Stimulated radiation cooling of ion beams

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In this paper the theory of the Stimulated radiation cooling (SRC) is developed. A method of formation of the broadband laser beam with given spectral distribution is discussed.

Introduction

Fast cooling of ion beams.

1. Linear version of the stimulated ion cooling.

2. Nonlinear version of the stimulated ion cooling.

Conclusion

References
Introduction

Beam cooling is the increase in six dimensional (6D) phase space density of the beam. The density is inversely proportional to the 6D beam emittance. That is why beam cooling is the reduction of its emittance as well. The brightness $B$ of the particle beam (the current density per unit solid angle and per unit energy spread) is proportional to the 6D phase space density of the beam. This is a very important parameter of the beam quality. It determines the luminosity of the colliding beams and brilliance of the light sources based on particle beams. It follows the importance of the application of the cooling process for the particle beam.

The rate of the beam density change in circular machines is determined by the 6D damping increment

$$\alpha_{6D} = \frac{2}{\beta^2} \frac{\bar{P}_{Fr}(\bar{p})}{\varepsilon} + \frac{\partial \bar{P}_{Fr}(\bar{p})}{\partial \varepsilon},$$

Where $\bar{F}_{Fr} = -\alpha(r, p, t) \cdot \bar{n}_p$ is the friction force, $\alpha(r, p, t) = \bar{P}_{Fr}(p, t)/c\beta$ is the frictional coefficient, $\bar{n}_p = \bar{p}/|\bar{p}|$ is the unite vector directed along the particle velocity, $\bar{p}$ is the particle momentum, $p = |\bar{p}|$, $\bar{P}_{Fr}(p)$ is the averaged rate of the particle energy loss owing to friction, $\varepsilon$ is the synchronous energy of the particle. It follows from the Robinson damping criterion [1], [2].
Cooling based on friction:

1) Radiation cooling,
2) Ionization cooling by energy losses in media (bremsstrahlung, ionization, excitation),
3) Electron cooling,
4) Cooling of relativistic ion beam by counter propagated monochromatic laser beam,
5) Radiative (Stimulated radiation) ion cooling by broadband laser beam.

Cooling based on interparticle spacing:

6) Stochastic cooling,
7) Optical stochastic cooling (OSC),
8) Transit-time OSC,
9) Enhanced optical cooling (EOC).
Robinson’s damping criterion follow from the continuity equation in the 6D phase space:

\[
\frac{d\rho}{dt} + \rho \, \text{div} \, \vec{v} = 0
\] (1)

Here components of the 6D velocity \( \vec{v} = (v_r, v_p) \) are \( x, y, z, p_x, p_y, p_z \), where \( \dot{r}_i = dr_i / dt, \dot{p}_i = dp_i / dt \). The divergence of the 6D-velocity in (1) is \( \text{div} \, \vec{v} = \text{div}_r \vec{v}_r + \text{div}_p \vec{v}_p = \text{div}_p \vec{v}_p \). The values \( \dot{p}_i \) are the components of the force \( \vec{F} \) acting upon the particle \( F = F_H + F_{Fr} = p = p_H + p_{Fr} \). The friction power is \( P_{Fr} = F_{Fr} \cdot \dot{v} = \alpha(\vec{r}, p, t) \cdot \vec{n}_p \cdot \vec{v} = \alpha(\vec{r}, p, t) \cdot c \beta \), where \( \beta = v / c \). It follows that \( \alpha(\vec{r}, p, t) = P_{Fr}(\vec{r}, p, t) / c \beta \), \( -\text{div} \, F_{Fr} = 2P_{Fr}(p) / \beta^2 \varepsilon + \partial P_{Fr}(p) / \partial \varepsilon \). The value \( \text{div}_p F_H = 0 \).

