PARTIALLY COHERENT EM RADIATION OF AN ELECTRON BUNCH

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Abstract

Peculiarities of electromagnetic radiation of a spatially non-uniform bunch of electron radiators are considered starting with a general discussion of their temporal and spatial coherence. The obtained spectral-angular characteristics permit to define a coherent part of the total radiation losses of the bunch. Their dependencies on the radiators density within the bunch, including a case of low particles number per one wavelength, permit to estimate the ultimate limit of possible coherent effects in rarified beams. The presented 3D visualization reveals radiators interference and permits to exclude a "useful" part of coherent radiation within a narrow angle along the particles relativistic velocity

INTRODUCTION

The physics of interference of two (sometimes of many) electromagnetic waves is well known and usually is discussed as appearance of interference lines with increased (or decreased) intensity. The same picture can be used for radiation of several sources separated by fixed distances of order of the wavelength. As a rule two limiting cases are considered. The first is an absolute regularity in positions and phases of N sources when the intensity in the maximum of an interference line is proportional to N^2. The second case is that of an absolute disorder in sources positions and/or phases when the radiation power is proportional to N and coincides in all spectral-angular characteristics with the radiation pattern of a single source (spontaneous radiation). However, the recent progress of new radiation sources of FELs type force to pay attention to intermediate cases of a system of artificially phased sources which positions are partially correlated (including the total disorder). The corresponding radiation keeping the features of coherent and spontaneous radiation will be called below as partially coherent one.

A successive theory of radiation of such electron bunch should answer the following non-clear (at least, for the authors) questions:

1. Spontaneous radiation is known to be caused by current fluctuations. So how and when does it disappear in the limiting case of an uniform steady state current which can not radiate at all?
2. To what extent the coherence of radiation is kept for very remote particles?
3. To what extent the N^2 law is valid for a bunch which size is much smaller than the wavelength and, hence, which "classical radius" appears N times larger than that of a single electron?
4. How is the response momentum distributed between two (or more) electrons radiating coherently a single photon?
5. In application to short-wave FELs: is there a lower limit for the radiation wavelength (not related to technical or financial reasons)? In other words, is there a limit of the classical induced radiation concept?
6. And more practical: what is a spectral-angular distribution of coherent radiation of a bunched beam in an undulator serving, for example as the second stage of a high power FEL?

Certain items of the list are discussed below.

DESCRIPTION OF THE MODEL

We choose as a model an infinitely thin train of N electrons moving along a plane undulator (x-direction) approximately equidistantly with some uncertainty of positions and with the same velocity βc. Such model permits to analyze a spectral-angular pattern of radiation for various rarefactions of radiators (including the physically important case of one particle per a wavelength). Besides, a freedom in particles positioning gives a possibility to trace an evolution of spectral-angular characteristics starting from a total disorder and finishing with an absolute regularity in a particle distribution.

Strictly speaking, any fixed realization of the point-like radiators distribution provides a certain degree of coherence. However, a smearing of interference lines and their relative intensity *in average* depend on the uncertainty in their positions. The ideal maximums $\sim N^2$ can be realized in the case of ideal phasing and regularity only. Any deviation from these conditions decreases the dependence.

For simplicity suppose that the particles are distributed independently around their ideal positions according the normal low with the dispersion σ. To find the coherence factor one has to calculate the sum [1]:

$$C = \frac{1}{N}\left\langle \left| \sum_{k=1}^{N} \exp(i\psi_k) \right|^2 \right\rangle \qquad (1)$$

where Ψ_k is a phase and the angular brackets denote averaging over probability. Actually, this summation takes into account mutual phasing of particles transverse (z-direction) oscillations in the undulator field. Note that

that every particle is “coherent” with itself by definition.

If the phase difference between adjacent oscillators *in average* is α/N the sum (1) can be easily calculated:

$$C(\alpha, A, N) = 1 + A\left(\frac{\sin^2 \alpha/2}{N \sin^2 \alpha/2N} - 1\right) \quad (2)$$

where $A = \exp\left(-\sigma^2/2\Delta^2\right)$ and Δ is the average distance between two neighbouring particles. The value of A changes from 0 for non-correlated particles positions up to 1 when the distribution is strictly regular. Eq. (2) is valid if the frequencies of all oscillators are the same. This is satisfied in the coordinate frame moving with electrons. Radiation is monochromatic in this frame and the wavelength is equal to L/γ where L is the undulator period in the lab frame and γ is Lorentz factor. As far as angular characteristics are concerned the radiation field looks like an undulator wave of phase velocity $-\beta c$ scattered at angles θ, φ in a spherical coordinate system with z-axis. It is easy to show that $\alpha=2\pi\Delta/\lambda$ where

$$\lambda = \beta L/\gamma^2 \left(1 + \beta \sin\theta \cos\varphi\right) \quad (3)$$

coincides with the wavelength of radiation at angles θ, φ in the lab frame.

