**WAKEFIELD UNDULATOR FOR GENERATING X-RAYS**

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Abstract

Conception of wakefield undulator (WFU) with very short period is presented. In the base of photon generation by the WFU lies a new mechanism of undulator-type radiation emitted by an ultrarelativistic electron bunch that undulates due to non-synchronous spatial harmonics of its wakefields while the bunch moves along a periodic waveguide. The creation of the WFU with sub-millimeter periods due to advanced accelerator technology opens possibilities to generate hard X-rays employing relatively low electron energies without external alternative fields.

INTRODUCTION

There is considerable current interest in developing insertion devices with sub-millimeter periods for production synchrotron radiation. Recent researches [1-3] of a new mechanism of radiation, emitted by a bunched beam of relativistic charged particles undulating in the alternating wakefields induced as it moves along a periodic waveguide, have specified new potential opportunities for generating ultra-short wavelength light. It has been shown that transverse components of nonsynchronous spatial wakefield harmonics acting on particles can give rise to their undulating motion and consequently to generating the undulator-type radiation (UR).

The goal of the present paper is to show potential opportunities of production of X-rays emitted from an ultrarelativistic electron bunched beam moves in WFU representing a sub-millimeter periodic corrugate waveguide.

WFU RADIATION MECHANISM

The essence of the radiation mechanism in WFU consists in the following. A short bunch of N charged particles with constant velocity \( v_z \) moving along a periodic waveguide/grafting of a period \( D \) induces wakefields acting on the particles. The wake force can be expressed as a spatial harmonic Floquet’s series

\[
\vec{F}(\vec{r}, t) = \sum_{\rho} \vec{F}^{(\rho)}(\vec{r}, t) e^{i \frac{2\pi \rho z}{D}}.
\]

Here \( \vec{F}^{(\rho)}(\vec{r}, t) \) is the \( \rho \)th harmonic of the wakeforce, \( \vec{F}^{(\rho)}(\vec{r}, t) \) is the \( \rho \)th harmonic of the force produced by a point charge \( eN \) with a transverse coordinate \( \vec{r}_e \) acting on a particle with a charge \( e \) having a transverse coordinate \( \vec{r}_e \) and moving at distance \( \sqrt{v_0} \) after the point charge; \( f_\rho(\vec{r}_e, t)N_e \) is the normalized charge-density distribution; \( S_\perp \) is the cross-section of the periodic rf structure.

The action of the synchronous spatial harmonic of wake force \( \rho=0 \) on the bunch particles results in energy losses associated with generating WF. The alternating transverse wake force \( \rho \neq 0 \) can give rise to undulating the particles with transverse velocity

\[
\vec{v}_\perp(\vec{r}, \tau) = \frac{eN}{2\gamma} \sum_{\rho \neq 0} \vec{F}^{(\rho)}(\vec{r}, \tau) e^{2i\pi \rho \tau/D},
\]

where \( \gamma > 1 \). \( \vec{F}^{(\rho)}(\vec{r}, \tau) \) is the undulator parameter

\[
\vec{K}^{(\rho)}(\vec{r}, \tau) = -\frac{\vec{F}^{(\rho)}(\vec{r}, \tau) D}{p\pi mc\nu_\gamma}.
\]

Here \( m \) is the particle mass of rest, \( c \) is the velocity of light, \( \tau = t - z/v_z \) is the time when a particle crosses the plane \( z=0 \).

As it is known undulation of charged particles with the velocity Eq.(2) can cause generating UR.

UR CHARACTERISTICS

Spectral-angular UR characteristics of an infinitely long periodic rf structure were analyzed in [3]. For WFU of a finite length consisting of \( N_p \) periods the average over period spectral-angular power density of hard UR emitted spontaneously by a single particle of the bunch has the following form in the limit \( |K^{(\rho)}| << 1 \)

\[
\frac{d\bar{P}}{d\Omega d\omega} = \frac{e^2}{8\pi c} \frac{\omega^2 N_p}{\omega_0} \sum_{\rho \neq 0} \left[ K^{(\rho)} \left( 1 - \sin^2 \theta \cos^2 \varphi R(\omega, \theta, \rho) \right) \right] - \Re \left( K^{(\rho)} K^{(\rho)} \right) \sin 2 \phi \sin \theta \dot{\varphi} R(\omega, \theta, \rho)
\]

\[
\times \left[ \frac{\omega(1 - \beta \cos \theta - \rho \omega)}{\omega_0} \right]^2 \left[ \frac{\pi N_p}{\omega_0} \right]^2
\]

where \( R(\omega, \theta, \rho) \equiv \left[ 1 - \frac{\omega}{\rho \omega_0} \beta \cos \theta \right] - \beta^2 \left( \frac{\omega}{\rho \omega_0} \right)^2 \), \( \omega \) is the UR frequency, \( \omega_0 = 2\pi \nu_c/D \), \( \beta = \nu_c/c, \varphi \) is the angle between the axis \( OX \) and \( XOY \) plane projection of UR wave vector \( \vec{k} \), \( d\Omega = \sin \theta \ d\theta \ d\varphi \), \( \theta \) is the polar angle between \( \vec{k} \) and the axis \( OZ \) of the WFU.

