AN ANALYTICAL APPROACH TO SOLUTION OF SELF-COORDINATED BEAM DYNAMICS IN DIELECTRIC WAKEFIELD ACCELERATING STRUCTURES*

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Abstract

Self-coordinated transverse dynamics of the high current relativistic electronic bunches used for generation of wake fields in wakefield accelerating structures with dielectric filling is investigated. An analytical approach to solution of self-coordinated beam dynamics is developed.

INTRODUCTION

Wakefield acceleration in a dielectric wakefield waveguide structures is one of the most intensively developed direction among new methods of particle acceleration. Linear accelerators are considered also as sources of sequence of electronic bunches for the free electron laser, which is considered now the major candidate for creation of ultra short impulses (of attosecond range) X-ray radiation. Waveguide structures with dielectric filling excited by a high current electronic bunch were investigated intensively for the last years [1] – [5]. The main purpose is of prospects of their use as high gradient linear accelerators.

One of the main problems in realization of the wakefield method is keeping of an intensive electronic bunch in the channel of a wave guide and prevention of subsidence of particles on its wall. In this regard, a key task in the wakefield method of acceleration is modeling of the self-coordinated movement of a relativistic electronic bunch passing through dielectric structure in fields of Vavilov-Cherenkov created by it.

In recent years in tasks of the analysis of self-coordinated dynamics of relativistic electronic bunches in wakefield accelerating structures methods of direct numerical modeling where developed. These methods are «particle – particle» and «particle – grid». These methods allow on the set parameters of accelerating structure and an initial condition of a bunch to simulate process of its movement. The results of calculations are determination of flight range of the bunch to a contact to them accelerating structure walls, emittance of the bunch, and also transferred or received by bunch energy of particles.

Shortcomings of these methods are considerable duration of calculations for ensuring accuracy of calculations, insistence to volume of random access memory and productivity of computer system. Let us note also that at change of parameters of the bunch and of accelerating structure complete recalculation of a problem of the bunch movement is necessary.

For design of accelerating structures, solutions of problems of optimization in which the structure and bunch parameters maximizing efficiency of accelerating process are determined are necessary. The solution of similar tasks based on direct numerical modeling of dynamics demands repeated carrying out numerical calculations. Creation of the analytical description of self-coordinated dynamics of the bunch allowing direct parametrical research of process in this regard is of interest.

BEAM DYNAMICS EQUATIONS

The description of movement of the electronic bunch was carried out on the basis of the equations of relativistic dynamics [3]:

$$F_r = d\left(m_e V_r \gamma\right)/dt \ ,$$

where

$$F_r = F_{focus} - eq\sum_{n,m}\left[\psi_{F_{r\,n,m}} I_n'\left(k_{r\,n,m} r(\zeta,t)\right)\cdot \cdot\int_0^{\zeta} f(\zeta_0)\sin\left(k_{z\,n,m}\left(\zeta-\zeta_0\right)\right) I_n(k_{r\,n,m} r(\zeta_0,t))d\zeta_0\right],$$

$r(\zeta,t)$ is a bunch deflection from waveguide axes, $\zeta = z - vt$ is a distance behind the bunch, F_f is a focusing force, e and m_e are charge and mass of electron, q and γ are charge and relativistic factor of the bunch, $k_{z\,i,j}$ and $k_{r\,i,j}$ are longitudinal and radial components of wave vector, $\psi_{E_{z\,i,j}}$ and $\psi_{F_{r\,i,j}}$ are coefficients of series, depending of geometry and wave guide filling permittivity, $f(\zeta_0)$ is a function describing longitudinal charge distribution, $I_n(x)$ are modified Bessel function of *n*-th order.

The task of the description of macroparticle movement is self-coordinated: the mutual provision of particles in ensemble influences a field created by particles which, in turn, leads to change of their position. Let's consider an analytical method of the solution of the integro-differential equation of self-coordinated dynamics at the following simplifying assumptions:

1. Let's consider that the charge of the bunch is distributed evenly in the longitudinal direction, thus $f(\zeta_0) = 1/l$, where l is a length of the bunch.

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2. Let's neglect change of size of a relativistic factor over time $\gamma(t)=\gamma_0$.

In considered case $k_{r\,i,j} r(\zeta,t) << 1$, in rejecting field at small deviations of the bunch from the wave guide axis the overwhelming contribution is brought by the 1st azimuthal mode $i=1$. Nonlinear component of a force is negligible. Thus, it is possible to consider that the force operating on charges in the radial direction, depends on r linearly $I_1(kr)\approx kr/2$, $I_1'(kr)\approx 1/2$.

