INFLUENCE OF THE VERTICAL CLOSED ORBIT DISTORTIONS ON ACCURACY OF THE ENERGY CALIBRATION DONE BY RESONANT DEPOLARIZATION TECHNIQUE

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Abstract

The series of the experiments on precise mass measurement of J/Ψ-, Ψ' mesons have been performed in 2002-2004. Energy calibration has been done with the help of the resonant depolarization technique. The present paper discusses the influence of the vertical orbit distortions on the accuracy of the energy calibration. The sources of the orbit distortions are misalignments of the quadrupoles and sextupoles in vertical plane and kicks of the vertical correctors.

INTRODUCTION

The series of the experiments on precise mass measurement of J/Ψ-, Ψ' mesons have been performed in 2002-2004. The following mass values have been obtained [1]:

$$M_{J/\Psi} = 3096.917 \pm 0.010 \pm 0.007 \; MeV \,,$$

$$M_{\Psi'} = 3686.111 \pm 0.025 \pm 0.009 \; MeV \,.$$

Energy calibration of the colliding beams has been performed by resonant depolarization technique. To achieve high accuracy of the mass measurements an analysis of possible errors have been performed [2, 3]. In particular, the effect of the vertical closed orbit distortions influence on accuracy of the energy calibration was preliminary estimated with the result of 14 keV correction for the mass of Ψ'-meson. Comparatively large value of possible energy bias stimulated the further analysis which is presented below.

The problem of the spin tune computation in presence of vertical orbit distortions has been addressed by other authors [4], which obtained different results from present work. Therefore the investigation of both approaches has been done.

The spin precession frequency Ω_S of the particle moving in vertical guiding field is described by

$$\Omega_S = \Omega_0 (1 + \gamma \frac{q'}{q_0}) \,, \tag{1}$$

where $\Omega_0 = q_0 B/\gamma$ is revolution frequency, B is average guiding field, γ is Lorentz factor, q', q_0 are anomalous and normal parts of gyro-magnetic ratio. Introducing the spin tune $\nu_0 = (\Omega_S - \Omega_0)/\Omega_0 = \gamma q'/q_0$ one will have a known relation between energy E and spin tune

$$E[MeV] = \nu_0 \times 440.64843(3) \,. \tag{2}$$

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The closed orbit in this case is assumed to be flat. In general, the radial and the longitudinal magnetic as well as vertical electric fields may exist in real accelerator that makes the given equation inadequate. In the first order of perturbation theory the modified relation between spin frequency and beam energy can be expressed in the form

$$\nu' = \gamma \frac{q'}{q_0} + \Delta\nu(\gamma, perturbations) \,. \tag{3}$$

The goal is to estimate spin tune shift $\Delta\nu$ by the given perturbations. This gives a possibility to find the correct energy value by the quantity $\nu' - \Delta\nu$, where ν' is a spin tune measured by the resonant depolarization technique and the following relation has to be used:

$$\gamma = \frac{q_0}{q'}(\nu' - \Delta\nu) \,.$$

The longitudinal fields arise from the errors of compensation of the detector's field. Consideration of these perturbations is most simple and presented in [2].We consider the influence of the radial fields which arise primary due to misalignment of quadrupoles in vertical plane.

SPIN FREQUENCY SHIFT DUE TO VERTICAL CLOSED ORBIT DISTORTIONS

To calculate the shift of the spin tune in the presence of the radial field we will assume sources of perturbations (including vertical correctors, quadrupole lenses and other sources of radial fields) to be point-like and rather weak. The calculations will be done in the second order of perturbation theory with the help of spinor matrices technique [5] using Pauli matrices (σ_x, σ_y, σ_z) and the unit 2×2 matrix I. Also, the coordinate system is related to the velocity vector of the equilibrium particle. Thus, rotation angle of the spin vector $2\chi = \nu\alpha$ is proportional to the rotation angle of the velocity vector α. The spinor matrix for the rotation around radial (x) basis vector on the angle $2\chi_i = \nu\alpha_i$ for the perturbation at azimuth θ_i is