As it follows from Eq.(4) the radiation have a line spectrum with the resonant frequencies

\[
\omega^{(\rho)}(\theta, \vec{r}, \tau) = 2\rho \omega_0 \gamma \left[ 1 + \sum_{\rho \neq 0} K^{(\rho)}(\vec{r}, \tau) \right] + \left( \gamma(\vec{r}, \tau) \theta \right) \gamma^2.
\]
(where \( p=1,2,\ldots \) are the numbers of radiation harmonics corresponding to wakefield spatial harmonics), and the spectral-angular flux density into a small bandwidth \( \Delta \omega=\omega-\omega_0^0 \) of the \( p^0 \) harmonics in the forward direction

\[
\frac{d\hat{n}^p(\rho,\varphi,\tau)}{d\Omega} = \frac{1}{\Delta \omega} \frac{d\hat{n}^p(\rho,\varphi,\tau)}{d\Omega} = \frac{\gamma}{\Delta \omega} \left| K^{(p)} \right| \left[ \frac{\sin \left( \frac{\Delta \omega}{\omega_0^p} \pi p N_b \right)}{\frac{\Delta \omega}{\omega_0^p} \pi p N_b} \right]^2
\]

where \( \alpha \) is the fine-structure constant, \( \left| K^{(p)} \right| \) and \( \gamma \) are the functions of particle position \((\mathbf{r}, \tau)\) in the bunch.

Let us consider the UR from a beam which represents multi-bunch trains with average current \( I_0 \). Since the bunch longitudinal dimension \( \sigma_z \) is by many orders of magnitude greater than wavelengths of hard UR radiation \( \sigma>\Delta \gamma \), so UR emitted by all particles of the bunch is spontaneous. It follows thence that the spectral-angular photon flux density of the \( p^0 \) harmonics emitted by the bunched beam in the forward direction is

\[
\frac{d\hat{n}^p_{\text{beam}}}{d\Omega} = \frac{1}{\Delta \omega} \frac{d\hat{n}^p_{\text{beam}}}{d\Omega} = \frac{\gamma}{\Delta \omega} \left| K^{(p)} \right| \left[ \frac{\sin \left( \frac{\Delta \omega}{\omega_0^p} \pi p N_b \right)}{\frac{\Delta \omega}{\omega_0^p} \pi p N_b} \right]^2
\]

where \( \langle \ldots \rangle \) is the bunch averaging.

**AXIALLY SYMMETRICAL WFU**

To calculate analytically a photon yield it is convenient to consider WFU which represents a weakly corrugated circular metallic waveguide shown in Fig.1.

Figure 1: Schematic representation of WFU.

For sake of simplicity, we consider a monochromatic ultrarelativistic electron bunch of length \( \sigma_z \) with uniform charge distribution which moves through the WFU at distance \( r_b \) from the axis. Wakefield spatial harmonics are found by the perturbation method [4]. The periodic radius of the waveguide can be written in Fourier series

\[
b(z) = b_0 + \sum_{m=1}^\infty e_m \exp \left( \frac{2\pi m z}{D} \right)
\]

where \( e_m \ll 1 \) is the relative depth of corrugations, \( b_0 \) is the average radius of the waveguide. As per this approach the first order undulator parameter of the \( p^0 \) spatial harmonics has the form

\[
K^{(p)}(r, \varphi, \tau) = -i \sum_{m=0}^\infty \int_{-\infty}^{\infty} \frac{\sin(\Delta \omega / \omega_0^p \pi p N_b \tau)}{\frac{\Delta \omega}{\omega_0^p} \pi p N_b \tau} \left[ \frac{\sin(\omega / \omega_0^p \pi p N_b \tau)}{\frac{\omega}{\omega_0^p} \pi p N_b \tau} \right]^2
\]

where \( \Delta \omega / \omega_0^p \tau \) is the relative depth of corrugations, \( b_0 \) is the average radius of the waveguide. As per this approach the first order undulator parameter of the \( p^0 \) spatial harmonics has the form

\[
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\]

where \( \delta_{m,n} \) is Kronecker's symbol,

\[
A_{m,n}(\eta b_0) = \frac{J_m(\mu_{m,n} b_0)}{J_m(\mu_{m,n} b_0)} \frac{\omega_0^p}{\omega_0^m} \sin(\omega_0^m \eta b_0 \tau / 2c) \left[ \frac{\sin(\omega_0^m \eta b_0 \tau / 2c)}{\frac{\omega_0^m}{\omega_0^p} \pi p N_b \tau} \right]^2
\]

