

# ENERGY ACCEPTANCE AND TOUSCHEK LIFETIME CALCULATIONS FOR THE SOLEIL PROJECT<sup>1</sup>

A. Nadji<sup>°</sup>, G. Flynn\*, J.-L. Laclare<sup>°</sup>, M.-P. Level<sup>°</sup>, P. Nghiem<sup>°</sup>, J. Payet<sup>°°</sup>

<sup>°</sup>Projet SOLEIL, DRIF CNRS, Av. de La Terrasse, Bât. 5, 91198 Gif sur Yvette, France

\*LURE, Centre Universitaire Paris-Sud, Bât. 209 A, 91405 Orsay Cedex, France

<sup>°°</sup>CEA Saclay, LNS, Groupe Théorie, 91191 Gif sur Yvette

## Abstract

Touschek scattering is an important beam lifetime limiting effect for the SOLEIL storage ring and the dominant one for high single bunch current operation. The Touschek relevant energy acceptance may be determined by the radiofrequency system, dynamic aperture or vacuum chamber physical aperture. In the latter two cases, Touschek induced betatron oscillations must be considered. We present energy acceptance and Touschek lifetime calculations for SOLEIL in the presence of these three limitations taking into account the non-linear chromatic orbit as well as energy variation of the optical functions. In light of these considerations, Touschek lifetime lattice optimization will be discussed.

## 1. INTRODUCTION

One approach to improving the Touschek lifetime ( $\tau_T$ ) is to increase the energy acceptance ( $\tau_T \propto \epsilon_{acc}^{>2}$ ) which may be determined by the RF bucket momentum height, by the aperture of the vacuum chamber or by the dynamic aperture if the induced amplitude after a Touschek scattering exceeds one of these two transverse limits.

To ensure a large longitudinal energy acceptance ( $\pm 6\%$ ), the RF system (using superconducting cavities) will provide a peak voltage of more than 3 MV.

The aim of this paper is to present calculations of the  $\epsilon_{acc}$  and the corresponding  $\tau_T$  in the presence of the three limitations indicated above. These calculations have been performed taking into account non-linear chromatic closed orbit effects as well as energy variation of the optical functions. In addition, the presence of a small vertical aperture in narrow gap undulators may also limit  $\tau_T$  in the presence of transverse coupling. Calculations supporting this result will also be presented.

Finally, we will shortly discuss a method for increasing  $\tau_T$  by acting on the so-called lattice H function.

## 2. NON-LINEAR CALCULATIONS OF THE ENERGY ACCEPTANCE

A Touschek scattering event results in an instantaneous change in particle energy  $\delta$ . As a result the particle reference orbit, initially assumed to be zero everywhere, is suddenly replaced by the chromatic orbit defined by :  $x_{ch}(s) = \eta(s)\delta$ , where  $\eta(s)$  is the dispersion

function. The well-known solution for the particle oscillation envelope subsequent to a collision located at  $s^*$  is given by :

$$x_{max}(s) = \eta(s)\delta + \sqrt{\beta(s)H(s^*)} \delta \quad (1)$$

where :  $H(s^*) = \gamma(s^*)\eta^2(s^*) + 2\alpha(s^*)\eta(s^*)\eta'(s^*) + \beta(s^*)\eta'^2(s^*)$

The first term in (1) is due to the chromatic closed orbit while the second results from induced betatron motion.

We have observed that straightforward application of (1), as done in ZAP [1], is in general insufficient for two reasons. First, with extremely large sextupoles, the chromatic closed orbit is not strictly proportional to  $\delta$  but contains higher order terms which, for the large  $\epsilon_{acc}$  of interest, can have a very significant effect. Secondly the optical functions (but not  $\eta$  and  $\eta'$ ) entering into the expression for  $H(s^*)$ , are those which hold for the energy  $\delta$ . Since these functions all vary with energy, this effect should also be taken into account.

In order to account for these higher order effects in  $\delta$ , a module for the automatic calculation of  $\epsilon_{acc}$  and  $\tau_T$  has been integrated into the BETA code [2]. The algorithm used is as follows :

The general non-linear closed orbit  $\Delta x$ , and the optical functions are all calculated as functions of  $\delta$  and  $s$ .

The dynamic aperture  $A_{dyn}$ , in betatron amplitude, is determined as a function of  $\delta$  using the turn by turn tracking module of BETA.