CRITERIA OF COHERENCE

Several important conclusions can be made directly from (2). First of all the expression in the brackets is a coherent addition to unity, i.e. to spontaneous radiation. It is to be multiplied by the “smearing” factor A always decreasing coherency but killing it only in the limiting case of $A=0$.

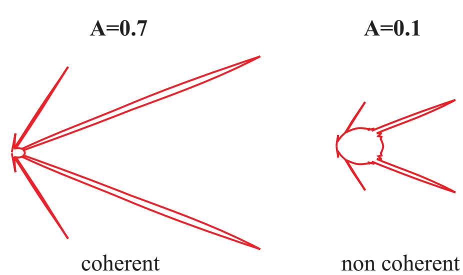

Fig. 1 Influence of “smearing” factor on coherence in the $\theta=\pi/2$ plane

If the distance between particles is much smaller than the wave length which corresponds to dense beams coherence is maximal and

$$C = 1 + (N-1)\exp(-\sigma^2 N^2 / 2l^2) \quad (4)$$

Note that this expression is independent of angles, so the system radiates as a single particle of an increased charge. However, this increase is limited by smearing and in the most interesting case of $\sigma/l \ll 1$ reaches its maximal value of $l/\sigma e^{1/2}$ at $N \approx l/\sigma$.

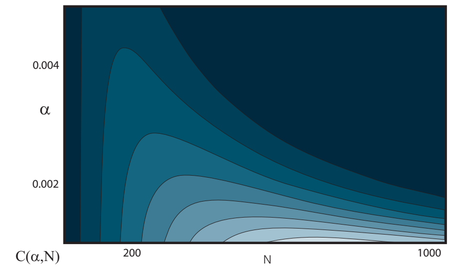

Fig.2 Domains of partial coherence for a dense bunch of smeared particles

The picture shows how crucial is the influence of random displacements of the particles even in the case when their number per a wave length is large. Moreover, an increase of particles within a bunch can even destroy coherence if the average distance between them becomes smaller than the position uncertainty.

For larger α radiation has coherent minimums and maximums $\sim N$ of the width of N^{-1} appearing firstly in forward direction at angles with $[\alpha/2\pi N]\approx 0$ ([] denotes a fractional part). Physically this corresponds to the case when a couple of interference angular maximums merge along the propagation direction (or to even number of wavelengths along the bunch). These maximums are again limited by the same smearing. This is illustrated in Figs (3) and (4).

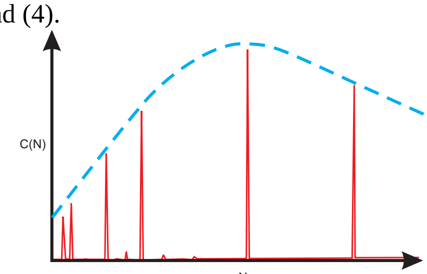

Fig. 3 Coherent maximums for a moderately dense bunch

As can be seen from (3), for a relativistic beam even small variation in angles around $\varphi=0$ and $\theta=\pi/2$ result in a sharp increase in α. In this context one should speak about *observable* coherence determined by the available angular (and spectral in the lab frame) experimental resolution. To illustrate that we show a result of averaging of the angular dependence over a period of oscillations. One can see that that curve is far below the N^2 low.