where the frequencies of resonant WF modes are the zeros of Bessel functions \( J_m(\mu_{m,n} b_0) = 0 \) and \( J_m(\mu_{m,n} b_0) = 0 \), respectively,

\[
\omega_{m,n} = \frac{\mu_{m,n} b_0}{2\pi p b_0}
\]

are the frequencies of resonant WF modes. Therewith, the distribution of electron energy loss in the WFU is

\[
\Delta W = \frac{dW/d\Omega}{d\Omega} = \frac{1}{\Delta \gamma} \sum_{m=1}^\infty \int_{-\infty}^{\infty} \frac{\sin(\omega_0^m \eta b_0 \tau / 2c)}{\frac{\omega_0^m}{\omega_0^p} \pi p N_b \tau} \left[ \frac{\sin(\omega_0^m \eta b_0 \tau / 2c)}{\frac{\omega_0^m}{\omega_0^p} \pi p N_b \tau} \right]^2
\]

Inserting Eq.(9) in Eq.(7) and taking into account the small beam energy dispersion it can obtain the spectral-angular flux density per unit of \( \Delta \omega / \omega_0^p \) emitted in the forward direction

\[
\frac{d\hat{n}^p_{\text{beam}}}{d\Omega} = \frac{1}{\Delta \omega} \frac{d\hat{n}^p_{\text{beam}}}{d\Omega} = \frac{\gamma}{\Delta \omega} \left| K^{(p)} \right| \left[ \frac{\sin \left( \frac{\Delta \omega}{\omega_0^p} \pi p N_b \tau \right)}{\frac{\Delta \omega}{\omega_0^p} \pi p N_b \tau} \right]^2
\]

To estimate possible values of hard X-ray fluxes it is convenient to consider the sinus-type corrugated waveguide of the radius \( b(z)=b_0=1+2t_0 \sin(2\pi \nu D) \). In Tab.1 it is shown one of sets of selected WFU and electron bunch parameters. The beam parameters are typical for the conceptual projects of synchrotron X-ray sources based on a new paradigm of electron accelerators, Energy Recovery Linacs (ERL) [4]. For the selected values of electron energy and waveguide period the fundamental energy of photons is 792 KeV.

As stated above the electron bunch moving along the WFU losses energy generating wakefields. The distribution of relative energy losses \( \Delta W/W_c \) (see Eq.(10))
per bunch charge $eN$ along the bunch is shown in Figs.2,3. In Figs.(2-4) $s$ is the particle distance from the beginning of bunches ($s = \pi + \sigma_z/2$).

Table 1: The WFU and beam parameters

<table>
<thead>
<tr>
<th>Period</th>
<th>$D$</th>
<th>300 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average radius</td>
<td>$b_0$</td>
<td>300 µm</td>
</tr>
<tr>
<td>Relative amplitude of corrugations</td>
<td>$\varepsilon_1$</td>
<td>0.05</td>
</tr>
<tr>
<td>Number of periods</td>
<td>$N_u$</td>
<td>1000</td>
</tr>
<tr>
<td>Energy of electrons</td>
<td>$W_e$</td>
<td>5 GeV</td>
</tr>
<tr>
<td>Average beam current</td>
<td>$I_0$</td>
<td>100 mA</td>
</tr>
<tr>
<td>Bunch length</td>
<td>$\sigma_z$</td>
<td>30, 300 µm</td>
</tr>
<tr>
<td>Bunch distance from axis</td>
<td>$r_b$</td>
<td>260 µm</td>
</tr>
</tbody>
</table>

Figure 2: The relative energy losses per bunch charge along the bunch of 30 µm length.

Figure 3: The relative energy losses per bunch charge along the bunch of 300 µm length.

The nonsynchronous transverse components of the induced wake force provides the following distributions of the undulator parameter Eq.(9) which in form normalized to bunch charge are shown in Fig.4,5.

Figure 4: The distributions of the undulator parameter $K$ per bunch charge within the bunch of 30 µm length.

Figure 5: The distributions of the undulator parameter per bunch charge within the bunch of 300 µm length.

The next Fig.6 demonstrates the dependences of angular flux densities of 792 KeV photons in the forward direction (see Eq.(11)) on bunch charge in terms of (photons/s/mrad/0.1% bandwidth).

Figure 6: The 792 KeV photon flux density v.s. bunch charge at the constant average current 100 mA. The lines upper and lower correspond to 30µm and 300 µm bunch lengths, respectively.

**SUMMARY**

Conception of the new insertion device, wakefield undulator is presented. The spectral–angular characteristics of hard radiation generated by the WFU have been obtained. Using, as the WFU-model, a weakly corrugated circular waveguide excited by an thin uniform electron bunch, it has been demonstrated the potential possibility of generating intensive fluxes of hard X-rays in the mode typical for synchrotron sources based ERLs.

**REFERENCES**