The horizontal invariant physical aperture is calculated as a function of  $\delta$  by :

$$A_{phys}(\delta) = \underset{s \in [0, s_{max}]}{\text{Min}} \left\{ \frac{(X_{vc} - \Delta x(s, \delta))^2}{\beta_x(s, \delta)} \right\} \quad (2)$$

where  $X_{vc}$  is the vacuum chamber half-width.

These two transverse acceptances are then combined into a single acceptance, which is symmetric and monotonic because of the synchrotron oscillation :

$$A(\delta) = \underset{\delta' \in [-\delta, \delta]}{\text{Min}} \left\{ \text{Min} [A_{phys}(\delta'), A_{dyn}(\delta')] \right\} \quad (3)$$

Next one calculates the invariant betatron amplitude induced by a change in energy occurring at the location  $s^*$  by :

$$a_{induced}(s^*, \delta) = \frac{(\Delta x(s^*, \delta))^2}{\beta_x(s^*, \delta)} + \beta_x(s^*, \delta) \left[ \Delta x'(s^*, \delta) - \frac{\beta'_x(s^*, \delta)\Delta x(s^*, \delta)}{2\beta_x(s^*, \delta)} \right]^2 \quad (4)$$

<sup>1</sup> Work supported by CNRS, CEA, MENESR.

The local transverse energy acceptance is then calculated by solving the equation :

$$A(\epsilon_t(s)) - a(s, \epsilon_t(s)) = 0 \quad (5)$$

Because of the symmetric and monotonic nature of  $A(\delta)$ , this equation can have at most two solutions which we call  $\epsilon_{t\pm}$ . The overall energy acceptance is then given by :

$$\epsilon_{acc\pm}(s) = \text{Min}[\epsilon_{RF}, \epsilon_{t\pm}(s)] \quad (6)$$

Finally the Touschek loss rates are calculated by the well known formula for each of the two solutions and the total loss rate thus becomes :

$$\frac{1}{\tau_{T1/2}} = \frac{1}{2} \left[ \frac{1}{\tau_{T1/2}^+(\epsilon_{acc}^+(s))} + \frac{1}{\tau_{T1/2}^-(\epsilon_{acc}^-(s))} \right] \quad (7)$$

which is averaged over the machine to give the global inverse Touschek lifetime.

### 3. RESULTS OBTAINED IN THE HORIZONTAL PLANE

The calculations have been performed for the standard low emittance (3 nm.rad) and Chasman-Green (9 nm.rad) lattices. Nevertheless, we will give here detailed results obtained only for the standard lattice and mention only briefly the Chasman-Green optics.

Figure 1 shows the dynamic and the physical apertures calculated as a function of  $\delta$ . One can note the asymmetry between the positive and the negative sides of  $\delta$ . For a  $X_{vc}$  less than 35 mm, the physical aperture is always smaller than the dynamic one.

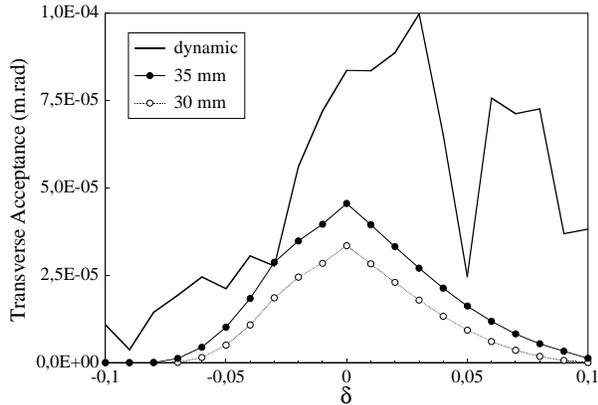


Fig. 1. Dynamic and physical apertures as a function of  $\delta$ .

The induced betatron amplitude calculated as a function of  $s$  and  $\delta$ , using equation (4) also shows asymmetric behaviour. The figure 2 gives a result obtained in the center of a long straight section. The amplitude for positive  $\delta$  is much greater than that for negative values. This is due to the large positive value of the second order dispersion  $\eta_2$  ( $\Delta x = \eta_1 \delta + \eta_2 \delta^2 + \dots$ ) at this location. In the center of the achromat, the amplitude curve is reversed because of the sign inversion of  $\eta_2$  (Fig. 2). Taking into account this asymmetry, the solution of equation (6) will give two different energy acceptance solutions. An example of this result can be seen in figure 3, where the difference is clearly explained by the considerations given above.

