SPACE CHARGE DOMINATED ENVELOPE DYNAMICS OF ASYMMETRIC BEAMS IN RF PHOTOINJECTORS

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While it has been proposed for several years that strongly asymmetric $(\sigma_x >> \sigma_y)$ beams can be employed in rf photoinjectors to obtain asymmetric emittances for linear collider applications[1,2], it is known that the emittances obtained directly from rf photocathode guns are not suitably small for this purpose. Because of this, simulational work has been performed in an attempt to apply the principle of emittance compensation to recover the small and asymmetric emittances after suitable focusing and acceleration of photoinjector beams. In order to guide this difficult three-dimensional analysis, we present here an extension of a previous theoretical model of the emittance compensation process in axisymmetric photoinjectors. In the extended model, we first analyze the general quadrupolar oscillations in a symmetric accelerating system, and then proceed to examine a propagation mode under asymmetric focusing. This mode is a generalization of the previously analyzed axisymmetric invariant envelope, allowing an optimization of the compensation process. Design philosophies, including rf cavity considerations, are included in the discussion.

I. INTRODUCTION

The envelope equation for an axisymmetric beam is

$$\sigma'' + \sigma'\frac{\gamma'}{\gamma} + \Omega^2\left(\frac{\gamma'}{\gamma}\right)^2\sigma = \frac{\kappa_s(\zeta)}{\sigma\ \gamma^3} + \frac{\varepsilon_{n,th}^2}{\sigma^3\ \gamma^2} \qquad (1.1)$$

where the perveance $\kappa_S(\zeta)$ explicitly keeps a dependence on the longitudinal position ζ of the particular slice, so that $\kappa_S(\zeta) = I(\zeta)/2I_0$ ($I_0 = 17$ kA), and $\Omega^2 = \eta/8 + b^2$. The *invariant envelope* solution $\hat{\sigma}$ reads[3],

$$\hat{\sigma} = \frac{1}{\gamma'}\sqrt{\frac{2I}{I_0\gamma\left(1+4\Omega^2\right)}} \qquad (1.2)$$

which is an exact solution of Eq. 1.1 in the laminar flow regime (*i.e.* $\varepsilon_{n,th} << \sigma^2\kappa_s/\gamma$).

For an asymmetric beam the two envelope equations for σ_x and σ_y become (Eqs. 1.3-4)

$$\sigma_x'' + \sigma_x'\frac{\gamma'}{\gamma} + \Omega_x^2\left(\frac{\gamma'}{\gamma}\right)^2\sigma_x = \frac{I}{I_0\left(\sigma_x+\sigma_y\right)\gamma^3} + \frac{\varepsilon_{nx}^2}{\sigma_x^3\ \gamma^2}$$

$$\sigma_y'' + \sigma_y'\frac{\gamma'}{\gamma} + \Omega_y^2\left(\frac{\gamma'}{\gamma}\right)^2\sigma_y = \frac{I}{I_0\left(\sigma_x+\sigma_y\right)\gamma^3} + \frac{\varepsilon_{ny}^2}{\sigma_y^3\ \gamma^2}.$$

II. ACCELERATING ASYMMETRIC BEAMS WITH SYMMETRIC FOCUSING

We again assume laminar flow, and proceed by ignoring the thermal emittance contributions to Eqs. 1.3-4. Let us define now the two quantities $\Delta \equiv \sigma_x - \sigma_y$ and $\Sigma \equiv \sigma_x + \sigma_y$. By linear combinations of Eq. 1.3 and 1.4 we obtain, in case of symmetric rf focusing $\Omega_x = \Omega_y = \Omega$ (solenoidal focusing is not relevant, as it introduces an unwanted beam rotation), the following:

$$\Delta'' + \frac{\gamma'}{\gamma}\Delta' + \Omega^2\left(\frac{\gamma'}{\gamma}\right)^2\Delta = 0, \qquad \text{and} \qquad (2.1)$$

$$\Sigma'' + \Sigma'\frac{\gamma'}{\gamma} + \Omega^2\left(\frac{\gamma'}{\gamma}\right)^2\Sigma = \frac{2I}{I_0\gamma^3\Sigma}. \qquad (2.2)$$

