

# SPIN TRACKING IN RHIC

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## Abstract

In the acceleration of polarized protons in RHIC [1] many spin depolarizing resonances are encountered. Helical Siberian snakes [2] [3] will be used to overcome depolarizing effects. The behaviour of polarization can be studied by numerical tracking in a model accelerator, That allows one to check the strength of the resonances, to study the effect of snakes, to find safe lattice tune regions, and finally to study the operation of special devices like spin flippers. In this paper we describe numerical spin tracking. Results show that, for the design corrected distorted orbit and the design beam emittance, the polarization of the beam will be preserved in the whole range of proton energies in RHIC.

## 1 SPIN RESONANCES

Spin resonances that may lead to the loss of polarization [4] [5] [6] are of two main species: *intrinsic* resonances that depend on the lattice structure of the ring and arise from the coupling of betatron oscillations with horizontal magnetic fields, and *imperfection* resonances caused by orbit distortions. In both cases, for vertical polarization (as in RHIC), the vertical motion of the beam is the main responsible for depolarization. If  $P$  is the superperiods of an accelerator, and  $\nu_y$  the vertical betatron tune, along the acceleration cycle intrinsic resonances are located at  $G\gamma = nP \pm \nu_y$  and imperfection resonances at  $G\gamma = k$ , with  $G = 1.7928$  the proton gyromagnetic ratio, and  $n$  and  $k$  integers. Resonance strength is proportional to the amplitude of the vertical motion of the beam, produced either by betatron oscillations or closed orbit distortion, then, ultimately, to the emittance of the beam. The strength is proportional to the energy of the beam  $\gamma$  for imperfections and to the square root of the energy for intrinsic.

Effects on depolarization are also due to the betatron horizontal and synchrotron longitudinal oscillations and to other causes, like the beam-beam interactions that take place in colliders. Resonances may also be enhanced in the vicinity of special fractional value of the betatron tune.

## 2 SPIN TRACKING

For spin tracking, we used an ad-hoc written computer code *Spink* [7], that follows an ensemble of particles through an accurate representation of the actual lattice of RHIC. Among special lattice modules are the helices of the Siberian snakes and spin rotators needed to bring the vertical polarization to a longitudinal orientation at the colliding points in RHIC. The most elegant representation of spin is

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in the 2-dimensional space through spinors and Pauli matrices. However, in the code we found it more convenient to represent the spin as a real vector in the ordinary space.

Tracking, hundreds particles for a number of revolutions of the order of 100,000 through a thousand matrices takes a computer time of the order of a few hours for a typical fast workstation. Some of the tracking with many particles has been performed on a supercomputer.

### 2.1 Orbit Matrices

*Spink* tracks a number of protons randomly generated in phase space, through the machine lattice. A proton is characterized by four transverse coordinates,  $x, x', y, y'$  and by two longitudinal coordinates  $dp/p, d\phi$ . Matrices used to transform orbit coordinates are built from the output of the code *Mad* [8]. We only retain the relevant ("active") modules for orbit tracking; everything else in the lattice is lumped in drift modules. Typically, for RHIC the number of *Spink* matrices is 981, each active element being surrounded by two drifts.

Acceleration by rf cavities is represented by a matrix in which the longitudinal momentum of a proton is changed at the expense of the transverse momentum [9]. In a cavity, the emittance of the beam changes as  $\epsilon = \epsilon_N / \beta\gamma$ , with  $\epsilon_N$  the normalized emittance.

### 2.2 Spin Matrices

In the spin space, a proton has three coordinates  $S_x, S_y, S_z$  (where  $S_x^2 + S_y^2 + S_z^2 = 1$ ). Spin is transformed by rotation, using matrices. In a bend, the rotation is around a vertical  $y$ -axis, in a quad, around a radial axis, in a snake, around an axis of orientation given as input, and in a spin flipper around a horizontal rotating or oscillating axis.

