

BUNCH SPACE CHARGE LIMITS IN AN RTM*

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Abstract

We consider analytically and by computer simulation the limits imposed by space charge forces on the charge per bunch, which can be accelerated in an RTM. For this we took into account ordinary space charge forces and coherent synchrotron radiation forces. We present numbers for space charge limit estimations especially for the MAMI-RTMs.

1 INTRODUCTION

CW RaceTrack Microtrons (RTMs) offer the possibility to get a very low transverse and longitudinal emittance beam with an average current appropriate for nuclear physics experiments. At the same time the peak current at CW operation, even for picosecond bunches, is too low for applications of these machines e.g. as an FEL-driver. Subharmonic, or as an extreme, single bunch RTM operation is a way to get high peak current. However, different instabilities, caused by space charge forces, transient beam loading of the accelerating structure and excitation of transverse parasitic modes can lead to emittance growth and beam blowup. Here we consider analytically and by computer simulation the limits for the charge which can be accelerated in an RTM, imposed by the space charge forces. We present space charge limit estimations especially for the Mainz Microtron (MAMI) RTMs.

2 MANIFESTATION OF THE SPACE CHARGE FORCES IN AN RTM

The cascade of MAMI CW RTMs consists of a 3.5 MeV injector linac and three RTMs, with output energies of 14, 180 and 855 MeV respectively [1,2]. Because of the low accelerating gradient in normal conducting CW linacs and the large number of orbits, the total path length in these accelerators is about, respectively, 90 m, 745 m and 2700 m. So, ordinary space charge forces, decreasing as a square of beam energy, should be taken into account when considering a high bunch charge especially for RTM1 and RTM2. These space charge forces do not change the bunch energy, but lead to an effective emittance growth: directly, owing to their nonlinear character and indirectly by the introduced energy spread. In the latter case a horizontal (bending plane) emittance growth takes place because of the

energy change inside and between the 180° end magnets, and a longitudinal effective emittance growth because of the strong nonlinearity of phase oscillations in the RTM.

Building a new injection system with appropriate beam energy, specially designed for high space charge operation, it is possible to make the role of ordinary space charge forces negligible, but other types of space charge forces, nearly independent of beam energy, come into play.

These forces are connected with the transverse particle acceleration when moving along the circular orbit in bending magnets. We consider here only the better studied and indirectly experimentally verified [3] longitudinal coherent synchrotron radiation (CSR) [4] force. The RTM with its short electron bunches, small bending radii and large number of orbits presents good conditions for a manifestation of this force. CSR forces change the average bunch energy and introduce a strong nonlinear bunch energy modulation. The average energy change leads to phase oscillations of the bunch as a whole, and energy change within it to a horizontal and effective longitudinal emittance growth.

Estimations based on the formulas given in [5] show, that for RTMs the transverse "centripetal and collective focusing" forces can also play a definite role in emittance growth, but less than the longitudinal CSR force.

3 SPACE CHARGE LIMITS FOR THE ORDINARY SPACE CHARGE FORCES

A rough limit for the bunch charge can be obtained from the consideration of longitudinal phase oscillations in an RTM, mostly sensitive to repulsive space charge forces [6]. The synchronous particle at the center of a longitudinally symmetric bunch is not influenced by these forces. Later particles at the rear of the bunch decrease their energy while those at the head increase it. In this respect space charge forces act on nonsynchronous particles similar to the RTM linac field. To estimate the energy change δE_{sc} per orbit by space charge forces, of a particle with phase deviation $\delta\phi$ from synchronous, we use as a model of the bunch a uniformly charged ellipsoid. In the MAMI RTMs transverse and longitudinal bunch dimensions are close to each other. So for a relativistic beam we have:

$$\delta E_{sc} \approx \frac{3q_b \pi L_{orb} \delta\phi}{\epsilon_0 \gamma^2 \lambda^2 \delta\phi_{max}^3} [\ln 2\gamma - 1] , \quad (1)$$

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where q_b - bunch charge, L_{orb} - orbit length, γ - mid of linac relative beam energy, λ - rf wavelength, $\delta\phi_{max}$ - maximum bunch halflength, defining a longitudinal parabolic charge distribution: $\psi_p(\delta\phi)=3[1-(\delta\phi/\delta\phi_{max})^2]/(4\delta\phi_{max})$. Formula (1) is valid under the assumption: $\gamma\lambda\delta\phi_{max}/2\pi\approx\gamma\sigma \gg \sigma$, where σ - average beam radius, which is always fulfilled in the cases considered below.

