An IMAGINARY-γ_t LATTICE WITH DISPERSION-FREE STRAIGHTS FOR THE 50 GeV HIGH-INTENSITY PROTON SYNCHROTRON

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Abstract

The 50 GeV proton synchrotron, proposed by the Institute of Nuclear Study of Japan (INS), requires zero-dispersion straight sections for polarized-beam operation. A new preliminary lattice that contains two straight sections with nonzero dispersion and two with zero dispersion is presented. The γ_t of the whole ring remains imaginary. Some analysis of the lattice is discussed.

1 INTRODUCTION

In order to reduce beam loss, the 50-GeV proton synchrotron of the Japan Hadron Project (JHP), designed by the INS, will operate with an imaginary-γ_t [1]. The lattice is 4-fold symmetric. Each quadrant consists of 6 flexible momentum-compaction (FMC) modules [2] and a long straight section of about 60 m in length. Each FMC module is 3 FODO-cell long. The dispersion in the long straight sections vary between -0.71 and 0.58 m. Although the dispersion is small, it is always more appealing to have zero-dispersion straights. This is especially true when the synchrotron is accelerating a polarized beam. To obtain zero dispersion in one straight section and in another straight section on the other side of the ring, a special excitation of the quadrupoles needs to be turned on so as to allow a dispersion wave and a betatron wave to flow through half of the ring. Aside from the inconvenience of having a special power supply, this excitation also brings about unwanted high betatron functions and high dispersion functions, which will eventually limit the performance of the accelerator at high intensities. In this paper, we suggest the introduction of dispersion suppressors. A preliminary lattice that contains two straight sections with nonzero dispersion and two with zero dispersion is presented. The details are given in Ref. 3.

2 DISPERSION SUPPRESSOR

The standard FMC module of the JHP ring is shown in Fig. 1 with its lattice elements, betatron functions and dispersion. To study its dispersion property, the module is plotted in the normalized dispersion space in Fig. 2(a), with $\sqrt{\beta_x}D' + \alpha_x D/\sqrt{\beta_x}$ versus $D/\sqrt{\beta_x}$ [2]. Here, D and D' are, respectively, the dispersion function and its derivative with respect to the longitudinal coordinate s, β_x and $\beta_x' = -2\alpha_x$ are, respectively, the horizontal betatron amplitude function and its derivative.

A thin dipole of bending angle θ will be represented by a horizontal advance of $\sqrt{\beta_x}\theta$. Outside the dipoles, the plot

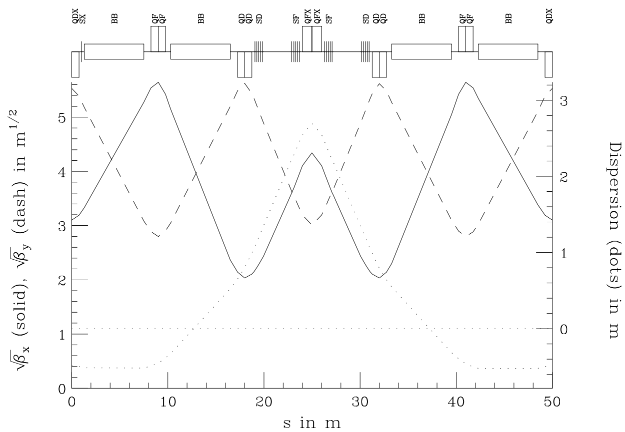

Figure 1: The standard FMC module of the JHP ring.

becomes an arc of a circle centered at the origin with phase advance equal to the horizontal Floque phase advance. We see that the module starts off from the quadrupole QDX with zero β_x' and D', but with dispersion -0.5213 m. The first two dipoles BB are represented by two long straight lines pointing mostly to the right. Note that these line are not exactly horizontal, because the dipole is far from a thin element and there is a phase advance across it. If we chop up the dipoles into smaller elements, this straight line will be curved. However, it will still be quite different from the arc of a circle with center at the origin of the plot. The deviation just represents the angle-bending nature of the dipole. After the dipoles, follows an arc of a circle centered at the origin leading to the center quadrupoles QFX, which have the largest $D/\sqrt{\beta_x}$. The other half of the module is just the mirror image of the first half, coming back to the starting point.

