WAKEFIELD DYNAMICS IN QUASI PERIODIC STRUCTURES

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Abstract

The behavior of longitudinal wakefields of very short bunches, excited in in multicell accelerating structures has been studied. Computations were performed in structures proposed for future linear colliders (like TESLA, SBLC and NLC) for bunch lengths down to 50 micrometers. The loss factor, which gives the global energy loss of the bunch, and the profile of the wake function, which is of major interest for bunch energy spread calculations, have been carefully studied. A strong modification of wakefields along the finite train of multicell cavities was clearly found for short bunchlengths. In particular, the wakes induced by the bunch, as it proceeds down the successive cavities, decrease in amplitude and become more linear around the bunch center, with a profile very close to the integral of the charge density. The loss factor per unit length, decreasing also with the number of cavities, becomes independent of bunchlength for very short bunches and goes asymptotically towards a finite value in the permanent regime. However, any break in periodicity (or multi-periodicity) will spoil the wake distribution and hence tends to increase the wakefield amplitude.

1 INTRODUCTION

The wakefields, excited by very short bunches in accelerating structures, are of major concern in the design of new projects for Linear Colliders and X-ray Free Electron Laser, because they can give rise to large energy spreads and transverse emittance growths. Up to now, wakefields calculations had been performed for single TESLA cavities[1]. They showed dramatic bunchlength dependence, scaling approximately inversely with square root of bunchlength. For the TESLA linac and especially for the SASE FEL mode[2], such wakefield behaviors could lead to relatively large uncompensated energy spreads. It is worthwhile noting that the ratio of iris radius over the bunch length, the relevant parameter for wakefield studies, is about 50 for the TESLA linac and even 700 for the SASE FEL parameters. Some analytical and qualitative results (see, for example[3]) predict the behavior of fields induced by short bunches in a single cavity and periodic structure. Unlike long bunches, wake potentials for very short bunches scale inversely with the square root of distance and become infinite at vanishing distance for a single cavity, whereas they are bound for periodic structures. It is then of outstanding importance to know when a chain of cavities behaves like a periodic structure or like a single cavity. The three regions, single cavity, transient and permanent regime were studied by means of numerical simulations on different structures. The results on the train of TESLA cavities will only be presented here. Unfortunately, conventional time-domain codes need very tiny mesh sizes to cope with the very high frequency part, generated by short bunches, leading to unreasonable long CPU times and poor accuracy[4]. For successful numerical wakefield calculations, a time-domain code, called NOVO, was then especially developed for very short bunches by one of the authors. For solving Maxwell's equations in time domain, the finite-difference method with improved characteristic of the dispersion curve in the region of minimum critical wavelength was used[5]. Lastly, an 'universal' analytical expression, valid for periodic structures, has been compared to the numerical results. We found that the couple of adjustable parameters, are very smooth functions of the exact geometry for multi-periodic structures like TESLA.

2 SINGLE-CELL APPROACH

Starting from the TESLA cavity, which is composed of 9 cells, we consider the single-cell approximation, where the different cells of a cavity are assumed independent. For relatively long bunches, this approximation is justified and can help in understanding the wakefunction evolution with bunchlength. Two parameters can be used to describe the "smallness" of the bunch length: the aperture radius a and s_0, which is defined for a cell with gap length g as $s_0 = a^2/2g$. In the case of a single cell with simple rectangular shape, for the distances smaller than the aperture radius $s < a$, it is possible to derive a good analytical estimation of the point-like wakefunction[6]

$$w(s) = \frac{Z_0 c}{a\pi^2}\{\frac{s+g}{\sqrt{s(s+2g)}}F(\frac{\sqrt{s(s+2g)}}{a}) - F(\frac{s}{a})\}$$

where the function $F(x)$ is given by

$$F(x) = \begin{cases} (2/x)\arcsin(x/2) & \text{if } x \le 2 \\ (\pi/x) & \text{if } x > 2 \end{cases}$$

The bunch wake potential $W(s)$ induced by a Gaussian bunch with a bunchlength σ = 0.5 mm passing through a TESLA-like cell, approximated by a rectangular shape (a=35 mm, g=90 mm and b=103.3 mm), has been calculated from the convolution of the above wakefunction formula with the bunch charge density and from time-domain simulations. Both results are shown on Fig. 1 and are in very good agreement. For much shorter bunches, when $s << s_0$ (s_0 = 5.3 mm for the TESLA cell), the well known result of K. Bane and M. Sands[3] can be found again from the above expression

$$w(s) = \frac{Z_0 c}{2\pi^2}\frac{1}{\sqrt{s_0 s}}$$

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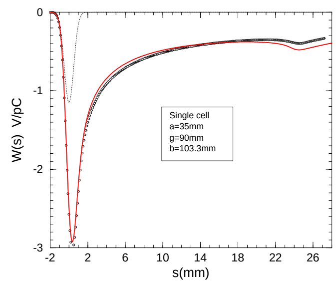

Figure 1: Wake potential for a rectangular TESLA-type cell: numerical (circles) and analytical results (solid lines).