In case the contribution of one of modes is overwhelming, and the others can be neglected, the equation essentially becomes simpler:

$$\frac{\partial^2 r(\zeta,t)}{\partial t^2} - B\int_0^{\zeta} \sin\left(k_z\left(\zeta-\zeta_0\right)\right) r(\zeta_0,t)\,d\zeta_0 = \frac{F_f}{\gamma_0 m_e}.$$

To reduction of the received integro-differential equation to the integrated equation we will apply Laplace's transformation on time. The received integrated equation has the known decision received on the basis of transformation of Laplace on longitudinal coordinate.

Expressing the image of required function, and finding the original Laplace's return transformation by image decomposition in a Laurent series, we receive for lack of focusing force $F_f=0$:

$$r(\zeta,t) = r_0 + v_{r0}t + \sum_{n=0}^{\infty}\left[\frac{(k_z\zeta)^{2n+2}}{(2n+2)!}\sum_{m=0}^{n}\left[\binom{n}{m}\frac{(-1)^m\left(t\sqrt{B/k_z}\right)^{2(n-m+1)}}{(2n-2m+2)!}\cdot\left(r_0+\frac{v_{r0}t}{2n-2m+3}\right)\right]\right],$$

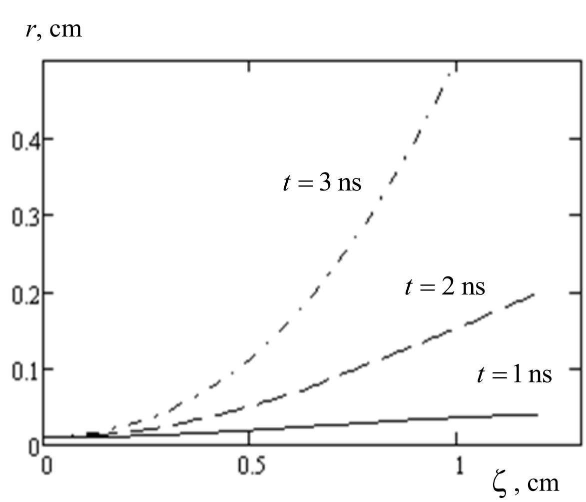

Figure 1: Characteristic dependences $r(\zeta)$ for the different moments of time.

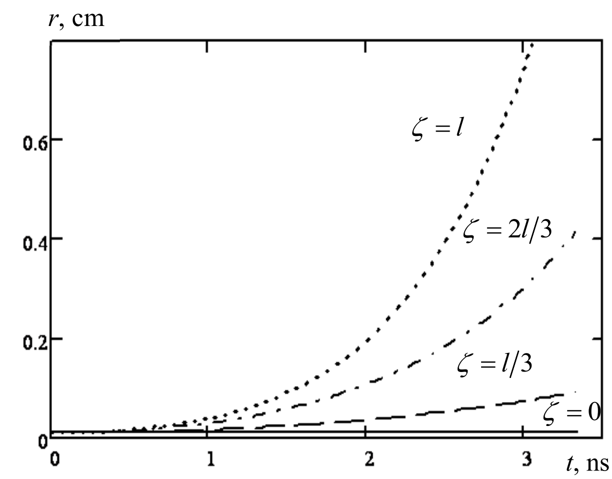

Figure 2: Radial bunch charge distribution for constant initial deflection of the bunch.

In Fig. 1, 2 for the purpose of an illustration of the received decision if $B=6.365\cdot10^{20}$ м$^{-1}\cdot$с$^{-2}$, $k_z=260$ м$^{-1}$, $r_0=0.01$ см, dependences $r(\zeta)$ for the different moments of time and $r(t)$ for the various ζ are given. A head – tail instability leads to growing deflection of the backward part of the bunch.

Comparison of the received analytical expression was carried out with numerical modeling of bunch dynamics by a method of macroparticles based on the BeamDynamics program. Comparative calculations were made at the following parameters of a waveguide and a bunch: $R_c=0.5$ cm, $R_w=0.634$ cm, $\varepsilon_1=16$, $W=16$ MeV, $Q=100$ nC, $l=6\sigma=1.2$ cm, $r_0=0.01$ cm, $v_0=0$.

The BeamDynamics program realizes modeling of Gaussian distribution of the bunch charge [3]. The bunch with the Gaussian profile of charge distribution exponentially suppresses excitation of high modes of the waveguide that allows its comparison to analytical calculation of dynamics of a homogeneous bunch taking into account only one main mode. Program finishing was made after a contact by the bunch of a wall of the vacuum channel. Results of comparison are presented on Fig 3. The dotted line shows radius of the channel of accelerating structure. Thus range of flight of the bunch to a contact of the waveguide wall in analytical and numerical calculations practically coincided and made $L=86$ cm.