$$T_i = I\cos(\chi_i) - i\sigma_x \sin(\chi_i) \,.$$

Spin evolution in vertical field is described by

$$M_i = I\cos(\Phi_{i+1,i}/2) - i\sigma_z \sin(\Phi_{i+1,i}/2) \,,$$

where $\Phi_{i+1,i} = \Phi(\theta_{i+1}) - \Phi(\theta_i)$ and $\Phi(\theta_i) = \int_0^{\theta_i} \nu_0 K d\theta$ is a rotation angle of the spin vector in the guiding field from the origin azimuth to given perturbation location; K is the orbit curvature in units of the inverse mean machine radius $1/R$. The total one-turn matrix of the spin evolution is obtained by multiplication of the subsequent spinor matrices

$$M = \prod_i T_i M_i \,.$$

The new spin tune ν' is obtained from the following formula $\cos(\pi\nu') = 1/2\,\mathrm{Sp}(M)$, while ν_0 denotes spin tune without radial fields. It is simple to calculate spin tune in the case of one perturbation (neglecting higher than second order terms)

$$\cos(\pi\nu_0) - \cos(\pi\nu') = \frac{\chi_1^2}{2}\cos(\pi\nu_0)\,,$$

of two perturbations

$$\cos(\pi\nu_0) - \cos(\pi\nu') = \\ = \frac{\chi_1^2+\chi_2^2}{2}\cos(\pi\nu_0) + \chi_1\chi_2\cos(\pi\nu_0 - \Phi_{2,1})\,,$$

of N perturbations

$$\cos(\pi\nu_0) - \cos(\pi\nu') = \\ = \cos(\pi\nu_0)\sum_{i=1}^{N}\frac{\chi_i^2}{2} + \sum_{j>i,i=1}^{N}\chi_i\chi_j\cos(\pi\nu_0 - \Phi_{j,i})\,.$$

This gives the spin tune shift

$$\Delta\nu = \nu' - \nu_0 = \frac{1}{2\pi\sin\pi\nu_0}\times \\ \times\left[\cos\pi\nu_0\sum\chi_i^2 + 2\sum\nolimits_{j>i}\chi_i\chi_j\cos(\pi\nu_0 - \Phi_{j,i})\right]. \quad (4)$$

The first term in the right part of the equation describes non-correlated part of the orbital distortions influence, the second one corresponds to their correlations.

Given z is the vertical closed orbit deviations in units of R; $z'' = d^2z/d\theta^2$. Obviously, $2\chi_i = \nu_0 z''\Delta\theta_i$, where $\Delta\theta_i$ is an interval of the i-th perturbation. Taking into account the last definition the equation (4) could be written in the integral form:

$$\Delta\nu = \frac{1}{16\pi\sin\pi\nu}\times \\ \times\int_0^{2\pi}\nu^{z''}d\theta\int_0^{2\pi}\nu^{z''}\left[e^{i(\pi\nu-|\Phi-\Phi'|)} + c.c.\right]d\theta'. \quad (5)$$

Introducing definition of the spin harmonic amplitude

$$\omega_k = \frac{1}{2\pi}\int_0^{2\pi}\nu z''\exp\left[-i(\Phi-\nu\theta) - ik\theta\right]d\theta\,, \quad (6)$$

it is possible to transform (5) to the form obtained by A.M. Kondratenko [6] (who derived this equation in different way)

$$\Delta\nu = \frac{1}{2}\sum_{k=-\infty}^{\infty}\frac{|\omega_k|^2}{\nu - k}\,. \quad (7)$$

Note, for correct application of this formula it is not enough to consider only contribution of the resonant harmonic with $|k-\nu_0| = |\varepsilon| \ll 1$. It is necessary to summarize over all essential harmonics.