Phase oscillations in the RTM, changing particles position within the bunch, decrease the energy spread caused by space charge forces. In linear approximation these oscillations are stable for the bunch charge less then [6]:

$$q_b^{lim} \approx \frac{4\epsilon_0\lambda^2\gamma^2\delta\phi_{max}^3\Delta E_s}{3\pi eL_{orb}[\ln 2\gamma - 1]}\xi, \quad (2)$$

$$\xi = \frac{1}{2\pi\nu} \left(1 - \frac{\pi\nu \tan \phi_s}{2} + \sqrt{1 + \frac{(\pi\nu \tan \phi_s)^2}{2}} \right) \quad (2a)$$

where ΔE_s - synchronous energy gain per orbit, ϕ_s - synchronous phase and ν - increase of orbit length per turn in number of wavelengths (for MAMI $\phi_s=16^\circ$, $\nu=1$). In estimations for q_b^{lim} the parameters of the first orbit should be used, as the influence of space charge forces decreases as $1/\gamma^2$. As can be seen from (2), the most essential factors which can be adjusted for a given RTM-design to increase the charge are bunch length and injection energy.

To get a more accurate estimation for charge per bunch limits and also an estimation of transverse emittance growth due to energy change in the 180° -magnets, we introduced into the RTMTRACE code [7] space charge calculations based on the model of the uniformly charged 3-D ellipsoid, and made beam dynamics calculations for the injector linac, RTM1 and RTM2. We should stress, that in this model space charge forces are proportional to the particle deviation from the bunch center, and thus directly do not contribute to the effective emittance growth. Effective emittance growth due to nonlinearity of space charge forces depends essentially on the details of the real charge distribution and can be estimated separately [8]. Table I presents limits for the bunch charge estimated with formulas (2,2a) and those calculated with RTMTRACE. In both cases the results were obtained for $\delta\phi_{max}=0.035 \text{ rad}\approx 2^\circ$. The injector linac was also calculated with the PARMELA code.

As criterion of the limit for bunch charge we considered an abrupt growth of the longitudinal emittance caused by nonlinear oscillations. Below this limit the transverse emittance growth is negligible, though just at the threshold the horizontal emittance grows by a factor of 2-4. Estimations of transverse emittance growth $\Delta\epsilon_{tr}$ due to the nonlinearity of space charge forces, based on [8], for the first orbits are also given at Table I.

Thus, to avoid effects of ordinary space charge forces at MAMI for a reasonable charge per bunch, say about 100 pC, a new injection system should be built with an energy of at least 30 MeV.

Table I. Injected beam normalized rms transverse and longitudinal emittances used in calculations, transverse emittance growth and limits in pC for bunch charge imposed by ordinary space charge forces.

	ILAC	RTM1	RTM2 ^c	RTM2 ^d	RTM3
$\epsilon_x, \mu\text{m}$	0.08	1	1	1.4	-
$\epsilon_y, \mu\text{m}$	0.08	1	1	1.4	-
$\epsilon_L, \text{kV}\cdot\text{deg}$	0.5	0.7	12.5	12.5	-
$\Delta\epsilon_{tr}, \mu\text{m}$	0.4-1 ^e	3	0.8	0.2	
$q_b^{lim \text{ a}}$	-	2.7	43	150	4000
$q_b^{lim \text{ b}}$	0.4-1.3	3-4	30-40	100-140	-