In order to be a dispersion suppressor, we must alter the lattice so that the end of the module stops precisely at the origin of the dispersion space. To accomplish this, we must first make the radius of the arc smaller in the upper half of the dispersion plane, and second we must use an exact amount of dipole to bring the module to $D = 0$ and $D' = 0$ at the point when this arc reaches roughly 180°. The suppressor constructed in this way is shown in Fig. 3 and its dispersion plot in Fig. 2(b). The construction starts from pulling out the second dipole, so that the module continues with a smaller arc until the quadrupole QFFX. To facilitate lattice matching, although unnecessary, we treat this temporarily as a point of symmetry, that is with $\beta_x' = \beta_y' = D' = 0$. After that we continue as in the case of the standard FMC module with the exception that the last dipole, called B4, is shortened so that the module lands exactly at $D = 0$, $D' = 0$. In order not to deal with a fractional dipole, the amount B4 has been shortened,

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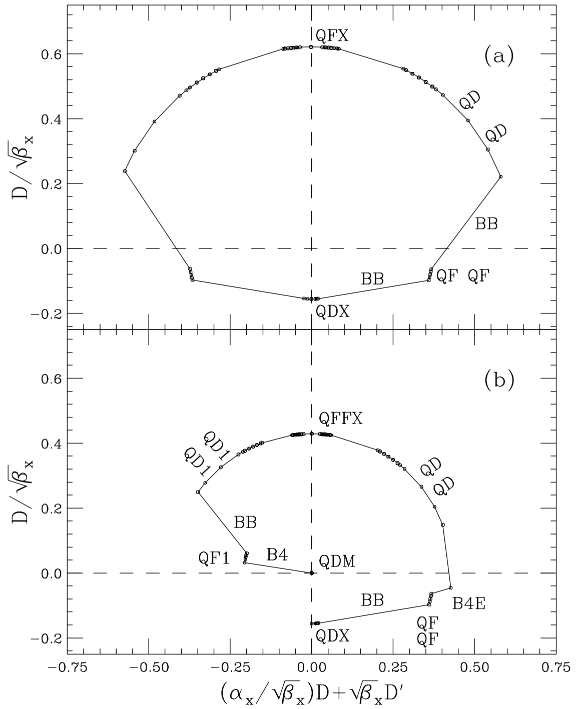

Figure 2: The standard FMC module and dispersion suppressor in the normalized dispersion plane.

called B4E, is placed in the space where the second dipole has been pulled out. In other words, one normal dipole has been pulled out, and another normal dipole has been chopped up into two parts B4 and B4E, 83% and 17% of the normal dipole. The chopping up of a normal dipole for the dispersion suppressor seems to be inevitable. This is very similar to the situation of the dispersion suppressor in a FODO-cell lattice, where one can avoid chopping up dipole only when the phase advance of each cell is exactly $\pi/3$.

As shown in Fig. 3, the dispersion suppressor is not very much different from the standard FMC module. It has a length of 48.7269 m, maximum/minimum dispersion of $1.8632/-0.5148$ m, maximum/minimum horizontal betatron function 31.84/4.22 m and maximum/minimum vertical betatron function 32.32/7.17 m. The vertical and horizontal tune advances are 0.721/0.545, which are very close to the 0.740/0.531 for the standard FMC module. Best of all, this suppressor has also an imaginary transition gamma of $\gamma_t = 69.05i$, so that the whole ring can still retain its imaginary-γ_t property.

3 LONG STRAIGHT SECTIONS

There will be two long straight sections that are dispersion-free and two that are not. The straight with dispersion, which joins two standard FMC modules together, consists of 4 FODO cells with a total length of 61.2524 m. The zero-dispersion straight is inserted between two dispersion-suppressors. It also contains 4 FODO cells with a total length of 62.2938 m.