Finally the above equation can be written in the form \( d\ln \rho / dt + \text{div}_p \vec{F}_{Fr} = 0 \). The solution of this equation is

\[
\rho = \rho_0 e^{-\int \text{div}_p \vec{F}_{Fr} \, dt} \Rightarrow \rho_0 e^{-\alpha_{6D} t}.
\]
\[ \alpha_{6D} = \frac{2}{\beta^2} \left( \frac{P_{Fr}(p)}{\varepsilon} \right)_{s} + \frac{\partial P_{Fr}(p)}{\partial \varepsilon} \bigg|_{s}, \quad \tau = \frac{1}{\alpha_{6D}}, \quad \tau^{\text{ord}} \approx \frac{\varepsilon}{P_{Fr}(p)} \bigg|_{s}, \quad \tau^{\text{stim}} \approx \frac{2\sigma_{\varepsilon}}{P_{Fr}(p)} \bigg|_{s}. \]
Linear version of the enhanced ion cooling

\[ \bar{P} = \bar{P}_m \frac{\varepsilon - \varepsilon_c}{\varepsilon_c - \varepsilon_c + \sigma_c} \quad \text{at} \quad \varepsilon > \varepsilon_c, \quad \varepsilon_c \leq \varepsilon_s - \sigma_c; \]

\[ \bar{P} = 0 \quad \text{at} \quad \varepsilon < \varepsilon_c, \quad \varepsilon > \varepsilon_s + \sigma_c. \]
Linear version of the enhanced ion cooling

$\epsilon - \epsilon_s$

$(\phi_s \neq \pi/2)$

$\phi - \phi_s$
Nonlinear version of the enhanced ion cooling

\[
\overline{P} = \overline{P}_m \frac{\varepsilon - \varepsilon_c}{\varepsilon_s - \varepsilon_c + \sigma_{\varepsilon}} \quad \text{at} \quad \varepsilon_c = \varepsilon_s, \quad \varepsilon > \varepsilon_c,
\]

\[
\overline{P} = 0 \quad \text{at} \quad \varepsilon < \varepsilon_c, \quad \varepsilon > \varepsilon_s + \sigma_{\varepsilon}.
\]
Nonlinear version of the enhanced ion cooling

\[ \varepsilon - \varepsilon_s \]

\[ \varphi_s = \pi/2 \]

\[ \varphi - \varphi_s \]
Consequences from the quantum nature of the light scattering

The equilibrium energy spread of the ion beam in the linear case is

$$\sigma_{\varepsilon, eq} = \hbar \omega \sqrt{2 \sigma_\varepsilon / \hbar \omega} >> \hbar \omega, $$

where $\hbar \omega = \hbar \omega_l \gamma^2$ is the average energy of scattered photons, $\hbar \omega_l$ is the energy of laser photons, $\gamma = \varepsilon / m_i c^2$ and $m_i c^2$ are the relative energy and the rest energy of the ion.

The equilibrium energy spread of the ion beam in the linear case is

$$\sigma_{\varepsilon, eq} = \hbar \omega,$$
The nonlinear version of the laser cooling considered above is not optimal one. Faster increase of the power losses like

$$\overline{P} = \overline{P}_m \frac{\varepsilon - \varepsilon_s}{\varepsilon - \varepsilon_s + \sigma_c}$$

at

$$\varepsilon_s < \varepsilon < \varepsilon_s + \sigma_\varepsilon,$$

$$\overline{P} = 0$$

at

$$\varepsilon < \varepsilon_s,$$

$$\varepsilon > \varepsilon_s + \sigma_\varepsilon,$$

and at $$\sigma_c << \sigma_\varepsilon$$ is preferable. Optimization of the nonlinear version is the topic for the future search.
METHODS OF FORMATION OF THE BROADBAND LASER BEAMS WITH GIVEN SPECTRAL DISTRIBUTION

1) The necessary power can be generated by broadband lasers filtered and then amplified by optical parametric amplifiers.
2) An undulator with a deflecting parameter \( K \sim 1 \) together with narrowband laser light can be used for the ion excitation in the interaction region (equivalent to the broadband laser light). Tapered undulator with the magnetic field varying by definite law and monochromatic laser beam will be equivalent to laser with a broad band and a very sharp frequency cutoff.
3) Successive frequency shift of a single mode laser by an acusto-optic modulator coupled to a passive ring cavity and other methods can also be used [4].
Conclusion

Fast (enhanced) 6D ion cooling occur in the case of stimulated radiation cooling by broadband laser beams. In this case the equilibrium energy spread and 6D emittance are extremely small. New generations of light sources can be based on ion storage rings if stimulated radiation cooling is used [7], [8].
References