One can also note that obviously,  $\epsilon_{acc}$  is the most reduced in the center of the achromat ( $\eta_{x\max}$ ) and in the medium length straight sections ( $\eta_x = 0.13$  m and weak  $\beta_x = 4$  m)[3].

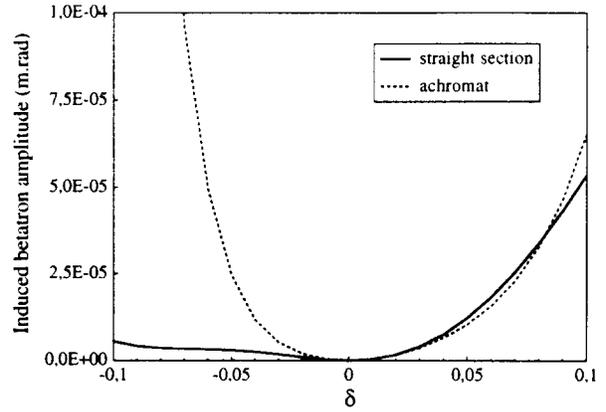


Fig. 2. Example of induced betatron amplitude.

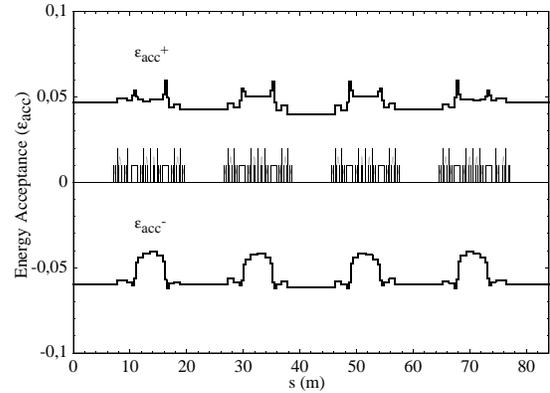


Fig. 3. Energy acceptance variation along the ring.

The difference between the linear and non-linear calculations is illustrated by the figure 4. This points out that the linear calculation of  $\tau_T$  is optimistic (42 h versus 27 h) and that, in our case the contribution of the sextupole effect cannot be neglected. The contributions of the non-linear chromatic orbit and the variation of optical functions with energy have been studied separately and the most important contribution is that due to the first effect.

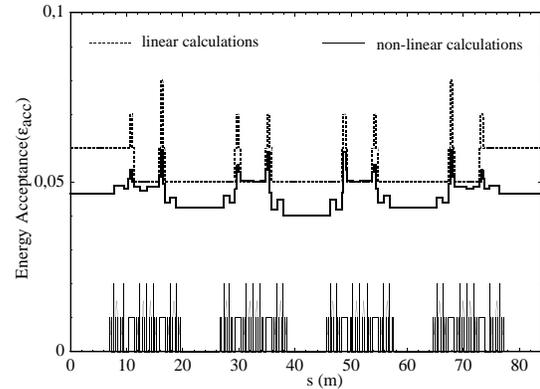


Fig. 4. Linear and non-linear calculations.

The results of  $\epsilon_{acc}$  calculations as a function of the physical aperture in the presence of the two other limitations suggest that a stay clear of  $\pm 35$  mm in the horizontal plane is a reasonable compromise. This corresponds to a  $\tau_T$  of 36 h for the multibunch operation mode [ $I_b = 1.26$  mA ( $I_t = 500$  mA) ;  $\kappa^2 = 0.01$  ;  $\epsilon_{RF} = 5.1$  % ( $V_{RF} = 3$  MV) and  $\sigma_{\ell N} = 13$  ps]. No bunch lengthening was taken into account.

For many technical reasons, the injection septum magnet will be placed inside the vacuum chamber. The calculations show that it must be located at 23 mm at least from the vacuum chamber center in order to avoid any related Touschek lifetime reduction.

In the case of the Chasman-Green optics, the induced betatron amplitudes are almost symmetric in  $\delta$  as are the energy acceptances and loss rates. The difference between the linear and non-linear  $\tau_T$  calculations is only 16 %.

#### 4. RESULTS IN THE VERTICAL PLANE

For the time structure mode, we will operate with a high current in a single bunch or in a few bunches. The  $\tau_T$  will be the dominant contribution to the overall beam lifetime. Normally, one expects that strong coupling between horizontal and vertical betatron motion will increase the bunch volume and therefore increase  $\tau_T$ .