In general, a matrix representing a rotation in the ordinary space can be written as [10].

$$R = I \cos \varphi + W(1 - \cos \varphi) + A \sin \varphi$$

where  $I$  is the unitary matrix,  $W$  a symmetric matrix,  $A$  an anti symmetric matrix, and  $\varphi$  the rotation angle. In the case of spin, if one writes the components of the rotation axis as follows

$$\begin{aligned} b_1 &= \cos \theta \sin \phi \\ b_2 &= \sin \theta \\ b_3 &= \cos \theta \cos \phi \end{aligned}$$

with  $\theta$  the latitude and  $\phi$  the azimuth, explicit expressions for the matrices are

$$W = \begin{pmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_2 b_1 & b_2^2 & b_2 b_3 \\ b_3 b_1 & b_3 b_2 & b_3^2 \end{pmatrix} A = \begin{pmatrix} 0 & b_3 & -b_2 \\ -b_3 & 0 & b_1 \\ b_2 & -b_1 & 0 \end{pmatrix}$$

### 2.3 Synthetic Modules and Field Maps.

Matrices for machine modules can be built directly with mathematical expressions (“synthetic” modules, e.g. a snake that rotates the spin exactly by  $180^\circ$  around an axis at  $45^\circ$ ) or can be calculated from field maps. In the latter case the technique is to integrate the differential equations of motion and the spin BMT equation [11] through the map for an ensemble of particles and averaging the results. For synthetic modules the orbit matrices are automatically symplectic and the spin matrices unitary, for field maps, some manipulation is needed, to make them so. This is important, since spin tracking is sometimes carried on for  $10^5$  to  $10^6$  turns, and systematic errors may lead to wrong results.

The orbit matrix  $M$  is symplectic if it satisfies the condition (equivalent to six independent conditions among the matrix elements)

$$M^T S M = S$$

where  $M^T$  is the transpose of  $M$  and  $S$  is the base anti symmetric matrix. The symplecticity condition in particular insures that the 4-dimensional emittance of the beam in transverse space is conserved.

If a syntehtic module, building spin matrices is immediate. In a field map, in terms of the elements of the spin matrices  $W$  and  $A$ , the angles of the rotation axis  $\hat{b}$  and the spin precession angle  $\mu$  are given by

$$\begin{aligned} \tan 2\phi &= \frac{2W_{13}}{W_{33}-W_{11}} \\ \tan 2\theta &= -\frac{A_{12}A_{13}\sin\phi}{A_{12}A_{23}-A_{13}^2\sin\phi\cos\phi} \\ \cos\phi &= \frac{A_{12}A_{23}}{W_{13}-1} \\ \cos\mu &= (\vec{R}\vec{S}_0) \cdot \vec{S}_0 \end{aligned}$$

with  $\vec{S}_0$  the spin vector before the rotation.

Note that the elements of spin matrices for snakes (unless synthetic) are a function of beam energy. The magnetic field in the helices should be varied during the acceleration cycle to optimize spin rotation angle and axis angles. However, since it is convenient to keep the snake field constant, and since the strongest resonances take place at high energy, we tune the fields in the snakes at their optimum value for high energy and maintain these values. It is expected to operate this way with the real magnets.

## 3 EXAMPLES

Reference values of the main parameters for spin tracking are given in Table 1 [12]

Examples of spin tracking follow, both for the acceleration mode and for storage mode at constant energy.