For a single cell, the wakefunction (bunch wake potential) varies inversely as the square root of the distance (bunchlength). A slight discrepancy between numerical simulations and theoritical scaling with bunchlength was found in bell-shaped cells like the TESLA cavity, and can be explained by the following. The more the bunch is short, the less the fields penetrate the elliptical iris region. We can define an "equivalent gap", which grows (from 88 to 100 mm) when the bunch shortens (from 1 to 0.1 mm).

3 MULTI-CELL APPROACH

Whereas the field pattern around the bunch is assumed unchanged before and after its travel through a cell in the single-cell approximation (the total energy loss in multi-cell structures would be then simply the sum of the losses in the individual cells), this assumption is not any more valid for short bunches. For example, the electric force lines left by the bunch in the exit beam tube of a single-cell cavity are represented on Fig. 2 for two bunchlengths σ = 2 mm and σ = 100 μm. The electric force lines are almost radial for the longer bunch, as they were before the cell, whereas a lot of lines lie horizontal and do not touch the wall of the beam tube in the vicinity of shorter bunch. The next cell will then be excited by the bunch of "extended length" and the radiation energy will be smaller than in the previous cell. After several cells, the electric force lines in the iris region reach a "steady-state regime", as if they were induced by an equivalent longer bunch. The number of cells N needed to achieve this periodic behavior depends on the bunchlenth σ : the more the bunch is short, the more the number of cells must be large. It be estimated from a simple geometrical model of the radiation process for a structure of period L : $N > a^2/2\sigma L$. Time domain simulations were performed

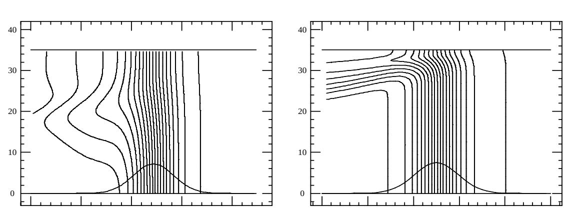

Figure 2: Electric force lines of the bunch field just after one TESLA cell for σ=2 mm (left), σ=0.1 mm (right)

on a periodic structure, consisting of regular TESLA cells. Fig. 3 shows the loss factor per unit length for a bunchlength of 0.2 mm as a function of the number of involved cells and converges to a finite constant value after a large enough number of cells (steady-state).

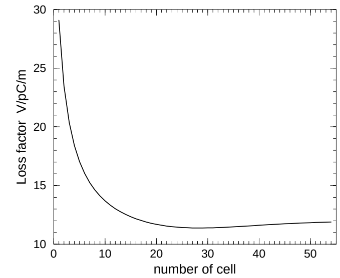

Figure 3: Loss factor vs number of cells in a chain of regular TESLA cells (σ = 0.2 mm).

In the region of very short bunches, the loss factor does not nearly depend on the bunchlength and is given by $k = Z_0 c/2\pi a^2$. However, more than 50 cells are needed to achieve the stabilization of the wakefunction. It was found that for any periodic structure, the wake potential can be very well described by the following expression, as suggested by K. Bane[7]

$$w(s) = \frac{Z_0 c}{\pi a^2}((1+\beta)\exp(-\sqrt{\frac{s}{s_0}}) - \beta)$$

The parameter β is 0.16 and $s_0 = a^2/2L$.

Figure 4 shows wake potentials, calculated for the 0.2 mm long bunch for different number of cells (from the sixth to the tenth 9-cell regular TESLA cavity, as well as the analytical expression. A very good agreement is clearly observed : from the head to about one σ after the bunch center with a fast convergence of the wakes and at the tail with damped oscillations of the wakes around the analytical curve.

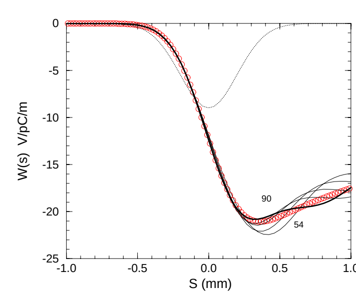

Figure 4: Wake potentials induced in cell numbers 54, 63, 72, 81, and 90 (dark line) and the analytical expression (circles) in the periodic TESLA structure (σ=0.2mm).