The forms of Eqs. 2.1-2 are gratifyingly simple; Eq. 2.1 is identical to that of single particle motion (no space-charge, $I = 0$) in this system[4], while Eq. 2.2 is formally identical to Eq. 1.1 in the case of laminar flow. Thus we immed-iately have, by analogy to previous work[4], the solution of Eq. 2.1 in terms of the initial conditions on the envelopes,

$$\Delta(z) = \Delta(0)\cos\left[\alpha(z)\right] + \Delta'(0)\frac{\sqrt{8}}{\gamma'}\sin\left[\alpha(z)\right], \qquad (2.3)$$

where $\alpha(z) = \frac{1}{\sqrt{8}}\ln\left(\frac{\gamma(z)}{\gamma(0)}\right)$. Likewise, the particular solution for Eq. 2.2 becomes (*a la* Ref. 3)

$$\hat{\Sigma} = \frac{2}{\gamma'}\sqrt{\frac{2I}{I_0\gamma\left(1+4\Omega^2\right)}}\ , \qquad (2.4)$$

which has the expected invariant envelope property that the phase space angle $\hat{\Sigma}'/\hat{\Sigma} = -\gamma'/2\gamma$, independent of current.

As Eq. 1.7 is purely oscillatory, it is clear that the sum invariant envelope given by Eq. 1.8 cannot be composed of invariant envelopes in both x and y. To illustrate this point, we assume the system is launched at $z = 0$ in a nominally invariant envelope configuration, with $\Sigma = \hat{\Sigma}$, and $\Delta'/\Delta = \sigma_x'/\sigma_x = \sigma_y'/\sigma_y = -\gamma'/2\gamma$, then the beam envelopes are given by

$$\sigma_x(z) = \frac{\hat{\Sigma}(0)}{2}\left[\sqrt{\frac{\gamma(0)}{\gamma(z)}} + \frac{A-1}{A+1}\left(\cos[\alpha] - \sqrt{2}\sin[\alpha]\right)\right] \quad (2.5)$$

and

Work supported by U.S. DOE grants DE-FG03-90ER40796 and DE-FG03-92ER40693, DOE SBIR program and Sloan Foundation grant BR-3225. *Perm. address: INFN-Milano, Via Celoria 16, Milan, Italy.

$$\sigma_y(z)=\frac{\hat{\Sigma}(0)}{2}\left[\sqrt{\frac{\gamma(0)}{\gamma(z)}}-\frac{A-1}{A+1}\left(\cos[\alpha]-\sqrt{2}\sin[\alpha]\right)\right]. \quad 2.6)$$

where $A=\sigma_x(0)/\sigma_y(0)$.

It should be noted that Eqns. 2.5-6 describe, in all generality, quadrupolar oscillations of intense, laminar relat-ivistic beams in an accelerating system with axisymmetric applied forces. These systems include standing wave ($\eta \cong 1$) and travelling wave ($\eta < 1$) rf linacs, and electrostatic accelerating systems with solenoid focusing (in the rotating Larmor frame, $b^2 > 0$, $\eta = 0$).

These solutions, because of there oscillatory character, obviously do not correspond to an invariant envelope condition, which is characterized by monotonically decreasing, well controlled size under acceleration. In fact, the small envelope is characterized by a cross-over (nonlaminar) trajectory for large enough A and/or γ, which violates the assumptions leading to Eqs. 2.1 and 2.2. We thus must look further, to asymmetric focusing systems, to find invariant envelope-like behavior for asymmetric beams.

III. ACCELERATING BEAMS WITH ASYMMETRIC FOCUSING: ASYMMETRIC INVARIANT ENVELOPES

We consider now the case of a flat beam, for which we assume that the horizontal size σ_x is always much larger that the vertical size σ_y, *i.e.* $\sigma_x >> \sigma_y$, and that the ratio of these beam sizes is constant for some values of external focusing strength. In this case Eqs. 1.3 and 1.4 become, in the laminar flow regime

$$\sigma_x''+\sigma_x'\frac{\gamma'}{\gamma}+\Omega_x^2\left(\frac{\gamma'}{\gamma}\right)^2\sigma_x=\frac{I}{I_0\sigma_x\gamma^3} \quad \text{and} \quad (3.1)$$

$$\sigma_y''+\sigma_y'\frac{\gamma'}{\gamma}+\Omega_y^2\left(\frac{\gamma'}{\gamma}\right)^2\sigma_y=\frac{I}{I_0\sigma_x\gamma^3}, \qquad (3.2)$$

where $\Omega_{x,y}^2=\eta/8\mp g$, with $g=\left(B'\gamma^2/B\rho\gamma'^2\right)$ the normalized vertical focusing gradient, assumed to arise from an applied quadrupole field. Note that the space charge term on the right hand side of Eqs. 3.1-2 ignores the (assumed small) contribution to the sum of the beam sizes from σ_y.