ESTIMATE OF THE SPIN TUNE SHIFT BY RMS ORBIT DISTORTIONS

Assuming accelerator without straight sections i.e. $\Phi = \nu\theta$ and given Fourier expansion of $z = \sum z_n e^{in\theta}$ and $z'' = -\sum z_n n^2 e^{in\theta}$ one can obtain that $\omega_k = -\nu k^2 z_k$ and corresponding spin tune shift is

$$\Delta\nu = \frac{1}{2}\sum_{k=-\infty}^{\infty}\frac{|\omega_k|^2}{\nu-k} = \frac{\nu^2}{2}\sum_{k=-\infty}^{\infty}\frac{|z_k|^2k^4}{\nu-k}\,. \quad (8)$$

To evaluate orbit harmonics z_n it is convenient to use the known variables $u = z/\sqrt{\beta_z}$ and $\phi = \int_0^\theta d\theta/(\nu_z\beta_z)$, where β_z is vertical beta function in units of R, ν_z is vertical betatron tune. Then the closed orbit equation is written as:

$$\frac{d^2u}{d\phi^2} + \nu_z^2 u = \nu_z^2\beta_z^{3/2}h(\phi) = F(\phi)\,, \quad (9)$$

where $h(\phi) = \Delta H_x/\bar{H}_x$. Performing Fourier decomposition on both parts of equation (9) one obtains

$$u_n = \frac{F_n}{\nu_Z^2 - n^2}\,,$$

where $u = \sum_{n=-\infty}^{\infty}u_n e^{in\phi}$, $F = \sum_{n=-\infty}^{\infty}F_n e^{in\phi}$. Assuming random and uniform kicks $F(\phi)$ i.e. $\langle F_iF_j^*\rangle = f^2\delta_{ij}$ we calculate the RMS orbit distortion

$$\langle u^2\rangle = \sum_{n=-\infty}^{\infty}|u_n|^2 = f^2\sum_{n=-\infty}^{\infty}\frac{1}{(\nu_z^2-n^2)^2} = f^2Q\,, \quad (10)$$

where

$$Q = \frac{\pi}{2\nu_z^3}\cot\pi\nu_z + \frac{\pi^2}{2\nu_Z^2}\csc^2\pi\nu_z\,. \quad (11)$$

In homogeneous approximation $\beta_z = const = \bar{\beta}_z$, $\phi(\theta) = \theta$, $\langle z^2\rangle = \bar{\beta}_z\langle u^2\rangle$ and $z_n = u_n\sqrt{\bar{\beta}_z}$ one can obtain a relation to estimate average spin tune shift by observed orbit RMS

$$\langle\Delta\nu\rangle = \frac{\nu^2}{2}\frac{\langle z^2\rangle}{Q}\sum_{k=-\infty}^{\infty}\frac{k^4}{(\nu_Z^2-k^2)^2(\nu-k)}\,. \quad (12)$$

In order to obtain the uncertainty of the above estimation one need to calculate

$$\langle\Delta\nu^2\rangle = \left(\frac{\nu^2\bar{\beta}_z}{2}\right)^2\sum_{k,n=-\infty}^{\infty}\frac{k^4}{(\nu_Z^2-k^2)^2(\nu-k)}\times \\ \times\frac{n^4}{(\nu_Z^2-n^2)^2(\nu-n)}\left\langle|F_k|^2|F_n|^2\right\rangle\,, \quad (13)$$

where averaging is performed on all possible orbits with the same RMS. Observing that $\langle F_k F_k^* F_n F_n^* \rangle = 3f^4(\delta_{k,n} + \delta_{k,-n}) + f^4$, hence

$$\sigma_{\Delta\nu} = \frac{\nu^2\sqrt{3}}{2}\frac{\langle z^2\rangle}{Q}\times \\ \times\sqrt{2\nu\sum_{k=-\infty}^{\infty}\frac{k^8}{(\nu_Z^2-k^2)^4(\nu-k)^2(\nu+k)}}\,. \quad (14)$$

In the next section comparison of simulation and estimation by (12) is presented.

SIMULATION

The Monte-Carlo simulation have been done to understand the possible energy shifts due to vertical closed orbit distortions. The sources of the distortions were alignment errors of quadrupoles and sextupoles and random kicks from vertical correctors. For each errors distribution closed orbit has been found and along the closed orbit the spin tune has been calculated using matrix technique. Comparison of the simulation with numerical estimations is presented on Fig. 1 and Fig. 2.