4 MULTI-PERIODIC STRUCTURE

If the multi-cell approach gave a good estimation of the wakes induced in periodic structures, the TESLA linac is far from being purely periodic. It is in fact a multi-periodic structure : a first elementary period is the regular bell-shaped cell, a second one is the 9-cell cavity, connected to a beam tube and a third one is the module, housing 8 cavities and a different connecting tube. In addition, some extra effects, like the larger tube diameter with respect to the iris aperture, must be taken into account. When the bunch leaves a cavity and enters the drift tube, some field must be created to fill in the space enclosed between the two different radii of 35 and 39 mm. First, wake potentials were calculated for a TESLA module, housing eight 9-cell cavities and beam tubes. The successive wake potentials, induced in each of the individual cavities and their beam tube, are shown on Fig. 5 (left) for a bunch of length σ = 0.2 mm. In the same way, the wake potentials, calculated for the periodic structure, composed of regular TESLA inner-cells, and without spacing between cavities are reproduced on Fig. 5 (right) for comparison. We note that the shapes are identical, but with larger amplitudes per cavity in the real multi-periodic TESLA structures. For the multi-periodic

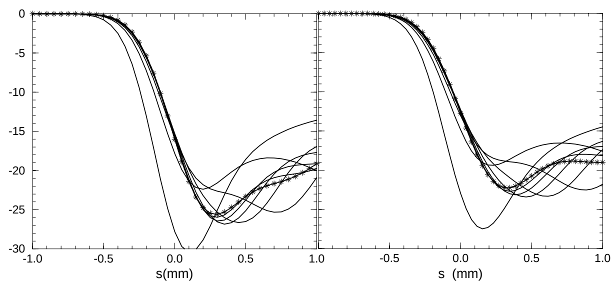

Figure 5: Wake potentials induced by each of the cavities (σ=0.2mm) in first real TESLA module (left) and in periodic structure composed of regular cells (right).

structure, we found that the wake potential per unit length can be described by the same previous expression, valid for the periodic structure, but with two additional parameters

$$w(s) = A\frac{Z_0 c}{\pi a^2}((1+\beta)\exp(-\alpha\sqrt{\frac{s}{s_0}}) - \beta)$$

The parameters $A = 0.94$, $\alpha = 1.33$ and $\beta = 0.18$ were found from the fit of the loss factors, computed for different short bunches. The wake potentials induced in the last 8th cavity of the module are plotted on Fig. 6, as well as the analytical results for two bunchlengths, 0.2 mm and 0.05 mm. We can note that the steady state is practically achieved for the 0.2 mm long bunch, but a larger number of cavities is needed for the shortest 0.05 mm bunch. However, in the reality, the spacing between the last cavity of a module and the first one of the next module will be larger than the standard spacing (3 rf half-wavelengthes) between two successive cavities. The bi-periodicity is broken again and as a result, the wakefields will be reinforced. This effect can be observed on Fig. 7. At the left side, the wake potentials for a 0.05 mm bunch are drawn for each of the eight consecutive cavities in a first module. It is worthy to note the amplitude, decreasing with the cavity index. At the right side, they are drawn for the next 9th cavity, with a same

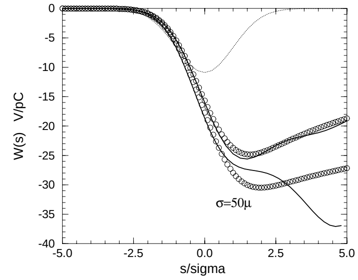

Figure 6: Wake potentials in the 8th cavity of the TESLA module for (σ=0.2 mm and σ=0.05 mm) and analytical results (circles).

standard spacing (as if it belonged to the previous module) and with the real additional spacing between two modules.

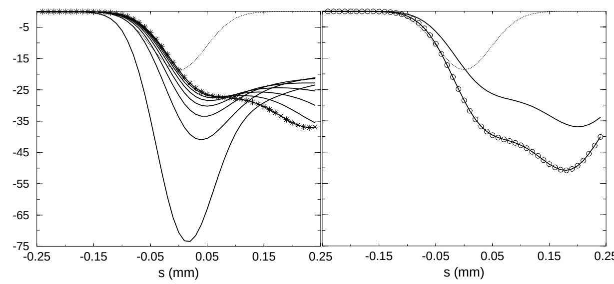

Figure 7: Wake potentials in the 8 cavities of a module (left) and in a 9th cavity (right), with standard spacing and with real spacing (circles)

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