Under this approximation, these two coupled equations have exact particular invariant envelope solutions for $\hat{\sigma}_x$ and $\hat{\sigma}_y$, as follows,

$$\hat{\sigma}_x=\frac{2}{\gamma'}\sqrt{\frac{I}{I_0\gamma\left(1+4\Omega_x^2\right)}}, \text{ and} \qquad (3.3)$$

$$\hat{\sigma}_y=\frac{2}{\gamma'\left(1+4\Omega_y^2\right)}\sqrt{\frac{I\left(1+4\Omega_x^2\right)}{I_0\gamma}}. \qquad (3.4)$$

To be consistent with the assumption of flat beam, the asymmetry ratio, which is dependent only on the external focusing, must satisfy

$$\hat{A}\equiv\hat{\sigma}_x/\hat{\sigma}_y=1+4\Omega_y^2\big/1+4\Omega_x^2>>1 \qquad (3.5)$$

In order to achieve high asymmetry ratios one has to apply some a second order (*i.e.* scaling as $(\gamma'/\gamma)^2$) defocusing force to the horizontal plane, so that $\Omega_x^2 \to -1/4$. This limiting condition is in fact the strength of the space charge defocusing in the cylindrically symmetric case; the invariant envelope is always a defocusing (hyperbolic) trajectory, which is monotonically decreasing only by virtue of adiabatic damping present during acceleration. The limit on horizontal defocusing is therefore also a limit on vertical focusing, in that the beam must not be so strongly focused in the vertical plane to overcome the defocusing of space charge — this produces a nonlaminar condition.

Eqs. 3.3-4 obey $\hat{\sigma}_x'/\hat{\sigma}_x=\hat{\sigma}_y'/\hat{\sigma}_y=-\gamma'/2\gamma$, implying an angle in both *x*- and *y*- phase space which is independent of current. This condition, should it be achieved for all longitudinal slices in the beam, would freeze all emittance oscillations. In practice, however, one typically matches the beam in an rms sense to the invariant envelopes, and so oscillations are performed around the invariant envelopes. These oscillations are described by a set of equations of the same form and solution type as Eq. 2.1

$$\delta\sigma_x''+\delta\sigma_x'\frac{\gamma'}{\gamma}+\left[2\Omega_x^2+\frac{1}{4}\right]\left(\frac{\gamma'}{\gamma}\right)^2\delta\sigma_x=0, \text{ and} \qquad (3.6)$$

$$\delta\sigma_y''+\delta\sigma_y'\frac{\gamma'}{\gamma}+\left[\Omega_y^2\left(1+\frac{1}{\hat{A}^2}\right)+\frac{1}{4\hat{A}^2}\right]\left(\frac{\gamma'}{\gamma}\right)^2\delta\sigma_y=0, \qquad (3.7)$$

giving oscillation frequencies again independent of current, which allows the entire ensemble of slices to perform coherent oscillations in phase space. This coherent phase space motion is a characteristic of emittance compensation, but in the highly asymmetric ($\hat{A}>>1$) case it can be seen that the two oscillation frequencies are quite different, $\alpha_y \propto \Omega_y$,, $\alpha_x \propto i\left|\Omega_x\right|$, and unstable small amplitude motion is found in the horizontal deviations. This implies that simultaneous compensation in both transverse phase planes is not possible in this mode, with the horizontal emittance left uncompensated.

Another characteristic is the secular diminishing of the phase space, and therefore emittance, oscillations. The trace space oscillations described by Eqs. 3.6 and 3.7 are Liou-villian, in the sense that a constant normalized emittance can be associated with this (offset from the origin) area. But, since this area is constantly moving towards the origin as the invariant envelope damps secularly as $\gamma^{-1/2}$, the projected normalized emittance damps by approximately this factor[3]. It should also be noted that the emittance ratio predicted by this model of compensation is $\varepsilon_x/\varepsilon_y \cong \sigma_x/\sigma_y=\hat{A}$.