As the charge of a Gaussian bunch is concentrated near its center, influence of radial forces created by it near the head of the bunch is expressed weaker, than in case of the homogeneous bunch. At the same time on a tail of the Gaussian bunch influence of radial forces is expressed stronger.

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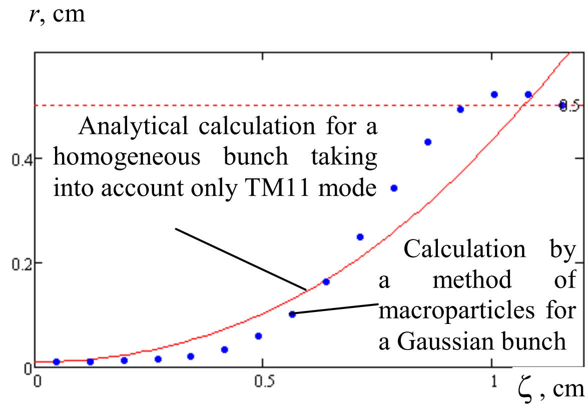

Figure 3: Comparison of analytical dependence to numerical calculation by a method of macroparticles.

BEAM DYNAMICS WITH FOCUSING

Significant amplitude of own rejecting fields generated by high current bunch affecting his tail, emphasizes focusing system necessity. To keep the high current beam is appropriate to use a rigid focusing system based on FODO focusing [4, 5].

The period of the radial force of the focusing system can be approximated by the harmonic dependence for taking part a potential "sagging" between quadruple lenses:

$$F_f = -k(z)r = -ec\frac{\partial B(z)}{\partial r}r = -ecr\frac{B_0}{2R_w}\cos\frac{2\pi z}{L_s}.$$

Such dependence may be written as

$$F_f = -g_1 r(\zeta,t) m_e \gamma_0 \cos\kappa(\zeta + vt),$$

To simplify the beam dynamics equation let us consider that restoring force is linear increasing with deflection from waveguide axis without alternating sign component.

$$F_f = -g_0 r(\zeta,t) m_e \gamma_0,$$

where $g_0 = g_1/4$.

Solving such integro-differential equation by Laplace transform method, we receive:

$$r(\zeta,t) = r_0\cos\left(\sqrt{g_0}t\right) +$$
$$\frac{r_0}{a^2}\sum_{n=1}^{\infty}\left\{\frac{(k_z\zeta)^{2n+2}}{(2n+2)!}\sum_{j=0}^{n-1}\left[\frac{j+1}{a^j}\sum_{m=0}^{(n-j-1)}\binom{n}{m}\frac{(-1)^{m+j}\tau^{2(n-m-j-1)}}{(2(n-m-j-1))!}\right]\right\} +$$
$$+r_0\cos\left(t\sqrt{g_0}\right)\left(\frac{\cos(\lambda\zeta)-1}{(a+1)^2} + \frac{k_z\zeta\sin(\lambda\zeta)}{2\sqrt{a}(a+1)^{3/2}}\right) -$$
$$-\frac{r_0 t\sqrt{g_0}}{2(a+1)}\sin\left(t\sqrt{g_0}\right)\left(\cos(\lambda\zeta)-1\right),$$

where $\tau = t\sqrt{B/k_z}$, $a = g_0 k_z/B$, $\lambda = k_z\sqrt{\frac{a+1}{a}}$.

Characteristic dependences $r(\zeta)$ for the different distances of bunch flight with focuser were presented on Fig. 4.

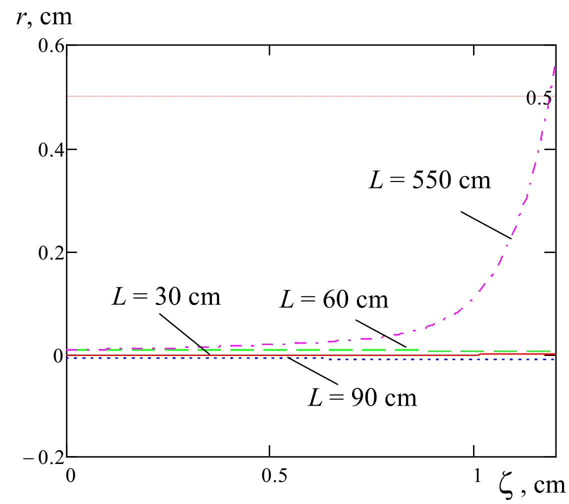

Figure 4: Characteristic dependences $r(\zeta)$ for the different distances of bunch flight with focuser.

It is visible that the bunch flight range significantly increases under the influence of the focuser.

CONCLUSION

Offered methods of analytical calculation of self-coordinated dynamics of bunches in wakefield accelerating structures open prospects for development of solving new optimization tasks of accelerators for physics of high energy and perspective sources of radiation in THz range of frequencies.

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