THE GEODESIC NETWORK CONSTRUCTION IN U-70 SYNCHROTRON BY EQUALIZATION VIA THE CORRELATES METHOD.

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Abstract

The construction procedure of the closed plane geodesic network is investigated on a new level for alignment of radial positions of magnets in accelerator U-70. It is confirmed that the means of magnetic element installation on the design orbit during the accelerator construction was optimal. Using the results of the fulfilled geodesic measurements the article presents the estimation of plane position stability of accelerator magnets over past 38 years.

INTRODUCTION

The fiducial points ($n = 60$) of the plane geodetic network established as far back as at the accelerator construction were anchored in the basement under the accelerator and are located approximately uniformly on a circle of radius R=233.4 m [1]. Since then this network relatively to which radial locations of magnetic blocks are defined was not exposed to verification because of methodological and technical complexity of this procedure. For past years the regular measurements of magnetic blocks locations have shown the appreciable distortion of accelerator ring in the vertical direction caused first of all by such man-caused actions on a geological basis of the machine arrangement as making of a radiation shielding, Booster and UNK tunnel constructions. Therefore there is no reason to believe that as a result of these and other actions within almost 40 years the plane geodetic network has remained immovable. In this connection the check becomes necessary both for the old existent network and for a new one that is supposed in the future to be disposed directly on the magnetic blocks. It would allow one to determine radial block locations more operatively and exactly.

For the solution of this problem the method of a high-accuracy network making tested at the accelerator construction was reanimated. The likelihood principle lays in a basis of this method [2]÷[4]. In this case the network represents the closed chain of overlapping triangles vertexes of which coincide with fiducial points as it is shown in Fig. 1.

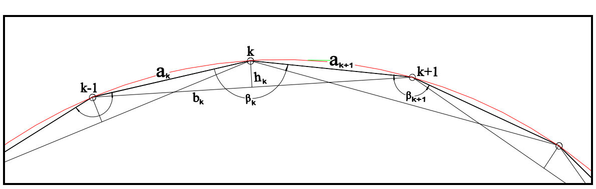

Fig. 1: A plane geodetic network – a system of overlapping triangles.

Actual arrangement of fiducial points and measurements of network parameters are executed with errors and to calculate the highly probable location of fiducial points the following procedure is used:

- the fiducial points of closed polygon network are supposed to be installed on all $n = 60$ magnetic blocks of the accelerator with the odd numbers (defocusing blocks);
- the theoretical lengths of all sides, quantities of obtuse angles or small heights of the overlapping triangles with vertexes in fiducials are determined;
- the differences of these side lengths and quantities of obtuse angles of these overlapping triangles from their theoretical values are measured with rms errors not exceeding $m_{a,b}$ for the sides and m_β for angles; the other variant of a network equalization assumes to measure only small heights with rms error m_h ;
- these measurements are used in the procedure of a network equalization according to likelihood principle to compute the highly probable displacements of all n fiducials and consequently the magnetic blocks from their theoretical positions;
- a radial position of each magnetic block with even number (a focusing block) is measured relative to next two blocks with odd numbers (defocusing blocks) with rms error m_{f} ;
- displacements $\Delta\mathbf{R}$ of all magnets are characterized both a magnitude of maximum displacement and maxima $\delta X = [\max(\mathbf{x}) - \min(\mathbf{x})]/2$ of orbit distortion $\mathbf{x}$ relative to the vacuum chamber axis caused by such displacements at betatron tunes $Q_{\mathrm{x,y}} = 9.750$.

All three sides and a obtuse angle are measured in each triangle of a plane network. (In alternative variant a small height is measured instead of an angle.) Thus in each triangle there is one redundant measurement and hence there is one limiting condition. The measured sides and angles (heights) of triangles form the closed polygon traverse which has three additional constraints for the sums of: (1)

the concluded angles of polygon (1a);
increments on Y (1b);
increments on X (1c).

Hence the considered network is described by $(n+3)$ restricting conditional equations. In addition it is necessary to notice that the measurement errors of triangle sides take the negligible effect on magnitudes of the defined angles. On this reason [2] it is expediently to “equalize” the measured sides, i.e. to find their highly probable values, only under the first n requirements not including the requirements (1). This circumstance essentially simplifies procedure of an equalizing of measurement results.

