# MEASUREMENT AND SIMULATION OF NONLINEAR BEAM-BEAM MODE COUPLING IN THE FERMILAB TEVATRON

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# Abstract

The Tevatron beam dynamics has been studied from the "weak-strong" representation of the beam-beam effects for various octupole field settings. A single particle is tracked using TEAPOT in the presence of the other beam. The nonlinearities introduced by the beam-beam forces introduces two unwanted effects: (1) it excites nonlinear resonances; (2) it introduces dispersion of the tunes with amplitude. The strength of nonlinear three mode coupling satisfying the sum rules  $v_3=v_1\pm v_2$  for various octupole settings are calculated. The higher order spectral analysis results are presented.

# **1. INTRODUCTION**

The Fermilab Tevatron is a 1.8 Tev/c center of momentum proton-antiproton collider, delivering a peak luminosity greater than 2.E31 cm<sup>-2</sup>sec<sup>-1</sup>. Tevatron operating with six protons and six bunches colliding at CDF and D0 interaction points does not seem to be limited by beambeam effects. Average intensity of proton and antiproton bunches are about 25E10 and 8E10 respectively. The Fermilab III accelerator complex upgrade, including the Main Injector will increase the peak luminosity to 10E32 cm<sup>-2</sup>sec<sup>-1</sup>. Higher luminosity will be achieved by injecting more proton and antiproton bunches with similar intensities. Number of bunches will increase from 6x6 to 36x36 and eventually to ~109x109, to keep the number of interactions per crossing at each high energy detectors at a manageable level.

The purpose of this paper is two folds: (i) to introduce higher order spectral analysis to visualizes turbulent or chaotic data to the accelerator physics community, (ii) to apply octupole field to reduce the beam-beam tune shift of high amplitude particles. We investigate the combined effect of long range interactions and higher order multipoles to calculate the nonlinearly generated frequency spectrum. In this paper we discuss three cases. First we restrict ourselves to syncro-betatron and betatron-betatron coupling of a single beam with betatron tunes close to the operating tunes at collision when higher order multipoles are present but no applied octupole field strength,  $(k_3)$  Second, beambeam with two positive settings of octupole field strength. Finally beam-beam with negative octupole field which results in resonance extraction are analyzed. A modified version of thin element tracking program TEAPOT has been used ro simulate the turn by turn data for Tevatron lattice.

#### 2. ANALYSIS

Higher order spectral analysis has been used to calculate the bicoherence spectrum of the three-wave coupling contributing to he characteristics of beam-beam coupling in the Tevatron. The *bicoherency* is defined as

$$b^{2}(v_{1},v_{2}) = \frac{B(v_{1},v_{2})^{2}}{\langle X_{v1}X_{v2} |^{2} \rangle P(v)}$$

where B(v1,v2) is the bispectrum defined as B(v1,v2)= $\langle X_{v1}X_{v2}X_{v3}^* \rangle$ , the power spectrum is  $P(v) = \langle X_{v1}X_{v2}^* \rangle$ , and Xv is the Fourier transform of the turn by turn data X(t), and  $\langle \rangle$  means ensemble averaging over many statistically similar realizations. The bicoherency measures the fraction of the fluctuating power at the frequency v3 which is phase correlated with the spectral components at frequencies v1 and v2 obeying the summation rule  $v_3=v_1\pm v_2$ . The bicoherency thus qualifies the degree of coupling among three waves (modes). The bicoherency is bound between 0 and 1: when  $b^2$  is equal to 1 then the oscillations at frequency v3 are completely coupled with frequencies v1 and v2 and completely decoupled for the zero value. The use of the bicoherence as a measure of three wave coupling is independent of any closure assumption. The bicoherence is nonzero if there is a statistically significant phase relation between the three modes.

#### 3. CALCULATIONS AND RESULTS

During Run II and TEV33 the Tevatron will be filled with three bunch trains each containing either 12 or 36 bunches respectively. The detector readout requires a gap between bunch trains of 2.5µsec. Due to this gap the bunches at the

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two ends of each bunch train will encounter different amont of beam-beam kicks. During Run II and Tev-33 the antiproton beam will occupy a much larger tune space due to larger tune shift and uneven bunch tune shifts. This will require us to adjust the tune of the Tevatron during the opposing beam injection process. Fig.(1) is an example of dependencies of tunes on intensity with and without headon collision.



Fig.1) Betatron tune as a function of bunch intensity.

As expected the head-on tune shift is larger by about a factor of four at the bunch intensity of 3.0E11. In the Tev-33 operations the number of bunches are going to increase from 36 to 108, hence the number of long range beam-beam interactions will increase from 70 to 214. If we consider that the long range tune shift will increase linearly, the long range beam-beam tune shift will be approximately of the same size as head-on tune shift. Further due to higher number of closely placed bunches the number of near misses and reduced separations between the two beams will increase, contributing more to beam-beam tune shift. As



Fig.2) Tune-Tune Foot-print of the beam in presence of beam-beam effects.

such, we need to consider ways to reduce the beam-beam tune shift. Fig.2 shows a tune foot print of the beam in presence of the beam-beam effects for 36x1 case.

## 4. REDUCTION OF TUNE-TUNE FOOT PRINT

We are investigating several options to reduce the tunetune foot print. The presence of the octupole component in the lattice causes an amplitude dependent tune shift. The octupoles can also be used to reduce the effect of beambeam tune shift. The octupoles can also be used to reduce the effect of beam-beam tune shift by introducing the tune shift of opposite sign. We have used the octupole magnets present in the Tevatron to study this idea. Of course, distribution of the octupoles can be optimized. In a detail study, we will need to place more octupoles at different locations in the lattice. Figure 3 shows the tune of launched particles as a function of its amplitude.



Fig.3) Tune-Tune footprint of the beam in presence of beam-beam effects with and without octupole fields.



Fig.4) Effect of octupole field on tunes.

It is clear from these calculations that octupoles can be used to reduce the tune-tune foot print, but the details of number, strength and their placement in the lattice need more detailed simulations. From Fig. (4), it is clear that octupole fields introduce coupling and as such, the usual phasespace plots are not a reasonable representation of visualizing beam structures. betatron coupling in horizontal plane due to beam-beam and the helical closed orbit (compare to Fig.6 which octupole fields are turned off). Peak (D) on the bottom section of figure 5 is due to the octupole field and arises at the difference vx-vy resonance line.



Fig.5) (Top) Horizontal self-coupling and synchrobetatron coupling, (middle) vertical self-coupling and synchro-betatron coupling, (bottom) cross-coupling of horizontal to vertical tune-tune coupling are shown.

In Figure 5, the octupole field strength is 15 units. This figure is composed of three parts, the top section is the representation of horizontal synchro-betatron coupling shown by Peak (A). Peak (B&C) shows the betatron-



Fig.6) Bicoherence of beam-beam without any applied octupole field.



Fig. 7) Applied negative octupole field causes beam loss due to difference resonance and ½ integer extraction.

If we change the sign of the octupole field, we observe particle losses and the bicoherency shows the difference tune resonance and ½ integer extraction (Fig.7).

## 5. SUMMARY

In this paper we have presented the initial results from an study to understand and visualize the effect of beam-beam as octupole field strength is varied. A closer attention needs to be paid by changing the Tevatron operating procedures to overcome the effects of increasing beambeam forces. We have introduced an idea to reduce he size of the tune foot-print.

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