hat{q}}\,\mathbf{K}(z)\,\delta x_{\text{P}}}{(1-z^{-1})(\hat{\beta}_{\text{K}}\hat{\beta}_{\text{P}})^{1/2}\text{K}_0}\,\widehat{M}_{\text{K}}\widehat{E}\Bigg),\quad (6)$$

where $\widehat{I}$ is the identity matrix; $\mathbf{K}(z)=\mathbf{K}_{\text{out}}H(z)\mathbf{K}_{\text{in}}$ is the transfer function of the feedback circuit; $H(z)=\mathcal{Z}\{h[n]\}$; $g=(\hat{\beta}_{\text{K}}\hat{\beta}_{\text{P}})^{1/2}\,\text{K}_0 S_{\text{K}} S_{\text{P}}$ is the feedback gain where K_0 is determined in such a way (see further) that $g>0$ corresponds to damped oscillations. Matrix $\widehat{\mathbf{M}}$ is

$$\widehat{\mathbf{M}}\equiv\widehat{\mathbf{M}}(z,s)=\widehat{M}(s)+\frac{g\,z^{-\hat{q}}\,\mathbf{K}(z)}{(\hat{\beta}_{\text{K}}\hat{\beta}_{\text{P}})^{1/2}\,\text{K}_0}\,\widehat{M}_{\text{K}}\,\widehat{T}\,\widehat{M}(s_{\text{P}}|s),$$

where $\widehat{T}$ is 2×2 matrix in which $T_{21}=1$ and other elements are equal to zero. Eigenvalues z_k at $a_{\text{D}}=0$ are solutions of equation $\det\left(z\widehat{I}-\widehat{\mathbf{M}}(z,s)\right)=0$, that is:

$$z_k^2-\left(2\cos(2\pi Q)+\frac{g\,z_k^{-\hat{q}}\,\mathbf{K}(z_k)}{\text{K}_0}\sin(2\pi Q-\psi_{\text{PK}})\right)z_k$$

$$+1-\frac{g\,z_k^{-\hat{q}}\,\mathbf{K}(z_k)}{\text{K}_0}\sin\psi_{\text{PK}}=0,\quad (7)$$

where Q is the betatron number of oscillations per turn and ψ_{PK} is the betatron phase advance from BPM to DK. If $g=0$ then $z_{1,2}=\exp(\pm\mathrm{j}\,2\pi Q)$, that is a well known result for free betatron oscillations. If $g\neq 0$ then the number $k>2$ of eigenvalues z_k depends on N_{F} and $\hat{q}$. The bunch state $\widehat{X}[n,s]=\widehat{M}(s|s_{\text{K}})\widehat{X}[n,s_{\text{K}}]$ can be calculated from components $\widehat{X}[n,s_{\text{K}}]$ after passing DK in accordance with (5) as the sum of harmonics with $A_k\hat{\beta}^{1/2}$ amplitudes, Φ_k phases, $\alpha_k\equiv-\ln|z_k|$ logarithmic decrements and $2\pi\{Q_k\}\equiv\arg z_k$ phase shifts per turn so that

$$x[n,s_{\text{K}}]=\sum_k A_k\sqrt{\hat{\beta}_{\text{K}}}\,\mathrm{e}^{-n\alpha_k+\mathrm{j}\,(2\pi n\{Q_k\}+\Phi_k)}\,.$$

It can be done if $|z_k|<1$ and

$$\lim_{n\to\infty}\widehat{X}[n,s]=\lim_{z\to 1}(z-1)\,\widehat{\mathbf{X}}(z,s)=0.$$

Consequently $\mathbf{K}(z=1)=0$. Thus, all poles z_k of function $\widehat{\mathbf{X}}(z,s)$ should lie inside the unit circle, and influence of δx_{P} in $\widehat{\mathbf{X}}(z,s)$ function should be excluded. The later condition is reached by using the feedback circuit with the notch filter whose transfer function is $H_{\text{NF}}(z)=(1-z^{-1})$.