^aCalculated with formulas (2,2a), ^bRTMTRACE ^c,^d 14 resp. 30 MeV injection, ^ePARMELA

4 EFFECTS OF THE CSR FORCES

The spectrum of synchrotron radiation, including the coherent part, for a bunch of N particles with Gaussian longitudinal distribution, moving in free space in a uniform magnetic field is given by [9,10]:

$$\frac{dW}{dy} = \frac{Nce^2}{6\pi\epsilon_0 R^2} \gamma^4 \varphi(y) \left(1 + (N-1)e^{-\frac{9}{4}(y\frac{\delta_z}{R}\gamma^3)^2} \right) \quad (3)$$

$$\varphi(y) = \frac{9\sqrt{3}}{8\pi} y \int_y^\infty K_{5/3}(x) dx, \quad y = \frac{4\pi R}{3\lambda_r \gamma^3}$$

Here δ_z - $1/2$ rms bunch length, R - bending radius, $K_{5/3}$ - modified Bessel function, λ_r - wavelength of radiation; and as bunch distribution function is supposed:

$$\psi(z) = \frac{1}{\sqrt{2\pi}\delta_z} e^{-\frac{1}{2}\left(\frac{z}{\delta_z}\right)^2} \quad (4)$$

Fig. 1 shows examples of synchrotron radiation spectra calculated for different orbits in RTM2. To have correspondence in bunch length with the ordinary space charge calculations, the parameter δ_z in (4) here and further was chosen such, that parabolic and Gaussian distributions have equal height ($\delta\phi_{max}=2^\circ$, i.e. $\delta_z\approx 0.36$ mm).

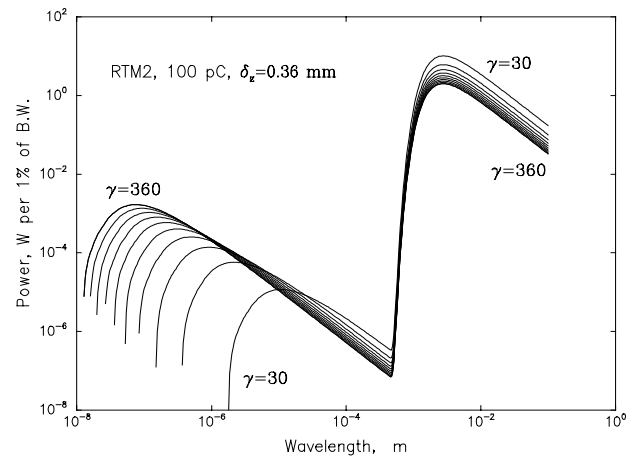


Fig. 1. Spectra of synchrotron radiation for MAMI RTM2 (100 pC, $\delta_z=0.36$ mm bunches).

The approximation of free space, used in (3), is valid for less than 1 mm long bunches moving in a vacuum

chamber with several cm height and several m width at the MAMI RTMs. The position of the maximum of CSR is beam energy independent and determined by bunch length, $\lambda_r \approx 2\pi\delta_z$. The total power radiated by the bunch (4) due to this coherent effect is also energy independent for constant bending radius and is given by [11]:

$$W_{coh} = \frac{\Gamma(5/3)q_b^2 c}{\Gamma(4/3)8\sqrt[3]{3}\pi\epsilon_0\delta_z^{4/3}R^{2/3}} \quad (5)$$

Because the RTM magnet field is the same for all orbits, the radiated power decreases as $1/\gamma^{2/3}$. For the first orbit of RTM2 with bunch parameters as in Fig.1, $W_{coh} \approx 1770$ W, while the power of incoherent radiation, $W_{in} \approx cq_b\epsilon_0^4/(\sigma\epsilon_0 R^2)$ is only 2.7 mW. Average particle energy losses by synchrotron radiation per one 180° -magnet, calculated with the relation $\delta E_{sr} = W\pi R/c$ are shown at Fig. 2 for RTM2 and RTM3.

