BEAM EXPANSION WITH SPECIFIED FINAL DISTRIBUTIONS*

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Abstract

The formation of nearly uniformly distributed beams has been accomplished by the use of multipole magnets. Multipole fields, however, are an inappropriate basis for creating precise distributions, particularly since substantial departures from uniformity are produced with a finite number of multipole elements. A more appropriate formalism that allows precise formation of a desired distribution is presented. Design of nonlinear magnets for uniform-beam production and the optics of an accompanying expansion system are presented.

1 INTRODUCTION

We consider the general problem of providing an arbitrary spatial beam distribution at a point starting from a given input beam. Such a technique is of interest in matching and for applications where material or power deposition must conform to a particular distribution. For some purposes the requirement is simply to sharply limit the beam extent on a target to prevent undesired radioactivation of surrounding areas. Often a uniform distribution is desired for medical purposes or for minimizing the cooling on a target. Other distributions may be useful in maximizing neutron flux from a spallation target. Although we here concentrate on producing a uniform distribution, the extension to other distributions is straightforward.

The problem of producing a uniform distribution has previously been attacked by using a multipole series to provide a nonlinear field that folds the beam in phase space [1-4]. The results of such processing of an initially gaussian beam by application of a strong octupole field and subsequent magnification optics are shown in Fig. 1.

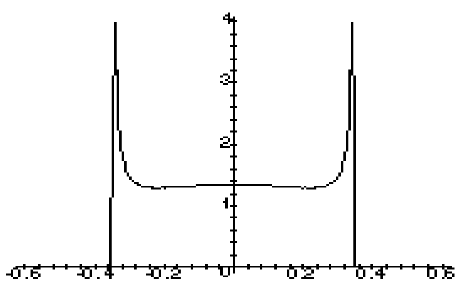

Fig. 1 Beam distribution versus transverse distance after nonlinear focusing by an octupole. Units are arbitrary.

In this process, the beam is expanded within the octupole in, say, the x dimension so that it is very narrow in y to eliminate coupling terms. for sufficient expansion, the effect of the emittance is small and the x, x' phase space may be represented by a line, as was done in the creation of Fig. 1. Figure 2 shows the phase space for the distribution of Fig 1. The "ears" on the distribution are caused by the fold shown in Fig. 2 and are smaller for finite emittance and momentum spread. The distribution is well confined except for the wings that form the lower and upper branches of the field. If further multipoles are added the confinement can be improved.

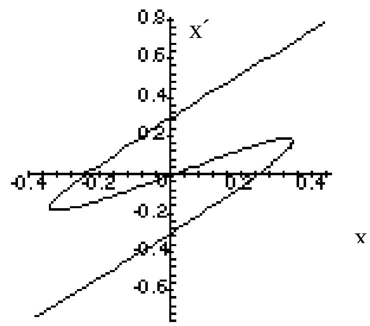

Fig. 2 Phase space for the distribution of Fig. 2. The ordinate is the position in Fig. 1 and the abscissa is the divergence in arbitrary units.

It has been noted [5] that a series of multipoles can be configured to tend toward an analytically exact uniform distribution, but the convergence is slow. We consider a magnetic element that is capable of providing the precise distribution required.

2 MAGNETIC FIELD

Since it is possible to effectively decouple the two transverse phase planes, a one-dimensional treatment is adequate. Beam in an element dx_0 contained in a distribution $\rho_0(x_0)$, described by initial phase-space coordinates x_0 and x_0', transformed to a set of coordinates x and x' obeys the relation $\rho\,(x)\;dx = \rho_0(x_0)\;dx_0$ if the two distributions are single valued. Thence,

$$\rho\,(x) = \frac{\rho_0(x_0)}{dx/dx_0}. \qquad 1)$$

This relation follows Batygin [5] and others.