FORMATION OF A PLANE NETWORK

For deriving the highly probable coordinates of fiducial points of the plane network in a cyclic accelerator by equalization via the correlate method [4] we shall consider at first the variant when in each triangle (see Fig. 1) all its three sides and a obtuse angle are measured. The presence of redundant measurement allows for each triangle to write the nonlinear equation of errors (a fundamental equation) relative to desired parameters specifying the triangle precisely

$$\sqrt{a_k^2 + a_{k+1}^2 - 2a_k a_{k+1}\cos(\beta_k)} - b_k = w_k\,, \quad (2)$$

where: a_k, b_k, β_k – the measured values of sides and an angle of a prolate triangle; k – a serial number of a fiducial point (number of a triangle). The misclosure $\mathbf{w}$ is calculated on the results of these measurements and generally differs from zero. Adding the highly probable corrections $\delta\mathbf{a}, \delta\mathbf{b}, \delta\boldsymbol{\beta}$ into the measured values makes it possible to reduce $\mathbf{w}$ to zero. This procedure consists in the following. Let A_k, B_k, Θ_k are the theoretical values of triangle parameters that are close to the measured ones such that

$$\sqrt{A_k^2 + A_{k+1}^2 - 2A_k A_{k+1}\cos(\Theta_k)} - B_k = 0\,,$$

$$A_k = a_k + \delta a_k\,,\ B_k = b_k + \delta b_k\,,\ \Theta_k = \beta_k + \delta\beta_k\,.$$

From here taking into account conditions (2) and assuming little corrections the fundamental equations it is possible to present:

$$F_k \equiv Z1_k\,\delta a_k + Z2_k\,\delta a_{k+1} + M_k\,\delta\beta_k + \delta b_k - w_k = 0\,, \quad (3)$$

where: $Z1_k = \left(-A_k + A_{k+1}\cos(\Theta_k)\right)/B_k$,

$Z2_k = \left(-A_{k+1} + A_k\cos(\Theta_k)\right)/B_k$,

$M_k = -A_k A_{k+1}\sin(\Theta_k)/B_k$.

It is obvious that the weighted measurement coefficients of sides $p_{a,b}$ and angles p_β are the same in all triangles. It is possible to put $m_{a,b} = m_S$, $p_{a,b} = 1$ and p_β to define as $p_\beta = m_S^2/m_\beta^2$. Then the determination of the corrections at network equalization is reduced to minimization $\sum_{k=1}^{n}\left\{\delta a_k^2 + \delta b_k^2 + p_\beta\,\delta\beta_k^2\right\}$ at the restrictions (3) using uncertain Lagrange multipliers which are called frequently as correlates in analysis of measurements. From here it is not difficult to find new adjusted values of triangles parameters: $\mathbf{a} + \delta\mathbf{a} \to \mathbf{a}$, $\mathbf{b} + \delta\mathbf{b} \to \mathbf{b}$, $\boldsymbol{\beta} + \delta\boldsymbol{\beta} \to \boldsymbol{\beta}$. However these new values to be substituted to expressions (2) generally can not provide zero misclosure $\mathbf{w}$ because they were received not from a solution of the original equations (2) but from the approximate linearized equations (3). For the new values $\mathbf{a,b,\beta}$ and corresponding $\mathbf{w}$ the above described procedure is applied again and again while the misclosure $\mathbf{w}$ becomes acceptably small.

As it was mentioned above at height measurements the equalizing procedure of the plane geodetic network by correlates method is practically the same as in variant with obtuse angle measurements. In this case the original equations (2) take now the form:

$$\sqrt{a_k^2 - h_k^2} + \sqrt{a_{k+1}^2 - h_k^2} - b_k = w_k\,.$$

The received above geodetic network should satisfy to three limiting conditions (1). For convenience we change numbering the triangles sides $a_{k+1} \to S_k$ (see Fig. 2) and designate Cartesian coordinates of the point with number k (vertex of a obtuse triangle angle) through (x_k, y_k). Then $\Delta x_k = x_{k+1} - x_k$, $\Delta y_k = y_{k+1} - y_k$ and the conditions (1) require the following zero misclosures:

$w_\beta = \sum_{k=1}^{n}\beta_k - \pi(n-2) \to 0$, $w_x = \sum_{k=1}^{n}\Delta x_k \to 0$, $w_y = \sum_{k=1}^{n}\Delta y_k \to 0$. Therefore it is necessary to add corrections $\delta\beta_k$, $\delta(\Delta x_k)$, $\delta(\Delta y_k)$ to each angle β_k and to each of projections Δx_k and Δy_k. The quantities of these projections depend on S_k and α_k:

$$\delta(\Delta x_k) \cong \delta S_k\cos(\alpha_k) - S_k\,\delta\alpha_k\sin(\alpha_k)\,,$$

$$\delta(\Delta y_k) \cong \delta S_k\sin(\alpha_k) + S_k\,\delta\alpha_k\cos(\alpha_k)\,,$$

and a correction to α_k is equal to $\delta\alpha_k = -\sum_{j=1}^{k}\delta\beta_j$.

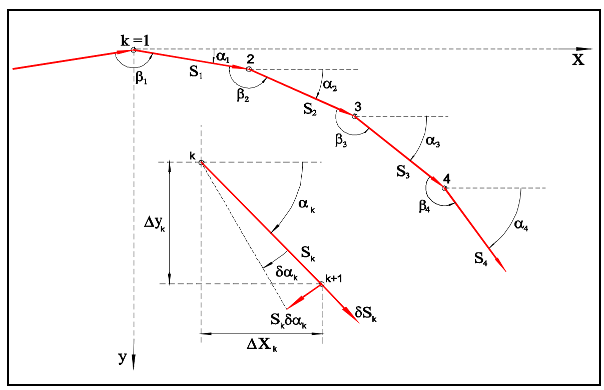

Fig. 2: Equalization of a completed polygon.

Finding of the corrections $\delta\boldsymbol{\beta}$ and $\delta\mathbf{S}$ is performed as before using the correlates method and proceeding from the requirement of minimization of the weighed norm of these corrections $\sum_{k=1}^{n}\left(p_\beta(\delta\chi_k)^2 + p_S(\delta S_k)^2\right) = \min$ at three restricting equations:

$$\sum_{k=1}^{n}\delta\chi_k = 0\,,$$

$$\sum_{k=1}^{n}\delta S_k\cos(\alpha_k) + \sum_{k=1}^{n}\delta\chi_k y_k + f_x = 0\,,$$

$$\sum_{k=1}^{n}\delta S_k\sin(\alpha_k) - \sum_{k=1}^{n}\delta\chi_k x_k + f_y = 0\,,$$

where: $\delta\boldsymbol{\beta} = -\delta\boldsymbol{\chi} - w_\beta/n$,

$$f_x = w_x - w_\beta\,y_1 + \frac{w_\beta}{n}\sum_{k=1}^{n}y_k\,,$$

$$f_y = w_y + w_\beta\, x_1 - \frac{w_\beta}{n}\sum_{k=1}^{n} x_k \ .$$

In this case it is possible to put $p_\beta = 1$ and hence $p_S = m_\beta^2 / m_S^2$.

Modification of angles and sides changes the coordinates of geodetic network points except for the first one:

$$x_k + \sum_{j=1}^{k-1}\delta(\Delta x_j) \to x_k \ , \ y_k + \sum_{j=1}^{k-1}\delta(\Delta y_j) \to y_k \ .$$

The displacements of network points relative to their theoretical positions are: $\delta x_k = x_k - X_k$ и $\delta y_k = y_k - Y_k$. From here it is possible to estimate the radial shifts of fiducial points as $\Delta R_k \cong \delta x_k \left(X_k / R_k\right) + \delta y_k \left(Y_k / R_k\right)$.

The considered method for the calculation of highly probable radial displacements of geodetic network points has some uncertainty. This uncertainty is a consequence of an arbitrary choice of point 1 as original and motionless one. Therefore optimal choice of magnitude of plane-parallel shift of the whole network (and the first point too) is determined by minimization of vector $\Delta\mathbf{R}$ norm.