IV. ASYMMETRIC ACCELERATING STRUCTURES

In order to implement this compensation scheme in an rf photoinjector we must specify the low energy section which precedes matching to the invariant envelopes. This process begins with the rf structure. We note that in a split photoinjector (gun plus linac[5]) that the for a highly asymmetric beam, the rf emittance associated with the exit electromagnetic kick from a cylindrically symmetric structure would be prohibitive. Therefore, we have designed an asymmetric structure, which from the point of view of the beam region has no dependence of the longitudinal electric field on *x*, and therefore has a transverse fields only in *y*. This is accomplished by use of an H-shaped structure, in which the central region is essentially a waveguide operated at cut-off, with slit regions ($L_x >> L_y$) for the beam holes.

This structure has been simulated using the 3-D electro-magnetic solver code GDFDIL[6], with the results of an S-band (2856 MHz) 1.5 cell gun modeling shown in Fig. 1. The field is balanced in both cells, and has only a 0.3% variation of E_z over 4 cm in x in the *x-z* symmetry plane.

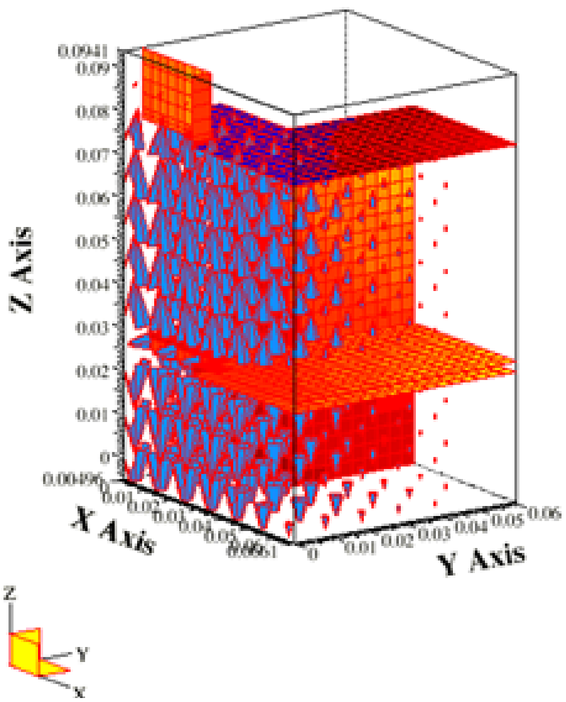

Figure 1. Electric field plot of 1/4 of H-shaped 2856 MHz rf gun structure π-mode with slit-shaped beam holes, obtained from GDFIDL simulation.

V. SYMMETRIC STRUCTURE OPTION

The original proposal for an asymmetric beam and emit-tance photoinjector was stimulated by the prospect of eliminating the electron damping ring for TESLA superconducting (SC) linear collider (LC). This motivation is strong because the emittances are not as small as for normal conducting (NC) linear collider designs. On the other hand, the duty cycle is very high, and so to save on rf power, only NC guns with SC linacs (split photoinjector configuration) have been considered.

If one is willing to look beyond the near term TESLA Test Facility demands and technical limitations (*e.g.* limits on rf power), this restriction is perhaps unnecessary, however. If it is relaxed, one can conceive of a more elegant solution to the asymmetric beam and emittance problem. The main problem which is ameliorated in this case is the large vertical kick at the gun exit, which is twice as large as in the symmetric gun, because the field fringes only in one direction. In the case of a long injector, one does not encounter an rf emittance problem, however, because the beam leaves the structure on the invariant envelope and with a small beam size. Thus there is no need of rf asymmetries, and no large asymmetric kick for the external quadrupole focusing to overcome (solenoids are forbidden because of a shearing due to differential $\vec{E} \times \vec{B}$ rotation, but quadrupoles are more difficult to work with in a compact space). Preliminary calculations indicate that asymmetric beam handling problems (*e.g.* preservation of small vertical beam size without introducing large horizontal angles) are indeed mitigated in this type of device.

Long photoinjectors have been developed by LANL[7], and are now undergoing further development, using the plane-wave transformer[8] (PWT) concept, at UCLA in collaboration with DULY Research[9]. Whether we choose to investigate asymmetric beam rf photoinjectors in the split (asymmetric rf gun) or integrated long photoinjector geometry depends on detailed simulation work. The simulation of 3D self-consistent, space-charge dominated, trans-relativistic beam dynamics is difficult, but we are now pursuing two new methods of attack: 3-D particle in cell codes (e.g. extending GDFIDL), and 3-D Lienard-Wiechert mesh calculators[10]. Both promise great improvements in performance over the point-by-point PARMELA simulations we have previously pursued[11].

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