We consider a small-emittance beam, extended in x_0, x_0' phase space with slope $a = dx_0'/dx_0$ and with small extent in y, that passes through a magnet of length l with y-directed magnetic field $B(x_0)$ on the x_0 axis. A subsequent linear optical system described by a matrix

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R transports the beam to the location described by the x coordinates. Transformation between x and x_0 is

$$x = (R_{11} + aR_{12})x_0 + l\, R_{12} B(x_0) / B\rho \qquad 2)$$

where $B\rho$ is the beam rigidity. Inserting this relation into Eqn. 1), the derivative of the magnetic field with respect to x_0

$$B'(x_0) = \frac{\rho_0(x_0) - (R_{11} + aR_{12})\rho(x)}{l\, \rho(x) R_{12}} B\rho\,. \qquad 3)$$

Integrating, the field becomes

$$B(x_0) = \frac{B\rho}{l\, R_{12}} \int_0^{x_0} du\ \frac{\rho_0(u)}{\rho[x(u)]} - \frac{B\rho(R_{11} + aR_{12})}{l\, R_{12}} x_0\,. \qquad 4)$$

Thus for various choices of beam transport, the fields required to produce a given distribution will vary by a linear term. One natural choice, given a limited beam distribution, is to minimize the peak field by adjusting the linear term. An attractive alternative sets

$$R_{11} + aR_{12} = 0\,, \qquad 5)$$

i.e., a point focus on the target.

For the case of a uniform target distribution of width $2w$, Eqn. 4) becomes

$$B(x_0) = \frac{2w\, B\rho}{l\, R_{12}} \int_0^{x_0} du\ \rho_0(u)\,. \qquad 6)$$

Note that for a limited initial distribution and the choice of Eqn. 5), the field at large x_0 is constant, a convenient field for magnet design. We consider an initially gaussian-distributed beam with rms width σ to obtain a field

$$B(x_0) = \frac{w\, B\rho}{l\, R_{12}} Erf\left(\frac{x}{\sqrt{2}\sigma}\right). \qquad 7)$$

An alternative choice for the initial distribution is $\sim 1/\cosh^2(x/c)$ which yields $w \tanh(x/c)/(2R_{12}c)$, a very similar function to 7) (along the x_0 axis) for $c \sim 1.25\sigma$, differing by only 3% in a limited range. The significance of this is that the latter distribution has been noted to represent a beam that is poorly matched in a linac whereas the gaussian distribution is representative of a well matched beam. Hence, a nonlinear magnet with some adjustability can presumably deal with a range of observed beams.

3 BEAM OPTICS

3.1 One-Dimensional Optics

Consider a drift of length L as an example of an optical system. Then $R_{11} = 1$ and $R_{12} = L$; accordingly set $a = -1/L$. For a given L the peak value of the magnetic field B_0 can be set from the asymptotic value of Eqn 7). Choosing L=1 and w=1 sets $a = -1$ and $B_0 l / B\rho = 1$. Setting $\sigma = 0.1$, the angular deflection through the magnet as a function of x_0 is shown in Fig. 3.

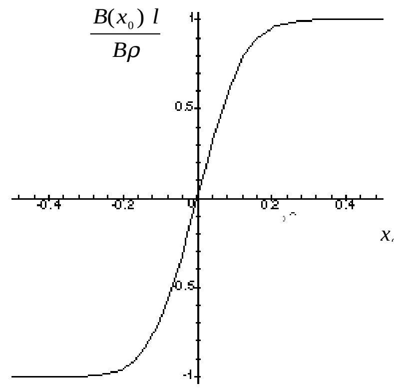

Fig. 3 Plot of the magnetic field along the x_0 axis needed to produce a uniform distribution for the parameters given in the text.

The phase space loci just after the nonlinear magnet and at the target are given in Fig. 4, where the calculation is limited to 6σ. Note that, at the target, the central 2σ of the beam is transformed to within $x = \pm 0.95$ while the remainder of the beam is placed at $|x| > 0.95$ along the trajectory tending asymptotically to the lines $|x| = 1.0$.

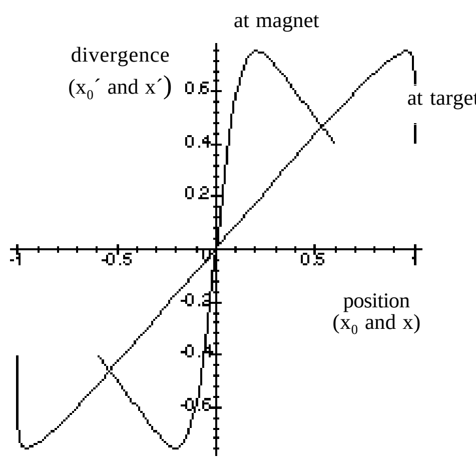

Fig. 4. Phase space at the nonlinear magnet and at the target for the example cited in the text.