The vertical vacuum chamber is restricted in several places of the ring by the undulators gaps to 15 mm (a vertical vacuum chamber half-height of 6.5 mm) in order to provide high brilliance for photon energies up to 10 keV [4].

Supposing zero vertical dispersion, all vertical betatron motion will be due to transverse coupling and thus the vertical betatron amplitude will be given by :

$$a_z(s^*, \delta) = \kappa^2 a_x(s^*, \delta) \quad (8)$$

The  $\tau_T$  has been calculated as a function of the coupling. In figure 5 we can see that the expected increase in  $\tau_T$  saturates rapidly and falls beyond  $\kappa^2 = 80$  %. This result is directly related to the vertical stay clear at undulator locations.

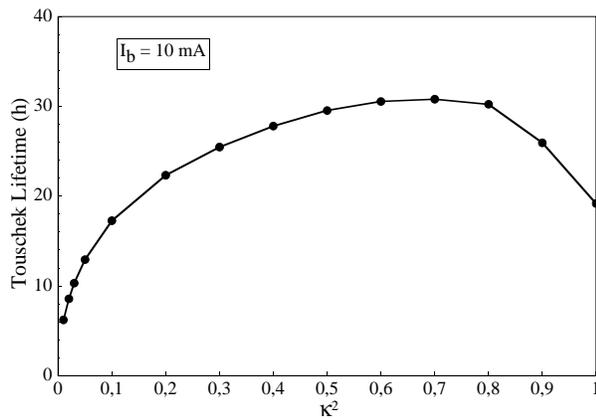


Fig. 5. Tauschek lifetime as a function of the coupling.

#### 5. EXAMPLE OF TOUSCHEK EFFECT OPTIMISED LATTICE

Starting from Touschek scattering calculation generalized to include the effects of dispersion [5], one can say that, for the motion around the chromatic closed orbit,  $\tau_T$  varies approximately as :

$$\frac{1}{\tau_T} \approx \int_s^{s+C} \frac{H^b(s)}{V_B(s)} ds \quad (9)$$

where  $b$  is a constant equal to values between 1 and 4 depending on the range of variation of the  $\xi$  parameter of the well-known function  $C(\xi)$  appearing in the Touschek lifetime expression.  $H(s)$  and  $V_B(s)$  are respectively the so-called  $H$  function (cf. 2) and the bunch volume both calculated everywhere in the ring.

Considering this interesting indication, simulations to minimize  $\int H(s) ds$  have been undertaken keeping the emittance, tunes and betatron functions nearly unchanged. Preliminary results show an 11 % decrease in  $\int H(s) ds$  and a corresponding 22 % increase in  $\tau_T$ , consistent with (9), where in our case,  $b$  is about 2 and  $V_B$  is nearly constant.

#### 6. CONCLUSION

We show that when the Touschek scattering occurs with large energy deviation in dispersion sections, it is necessary to consider the non-linear chromatic closed orbit as well as the variation of the optics with energy. If only the linear effects are considered (ZAP code for example), the calculations can be optimistic by a factor of 2.

In the high intensity bunch mode, the increase of the coupling between horizontal and vertical betatron motions do not give the hoped for increase in Touschek lifetime because of the limited vertical stay clear. Notice that there is no IBS effect at the nominal energy.

To increase the single bunch current threshold we prefer using a feedback system instead of increasing the vertical chromaticity in order to avoid reducing the large energy acceptance.

An interesting reflexion concerning Touschek effect lattice optimization shows that if the dispersion is distributed in such a way that the so-called  $H$  function is minimized everywhere in the machine, the Touschek lifetime can be substantially increased.

Nonetheless further work on this subject is needed as calculations done so far do not completely take into account the effects of non-linear betatron motion nor tune modulation resonances due to synchrotron oscillations.

#### 7. ACKNOWLEDGEMENTS

We would like to mention the fruitful discussions on this subject with our colleagues of the SLS project.

#### 8. REFERENCES

- [1] M.S. Zisman et al., ZAP code.
- [2] J. Payet et al., BETA code LNS version.
- [3] P. Nghiem et al., this conference.
- [4] J.-L. Laclare et al., this conference.
- [5] J. Le Duff, LAL-RT 88-08.