If $g\ll 1$ and $\mathbf{K}_{\text{in}}(\omega)\mathbf{K}_{\text{out}}(\omega)$ depends weakly on frequency then from (7) one can obtain the following:

$$\alpha_m=\frac{g\,|\mathbf{K}(\omega_m)|}{2\text{K}_0}\sin\Psi_{\text{PK}}\,,\quad (8)$$

$$\{Q_m\}=\{Q\}-\frac{g\,|\mathbf{K}(\omega_m)|}{4\pi\text{K}_0}\cos\Psi_{\text{PK}}\,,$$

$$\Psi_{\text{PK}}=\psi_{\text{PK}}+2\pi\hat{q}\,Q-\arg\mathbf{K}(\omega_m)\,,$$

$$\mathbf{K}(\omega_m)=\mathbf{K}_{\text{in}}(\omega_m)\,\mathbf{K}_{\text{out}}(\omega_m)\,H\left(z=\mathrm{e}^{\mathrm{j}\,2\pi Q}\right),$$

$$|\text{K}_0|=|\mathbf{K}(\omega_{\min})|,\qquad \text{K}_0\sin\Psi_{\text{PK}}(\omega_{\min})>0,$$

where $\omega_m=2\pi(Q+m)f_{\text{rev}}$ and m is a positive or negative integer. For the fractional tune $\{Q\}$ the following definition is used: $-0.5<\{Q\}\leqslant 0.5$. The $|\text{K}_0|$ value is equal to the gain transfer characteristic of the feedback circuit at the minimal frequency of ω_m. The sign of K_0 is chosen in such a way that $g>0$ corresponds to the damped oscillations.

The optimum damping corresponds to phase balance Ψ_{PK} of $|\sin\Psi_{\text{PK}}|=1$, so that $\{Q_m\}=\{Q\}$.

Hence, it is necessary to tune the TFS in such a way to provide two conditions: the BPM signal should correspond to the decay amplitude mode and zero frequency shift $(\{Q_m\}-\{Q\})f_{\text{rev}}$ should be tuned for optimum damping.

BEAM RESPONSE TO DRIVING FORCE

From Eq. (6) for $\widehat{\mathbf{X}}(z,s)$ it is clear that the bunch state $\widehat{X}[n,s]$ in accordance with Eqs. (5) depends on initial conditions $\widehat{X}[0,s]$, the deviation δx_{P} of the BPM electrical centre from the reference orbit and the driving force. In the case of damped oscillations after some time the bunch state is determined by the driving force only.

If the driving force is the $\delta[n-n_{\text{D}}]$ function then $\widehat{\mathbf{X}}(z,s)$ includes the term $\mathcal{Z}\{\Delta x'_{\text{D}}[n]\}=a_{\text{D}}\,z^{-n_{\text{D}}}/(\hat{\beta}_{\text{P}}\hat{\beta}_{\text{K}})^{1/2}$ that leads to damped oscillations starting at the moment $n_{\text{D}}T_{\text{rev}}$ with decrements and frequencies in accordance with z_k from the characteristic equation (7). Fig. 2 (left) shows an example of the beam response signal at BPM in the case of two δ-kicks at different gains g. Calculations were

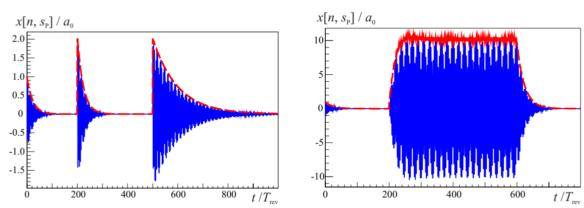

Figure 2: Beam response signal (solid blue lines) and its envelope (dashed red lines) on two δ-kicks (left), harmonic impulse (right) and damped injection oscillations at $t=0$