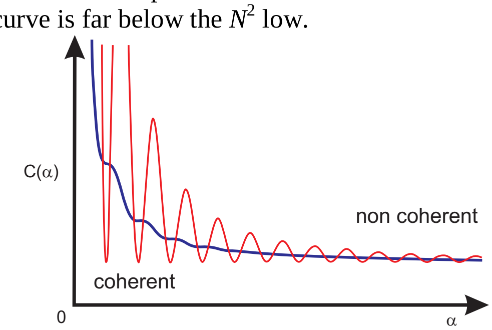

Fig.4 Angular resolution of the first interference maximum

POYNTING VECTOR AND DIRECTIONALITY OF RADIATION IN THE LAB FRAME

To get the angular distribution of the radiated power inn the lab frame one should take into account that Poynting vector is proportional to the 4th column of Maxwellian tensor. For radiation field of a harmonic oscillator in the proper frame according usual rules of Lorentz transformation [2] one gets the coefficients to multiply the coherence factor *C*. In Cartesian coordinates the power flux density components in the lab frame are:

$$W_x = \gamma^2 \sin^2\theta\left[\left(1+\beta^2\right)\sin\theta\cos\varphi + \beta\left(1+\sin\theta^2\cos^2\varphi\right)\right]$$

$$W_y = \gamma\sin^3\theta\sin\varphi\left(1+\beta\sin\theta\cos\varphi\right)$$

$$W_z = \gamma\sin^2\theta\cos\theta\left(1+\beta\sin\theta\cos\varphi\right)$$

$$W_r = W_x\sin\theta\cos\varphi + W_y\sin\theta\sin\varphi + W_z\cos\theta$$

Taking this into account one can write down the expression for the radial component of the radiation flux density in the lab frame normalized per one particle:

$$P_r \cong C\left(\alpha, A, N\right)W_r \qquad (5)$$

By the way, using Cartesian presentation and the equation dx/W_x=dy/W_y one can get the flux lines on the plane θ=π/2 showing the relativistic directness of undulator radiation:

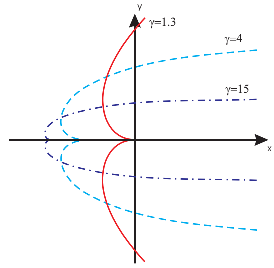

Fig. 5 Poynting vector current lines

A typical angular 3D distribution of the radiated power is shown in Fig 6.

These dependencies show that radiation propagates mainly within a narrow cone containing almost all interference maximums:

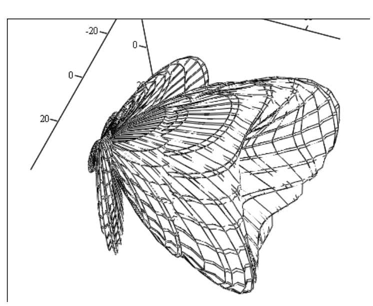

Fig.6 Angular distribution of power

The set of parameters of Fig. 6 has been chosen deliberately outside of the interference maximum. The power density along the *x* axis is here far below the N^2 dependence but the additional petals show that even this case keeps a certain degree of coherence.

CERTAIN CONSEQUENCES

Putting aside the academic theory, at least two important features should be mentioned here. For practical applications not an interference pattern is of importance but rather a maximal intensity along the particles flow. In spite of radiation is concentrated within a narrow cone $\approx\gamma^{-2}$ a distant receiver can intercept only small part of it (in the lab system the same is valid for a spectral sensitivity of the receiver). This is this part, for example, which is responsible for induced effects in FELs where one can take as a receiver acceptance the ratio of transverse beam size to the radiation length. To illustrate that one can calculate the power flux within a narrow solid angle $\Omega<<\gamma^{-2}$ as shown in Fig 7. The "knees" of the curve correspond to additional interference maximums merging close to *x*-direction.

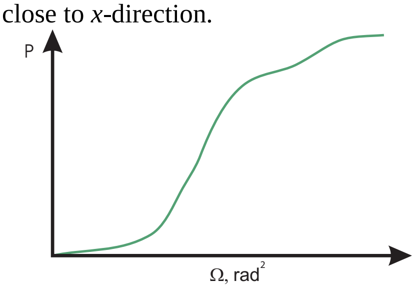

Fig. 7 Total intensity within a narrow cone of observation

The first "knee" can be treated then as a real threshold of coherence while all remaining part is to be considered as useless and even damaging spontaneous radiation (see [3]). A sharp dependence of the thresholds upon the number of particles could explain the random, noisy structure of shortwave FELs output.

The second comment is to be made on the sudden lost of the coherence pattern for a point- like bunch of *N* particles still radiating N^2 times larger than a single particle. Note that all arguments above are valid for particles independent distribution only. It is not the case of an infinitely dense bunch where internal degrees of freedom appear with proper frequencies comparable with those of radiation. We will not discuss here the problem which is relevant for coherent acceleration methods rather for FELs. To make the long story short: a point–like bunch never behaves as an elementary particle of an increased charge.

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