Now the whole ring can be assembled. We start from

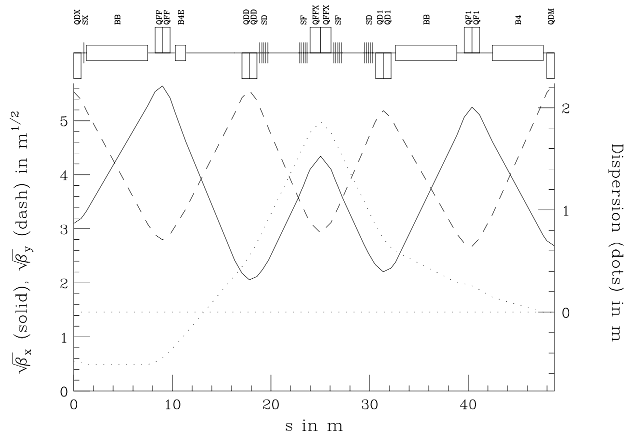

Figure 3: The lattice structure of the dispersion suppressor.

the center of the non-dispersion-free straight section, then 5 standard FMC modules, the dispersion suppressor, and then the dispersion-free long straight section. We make a mirror reflection about the center of the dispersion-free straight to arrive back at the center of the other non-dispersion-free straight. This complete one half of the ring. The whole ring has now only 92 dipoles each of length 6.2 m. Since the beam particles are to be accelerated to the maximum total energy of 50 GeV, the maximum bending field of the dipole becomes 1.837 T, which is high but is still possible. There is enough space to increase the length of the dipoles from 6.2 m to 6.3 m, thus reducing the bending field to 1.808 T.

4 SEXTUPOLE CORRECTION

In general, quadrupoles of the FMC modules are stronger than those in the usual FODO lattice. As a result, larger natural chromaticities will be generated and sextupoles of larger strengths will be required for their corrections. The corrections are mainly made by the two families of sextupoles SF and SD as shown in Figs. 1 and 3. There each SF or SD is represented by 5 thin sextupoles in the lattice code. Just after the entrance defocusing quadrupole of each FMC module, there is a third family SX, which is used for fine adjustment. For example, the chromaticity corrections have been made by setting the strength of each SX to be 0.105 m^{-2} and each of the thin SF and SD 0.0572 m^{-2} and -0.0933 m^{-2}, respectively. The amplitude-dependent betatron tunes are $\nu_x = 21.0954 + 130\epsilon_x - 116\epsilon_y$ and $\nu_y = 15.4433 - 116\epsilon_x + 134\epsilon_y$, where the emittances ϵ_x and ϵ_y are measured in πm. We see that with $\epsilon_x = \epsilon_y = 50\times10^{-6}\,\pi$m, the largest tune spread is only 0.0067, which is quite acceptable for a non-storage ring.

Another measure of nonlinearity introduced by the correction sextupoles is the single-particle smears, which are defined as the fractional rms distortion of the Poincaré torus at any phase advances in the horizontal and vertical phase spaces. The smears can be expressed analytically in terms of the distortion functions [4]. We see from Fig. 4 that the rms vertical smear reaches only about 0.1%, which is very small, and the horizontal smear is still smaller. The full smears will be roughly $\sqrt{2}$ times the rms values, which are

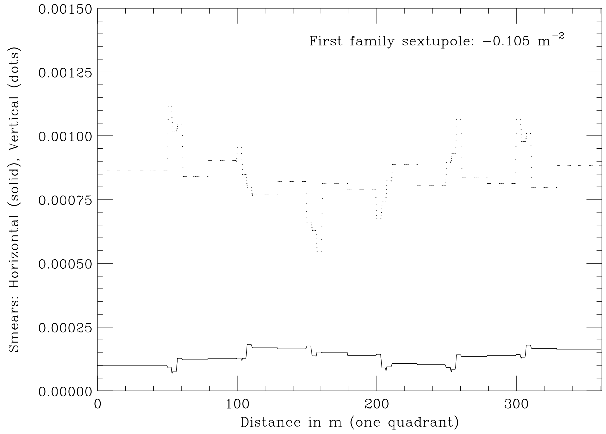

Figure 4: The horizontal and vertical single-particle smears for one quarter of the accelerator ring.