done according to (1), (2), (3) and (4) at optimum damping, $x[0,s_{\text{P}}]=a_0$, $x'[0,s_{\text{P}}]=0$, $\delta x_{\text{P}}=a_0$, $Q=59.31$, $\psi_{\text{PK}}=2\pi\times 59.25$, $\psi_{\text{PD}}=2\pi\times 58.30$, $a_{\text{D}}=2a_0$, $\hat{q}=0$, $\text{K}_0=1.576$, $H(z)=(1-z^{-1})(1+0.576z^{-1})$; the Twiss functions in $\widehat{M}(s_2|s_1)$ are $\hat{\beta}_{\text{P}}=\hat{\beta}_{\text{K}}=\hat{\beta}_{\text{D}}=71.538$ m, $\hat{\alpha}_{\text{P}}=\hat{\alpha}_{\text{K}}=\hat{\alpha}_{\text{D}}=0$. The corresponding dependences of

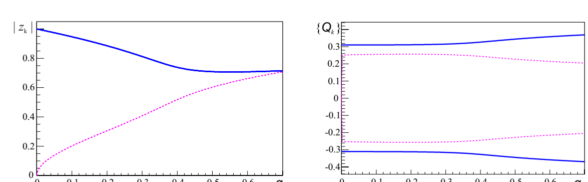

Figure 3: Dependences of $|z_k|$ and $\{Q_k\}$ on gain g

$|z_k|$ and $\{Q_k\}$ on gain g are shown in Fig. 3. There are four

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solutions of the characteristic equation (7) because of $H(z)$ chosen. Two solutions (solid lines) correspond to eigenvalues $z_{1,2}$ with frequencies $|\{Q_{1,2}\}|f_{\rm rev}$ close to $|\{Q\}|f_{\rm rev}$ at $g < 0.3$. Eigenvalues $z_{3,4}$ (dotted lines) correspond to fast damped modes at $g < 0.3$.

The rf-signal at $n_{\rm D} = 200$ in Fig. 2 (left) corresponds to $g = 0.08$. In this case the decrement value from (7) is $\alpha_{\min} = -\ln|z_k^{(\max)}| = T_{\rm rev}/\tau = 0.0429$. Consequently the amplitude decay is faster in comparison with the linear approximation $g/2$ in (8). The rf-signal at $n_{\rm D} = 500$ corresponds to $g = 0.02$. The decrement $\alpha_{\min} = 0.01$ from (7) is in a good agreement with the linear approximation (8).

The BPM signal in the case of the harmonic impulse in TFS with gain $g = 0.08$ is shown in Fig. 2 (right) at $Q_{\rm D} = 0.315$, $a_{\rm D} = a_0$, $n_{\rm D} = 200$ and $N_{\rm D} = 300$. It is visible that in 100 turns after the excitation start the vibration amplitude approximately by 12 times more than $a_{\rm D}$ is achieved. The amplitude of the forced oscillation also increases at $Q_{\rm D} \to \{Q\}$, but it is not converted in infinity as it occurs at a resonance without damping.

Function $a(n, Q_{\rm D})$ of two variables n and $Q_{\rm D}$ can be presented on a plane in the form of a graph of isoamplitudes. Each line is characterized by constant amplitude in any point. For example, for isoamplitudes one can choose:

$$a(n, Q_{\rm D}) = a_i, \qquad -0.01 < \{Q_{\rm D}\} - \{Q\} < 0.01, \quad (9)$$
$$a_i = a_{\max}\sqrt{i/10}, \quad i \in [0, 10),$$

where $a_{\max}(Q_{\rm D}^{(\max)})$ is the peak amplitude of steady state oscillations in the specified frequency range (Fig. 4, left).