TOLERANCES ON MEASUREMENT PRECISION

The simulation of measurement and equalizing process for definition of necessary precisions $m_{S,\beta}$ was made. It was supposed that points of a network "are fixed" rigidly on magnetic blocks forming the regular $n = 60$ polygon located on a circle of average radius of U-70 orbit ($R = 236.138$ m). In such conceptual ideal network: $a_k^{id} = 2R\sin(\pi/n)$, $b_k^{id} = 2R\sin(2\pi/n)$, $\beta_k^{id} = \pi - 2\pi/n$. We assume that these sides and angles are measured with random independent errors Δa_k , Δb_k , $\Delta\beta_k$ distributed normally with rms deviations $m_{S,\beta}$:

$$a_k = a_k^{id} + \Delta a_k \ , \ \ b_k = b_k^{id} + \Delta b_k \ , \ \ \beta_k = \beta_k^{id} + \Delta\beta_k \ .$$

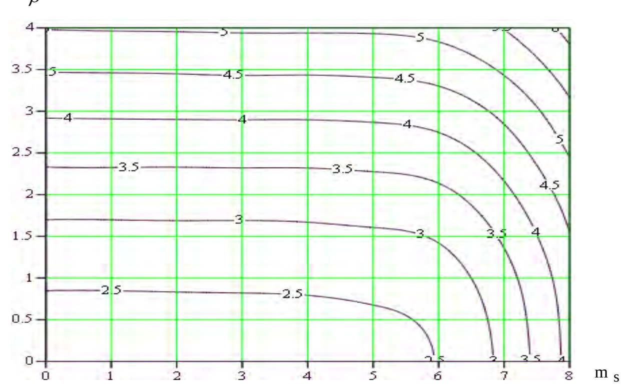

Fig. 3: Level lines of the contribution to maximum radial distortions of U-70 orbit (in mm, 90 % probability) at $m_f = 50$ µm: m_β is measured in seconds, m_f – in mm.

The distributions of maximum displacement of fiducial points and maximum orbit distortions were determined on calculation of 5000 realizations of accelerator geodetic network. For each realization of random errors this method computes the highly probable displacements of points on odd (defocusing) blocks relative to their theoretical positions. The position of fiducial points on even (focusing) blocks is determined by measurements relative to next two odd blocks with rms error m_f . From the results shown in Fig.3 it is possible to make the following conclusion. If one establishes a limitation on the contribution to radial orbit distortions with probability 90 % which not exceeds 3 mm then rms errors of measurements should not exceed $m_\beta \approx 1.5''$ and $m_S \approx 5$ mm at $m_f = 50$ µm. Such tolerance requirements to geodetic measurements are quite acceptable.

CONCLUSION

The measurements of points positions of the existing plane geodesic network were fulfilled on the described above technique in 2003. The definition of radial displacements of geodesic network points ΔR_β was performed at precisions $m_S = 500$ µm and $m_\beta = 0.44''$. In this case the maximum error of points position determination does not exceed 1.2 mm with probability 90 %.

The described technique of equalization was applied to the measurements data of 1966 when the sides and small heights were measured. The calculation of radial point displacements ΔR_h was executed at $m_S = 500$ µm and $m_h = 40$ µm. The maximum error in determination of points positions with probability 90 % does not exceed 1.7 mm. The magnitudes $\Delta R_\beta - \Delta R_h$ determining displacements of the existing geodetic points for last 40 years are shown in Fig. 4.

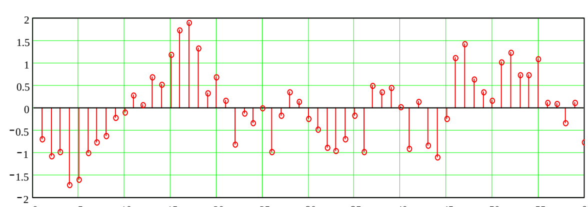

Fig. 4: Radial displacements of the fiducial points (mm) for last 40 years.

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[2] See [1], page 102.

[3] See [1], page 123.

[4] Yu.V. Linnik, Method of least squares and bases of the theory of handling of observations, M.: Fizmatgiz, 1962, (in Russian).