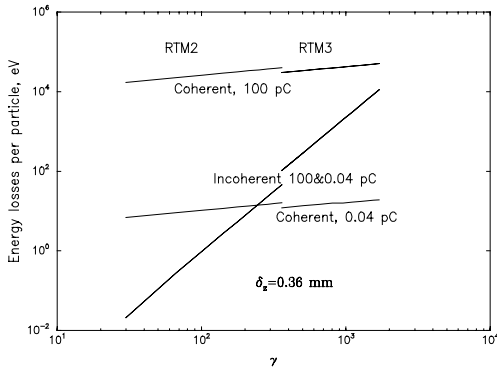


Fig. 2. Average particle energy losses due to coherent and incoherent synchrotron radiation for RTM2 and 3.

The average CSR energy losses are growing as $\gamma^{1/3}$, with a jump from RTM2 to RTM3 because of the magnet field change. These losses for 100 pC charge change from 17 to 40 KeV for RTM2 and from 30 to 50 keV for RTM3, as compared with maximum incoherent radiation losses of about 11 keV. The CSR losses will lead to bunch phase oscillations, which however can be partially compensated by RTM tuning.

More important is the fact that, in contrast to incoherent radiation, CSR energy losses strongly depend on particle position within the bunch. The longitudinal force acting on a particle at position z with respect to the center of a Gaussian bunch is [12]:

$$F_L = \frac{q_b e \chi(p)}{2^{5/3} 3^{1/3} \pi^{3/2} \epsilon_0 \delta_z^{4/3} R^{2/3}}, \quad p = \frac{z}{\sqrt{2}\delta_z} \quad (6)$$

$$\chi(p) = \frac{e^{-p^2}}{6} \sum_{n=0}^{\infty} \frac{\Gamma(n/2 - 1/6)}{n!} (2p)^n$$

Fig. 3 shows a comparison of the particle energy change by CSR forces and by ordinary space charge forces for RTM2. Results are given per one orbit: two 180° -magnets + two drifts between them. The distance between RTM2 end magnets is about 5 m, first and last orbit bending radii are about 0.09 m and 1.1 m. One can see, that the maximum energy change by ordinary space

charge forces for the existing injection energy (14 MeV, $\gamma \approx 30$) exceed that by CSR forces for all orbits, but for energies above 70 MeV ($\gamma \geq 140$) the CSR forces will play a major role.

The change of the particle energy within the bunch will modify RTM phase oscillations, but now a linear approximation cannot be used to get limits for the bunch charge; this can only be done with a computer simulation which is in progress by introducing into the RTMTRACE code both longitudinal CSR forces and the transverse space charge forces arising in bends.

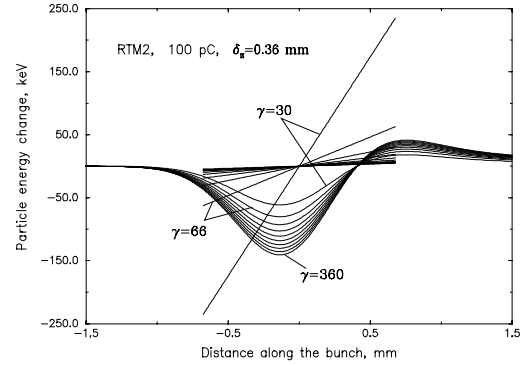


Fig. 3. Particle energy change within bunch, per one RTM2 orbit. Straight lines - ordinary space charge forces, curved - CSR forces.

To get crude numbers for charge per bunch limits by CSR forces we can compare the energy spread introduced by them with that by ordinary forces. The CSR-spread is maximal for the last orbit of RTM3, and is about 1.5 times higher than that from ordinary forces at the first RTM2-orbit with 30 MeV injection. Thus the limit for charge per bunch can be at least 1.5 times lower. - In principle the influence of CSR forces can be essentially reduced by decreasing the vacuum chamber height to the order of the bunch size.

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