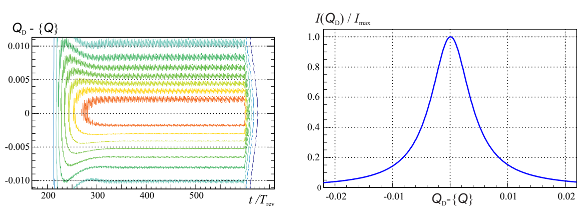

Figure 4: Graph of isoamplitudes (left) and resonance curve (right)

When the bunch makes forced oscillations, its transverse energy $I(Q_{\rm D})$ on the average remains invariable in the field of steady state oscillations: on each turn after the driving kicker the bunch gains an additional transverse impulse which is compensated by the damper kicker. Quantity $I(Q_{\rm D})$ is proportional to a quadrate of the amplitude of steady state oscillations. Consequently for a resonance curve (see Fig. 4, right) one can write in accordance with (6) and (5):

$$\frac{I(Q_{\rm D})}{I_{\max}} = \frac{1}{I_{\max}}\left|\det\left(z_{\rm D}\widehat{I} - \widehat{\mathbf{M}}(z_{\rm D})\right)\right|^{-2}, \qquad (10)$$
$$z_{\rm D} = \exp({\rm j}\,2\pi\{Q_{\rm D}\}), \quad I_{\max} = I\left(Q_{\rm D}^{(\max)}\right).$$

In $g \ll 1$ then $I(Q_{\rm D})/I_{\max}$ in accordance with Eqs. (8) is

$$\frac{I(Q_{\rm D})}{I_{\max}} = \frac{\alpha_m^2}{4\pi^2(\{Q_{\rm D}\} - \{Q_m\})^2 + \alpha_m^2}.$$

This function coincides with a resonance curve which is used for the analysis of forced oscillations in the presence of friction. The function $I(Q_{\rm D})/I_{\max}$ has a maximum at

$$\{Q_{\rm D}^{(\max)}\} = \{Q_m\} = \{Q\} - \frac{g\,|\mathbf{K}(\omega_m)|}{4\pi {\rm K}_0}\cos\Psi_{\rm PK}.$$

Thus, the resonance curve maximum is shifted relatively the betatron frequency under violation of the phase balance condition $|\sin\Psi_{\rm PK}| = 1$.

CONCLUSION

The beam tests based on measurements of a bunch response to the δ-impulse were used at the LHC for tuning the LHC Damper [1] at injection and collision [7]. Good

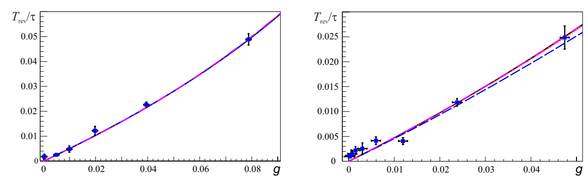

Figure 5: Calculated decrements (curves) and results of measurements for vertical (left) and horizontal (right) feedback chains of the LHC Damper at the energy of 3.5 TeV

agreement on the measured and calculated decrements has been obtained (see, for example, Fig. 5).

Analytical expressions have been obtained for the bunch response to the harmonic excitation impulse taking into account real performances of a TFS. The dependences based on the resonance curve can be used for thin adjustment of the TFS and selective measurements of circulated bunches.

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REFERENCES

[1] V. M. Zhabitsky et al. Beam Tests of the LHC Transverse Feedback System. Proc. of XXII Russian Particle Accelerators Conference RuPAC-2010, 27 September–1 October 2010, Protvino, Russia. IHEP, Protvino, 2010. Pp. 275–279.

[2] V. M. Zhabitsky. The Z-Transfom Application for Solving the Problem of Free Beam Oscillation Damping in the Accelerator. JINR Communications. P9-91-91. Dubna. 1991.

[3] V. M. Zhabitsky. Stability of an Ion Beam in Synchrotrons with Digital Filters in the Feedback Loop of a Transverse Damper. Physics of Particles and Nuclei Letters. 2010. Vol. 7, No.7(163). Pp. 742–747.

[4] E. D. Courant, H. S. Snyder. Theory of the Alternating-Gradient Synchrotron. Annals of Physics. 2000. Vol. 281. P. 360.

[6] W. McC. Siebert. Circuits, Signals, and Systems. The MIT Press, 1986. 651 p.

[7] U. Wienands, W. Höfle, D. Valuch. Gain measurements of the LHC transverse feedback system at 3.5 TeV beam energy. CERN–ATS–Note–2011–131. Geneva. 12 December 2011.

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