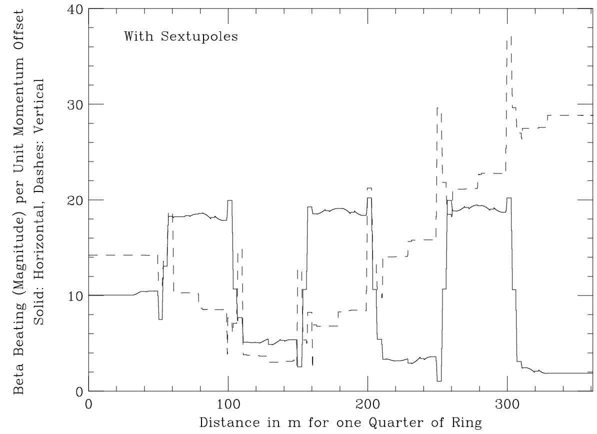

Figure 5: The magnitudes of the horizontal and vertical beating vectors for one quarter of the accelerator ring.

much less than the 7% nonlinear criterion of the former Superconducting Super Collider. We also see that the smears are step-like, constant over a region and exhibit a jump only when a sextupole is encountered.

5 BETATRON BEATINGS

In this FMC-type lattice, it is impossible to place a sextupole beside every quadrupole to correct for local chromaticities. As a result, particles with a momentum offset will see a different set of betatron functions. The fractional change in the betatron function per unit momentum deviation $\Delta\beta/\beta|_\psi$, horizontal or vertical, at phase advance ψ, is called "beat factor" [5]. Each beat factor can be made complex by introducing the imaginary part $-\frac{d}{d\psi}\frac{\Delta\beta}{2\beta}|_\psi$. As a vector, the complex number just rotates at a tune of 2ν, except when crossing a field gradient k of infinitesimal length ℓ where the real part jumps by $\frac{1}{2}\beta k\ell$. Note that the field gradient can come from quadrupoles, sextupoles, the centripetal force of the dipoles as well as the edges of the dipoles. The magnitudes of the beat vectors are plotted in Fig. 5, the largest being around 30. Considering that the momentum spread of the beam is only 0.5% at injection, the relative change in betatron function is at the most 15% which is not excessive at all.

The harmonic analysis of the beat factors is also important, because it gives us some clues to reduce the beat factors. Choosing the mid-point of a non-dispersion-free straight as the point having zero phase advances, the lattice is almost left-right symmetric (aside from the sextupoles SX). The Courant-Synder J_p's then become almost real:

$$J_p=\int_{-\pi\nu}^{\pi\nu}k(\psi')\beta^2(\psi')\cos\frac{p\psi'}{\nu}d\psi'\ .$$

Each beat factor can be expanded as

$$\left.\frac{\Delta\beta}{\beta}\right|_\psi=-\frac{J_0}{\pi\nu}-\frac{2\nu}{\pi}\sum_{p>0}\frac{J_p\cos\frac{p\psi'}{\nu}}{4\nu^2-p^2}\ .$$

We find that the sextupoles do produce beat waves in the harmonic space. This is because they have not been placed at the proper phase advances for confinement or cancellation. The tunes of the lattice are $\nu_x=21.0954$ and $\nu_y=15.4433$, so that the important Fourier components are $p=42$ for the horizontal and $p=31$ for the vertical. Because of the two-fold symmetry of the lattice, $p=31$ does not occur. As for $p=42$, the horizontal beat factor is not large because $2\nu_x$ is still far from 42. However, we do see the beat waves exhibit large magnitudes at $p=28$. This comes about because each of the two straight sections has a vertical tune advance of ~ 0.60 which is not too far from the vertical phase advance of 0.53 for each FMC module. On the other hand, their horizontal tune advances are 1.00 and 0.70, respectively, for the straights with dispersion and the one without. They average out to roughly the horizontal tune advance of a FMC module. Thus, the contributions of the sextupoles add up. As a result, there appears to be roughly a 7-fold symmetry in a superperiod. Hopefully, $p=28$ is quite far away from a multiple of the tunes and the contribution of this harmonic has not been too large. To reduce this contribution, the vertical phase advance of the straight section must be increased.

6 SUM RESONANCES

Sum and difference resonances should also be studied. We note that $2\nu_y+\nu_x=51.982$ is too close to the third integer sum resonance. This happens because this preliminary lattice is just a simple modification of the original INS lattice [1], which also sits at this third integer sum resonance. However, we believe that this resonance can be avoided by a more careful design of the lattice.

7 REFERENCES

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