Initial and final distributions are shown in Fig. 5. The final distribution departs negligibly from constant within $-w \le x \le w$. Finite bin sizes in the calculation blur the final-distribution edges slightly.

3.2 Errors

We consider two possible errors that affect the final distribution. The first is departures from the initial distribution. In general, if the initial distribution is "wider" than the distribution for which the magnet was constructed, the final distribution will grow "ears" (as in

Fig. 1); "narrower" distributions will experience rounding on top. This is illustrated in Fig. 6 that shows the output distributions for $\sigma = 0.09$ and 0.11. If a $1/\cosh^2$ input distribution is used, similar changes occur in the distribution.

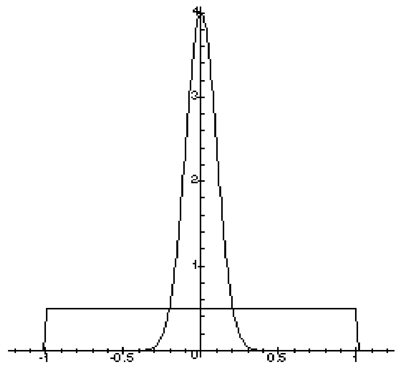

Fig. 5 Normalized initial and final distributions.

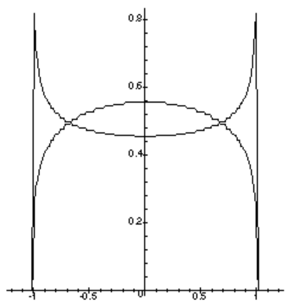

Fig. 6 Final distributions for ±10% width errors in the initial distributions.

Steering errors also change the distribution. This is shown in Fig. 7 where the beam enters the nonlinear magnet off axis by 0.1σ.

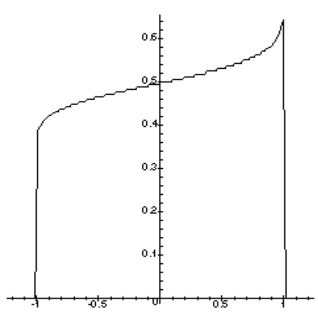

Fig. 7 Final distribution for a steering error of 0.1σ.

Note that despite marked changes in the distribution, the beam remains entirely confined for both these cases because of the well-defined beam limits due to the vertical part of the phase-space trajectory seen in Fig. 4.

5.3 Two-Dimensional Optics Design

Two nonlinear magnets are used in producing a beam that has a given two-dimensional profile on the target. In the first magnet, the beam is made small in the y dimension so that interplane coupling by the field x components is negligible and the beam is made large in the x dimension with large correlation and slope a. A subsequent focusing section provides condition 5) in the x direction with small w in the second linear magnet that is rotated 90° from the first. This limits the beam in the magnet gap. The beam in the y direction is highly correlated (as specified in the x direction of the first magnet). A second focusing section provides the condition 5) from the second magnet to the target in the y direction and from the first magnet to the target in the x direction. For each plane, the values of R_{12} and R_{34} are determined by the desired values of w and the value of B_0, in the respective planes. Such simulations have verified the calculations shown here.

For a given value of B_0 four quads are in general needed in each focusing section to meet all conditions. The number of quads may be reduced by using symmetry, system lengths, and choice of input beam.

5 MAGNET DESIGN

We treat the magnet design only briefly. The magnet has quadrupolar symmetry. For sufficiently low fields, the pole shape along a scalar equipotential is determined from the field in the complex plane. The complex potential provides the conformal map into a dipole geometry, to be used in specifying the pole width or shimming that provides a homogeneous field to the extreme particles of the beam. Variability in the magnet field shape to fit off nominal distributions is provided by dividing the pole into individually excited segments or by current sheets on the pole surfaces.

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