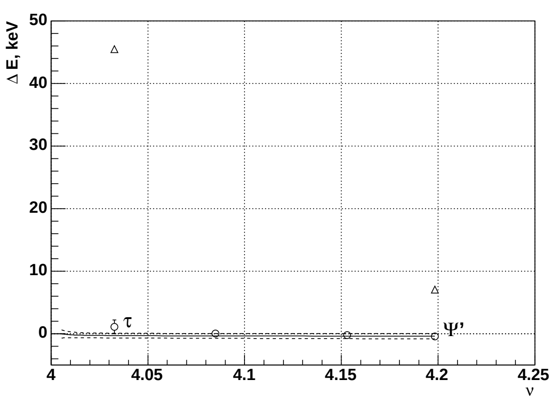

Figure 1: Energy shift versus spin tune at 1 mm vertical orbit RMS. Triangles are calculations neglecting correlation term, circles with errors are results of simulation, solid line presents the estimation by equation (12) and dashed line is the estimation uncertainty by (14).

In previous our publications [2, 3] we used formula (4), where the second term responsible for correlation of the orbital distortions has been neglected relying on the oscillations of $\cos(\pi\nu_0 - \Phi_{j,i})$. Then it was assumed that $\sum\chi_i^2 \approx N\nu^2\langle z^2\rangle P^2$, where $N \approx 100$ is a number of quadrupoles, $P \approx 1/3.7$ m^{-1} is an average quadrupole strength. The results were $\Delta E = 7.068$ keV for $E = 1850$ MeV ($\nu = 4.198$, Ψ') and $\Delta E = 45.485$ keV for $E = 1777$ MeV ($\nu = 4.033$, τ) (see Fig. 1 triangle dots). Such a big difference shows inadequacy of neglection the correlation term.

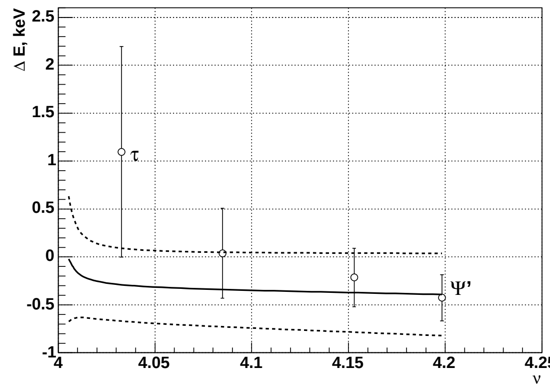

Figure 2: Energy shift versus spin tune at 1 mm vertical orbit RMS. Circles with errors are results of simulation, solid line presents the estimation by equation (12) and dashed line is the estimation uncertainty by (14).

In paper [4] the correlation term of formula (4) was neglected by assumption that orbit deviations z_i at different quadrupoles are uncorrelated: $\langle z_i z_j\rangle \approx \delta_{ij}$ and therefore $\langle\alpha_i\alpha_j\rangle \approx \delta_{ij}$. Such an assumption is not correct because it breaks closed orbit condition $\sum\alpha_i = 0$, taking into account the following relation

$$\sum\alpha_i^2 = \frac{1}{2}\left(\sum\alpha_i\right)^2 - \sum_{i\neq j}\alpha_i\alpha_j\,.$$

CONCLUSION

Vertical orbit distortions introduce an energy bias in energy calibration done by resonant depolarization technique. Which could be estimated by using formulas (12) and (14) in the region far from the integer spin resonance. However, assumptions used in obtaining (8) and (12) limit the accuracy of the estimation (see Fig. 2). For more accurate analysis a computer simulation is needed. The energy shift was -0.6 ± 0.4 keV at RMS of vertical orbit distortions of 1.2 mm for experiment of Ψ' mass measurement and was estimated to be 1.5 ± 1.5 keV for τ lepton mass measurement experiment.

The further improvement of analytical estimations will require consideration of straight section in accelerator and varying beta function.

REFERENCES

[1] V.M. Aulchenko et al., Phys. Lett. B 573(2003)63-79.

[2] V.E. Blinov et al, proceedings of PAC'01, Chicago, USA.

[3] V.E. Blinov et al., NIM A 494(2002)68-74.

[4] R. Assmann, J.P. Koutchouk, CERN SL/94-13 (AP).

[5] Ya.S. Derbenev, A.N. Skrinsky, Preprint INP 71-72(1972) (in Russian).

[6] A.M. Kondratenko. Doctoral Thesis. Novosibirsk, 1982.