### 3.1 Acceleration Mode

Figure 1 shows an example of spin tracking of one proton during the entire acceleration cycle in RHIC, for proton energy from 25 GeV to 250 GeV and snakes on. Initial transverse coordinates were taken at random on the contour of progressively larger emittances. In addition to the

Table 1: RHIC parameters for spin tracking

<b>Acceleration mode</b>		
Acceleration rate	$d\gamma/dt$	3.9/s
Betatron tunes	$\nu_y, \nu_x$	29.18, 29.19
Harmonic number	$h$	360
RF voltage	$V_{RF}$	300 kV
Beta-star (all interactions)	$\beta^*$	10 m
Normalized emittance	$\epsilon_N$	$20\pi 10^{-6}$ m-rad
Momentum spread	$dp/p$	$\pm 0.00056$
<b>Storage mode</b>		
Beam energy	$E$	250 GeV
Harmonic number	$h$	2520
RF voltage	$V_{RF}$	6,000 kV
Beta-star (6, 8 o'clock)	$\beta^*$	1 m
Momentum spread	$dp/p$	$\pm 0.00019$

unavoidable intrinsic resonances, also imperfection resonances were present, induced by closed orbit distortions of different rms values. At a constant acceleration rate the full process took several million turns.

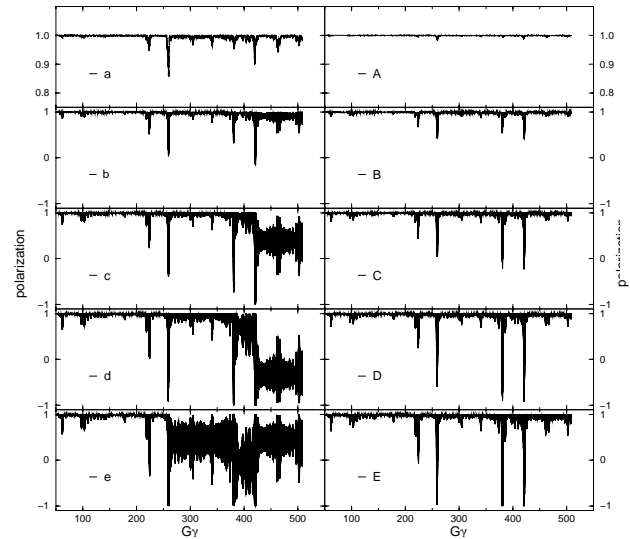


Figure 1: Spin tracking in acceleration mode. Plots on the left for vertical closed orbit deviation of 0.7 mm rms, on the right for 0.2 mm rms. From top to bottom, beam emittance of 0, 5, 10, 15 and 20  $\pi$  mm-mrad.

The example showed that with a distorted orbit of 0.7 mm rms, the polarization is substantially reduced at high energies, while at 0.2 mm (the design value in RHIC, after corrections) the polarization will be preserved even at large values of the emittance. However it should be noted that a good orbit correction scheme for spin is not necessarily the best scheme for unpolarized beam, since the correcting dipoles may create additional spin resonances.

### 3.2 Storage Mode

Beam-beam collisions in RHIC for experiments will be performed at fixed energy and mostly with longitudinal polar-

ization. Spin rotators across the two collision regions will be used to rotate the spin from the vertical to the longitudinal and back to the vertical after the interaction. We studied by spin tracking the survival of beam polarization at fixed energy. In Figure 2 the longitudinal spin at an interaction point is plotted vs. number of turns (up to  $10^6$ ), for one particle extracted at random on a  $20\pi 10^{-6}$  m-rad emittance on an uncorrected distorted closed orbit and snakes on. The polarization is shown to oscillate, but survive.

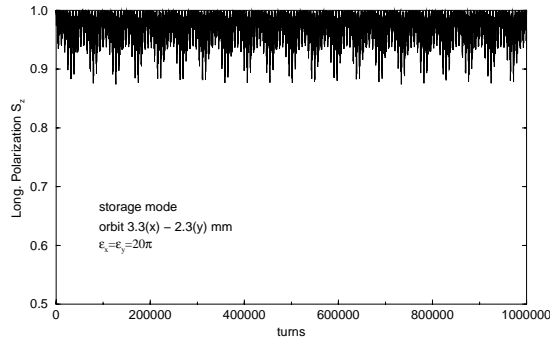


Figure 2: Spin Tracking at  $\gamma = 250$ . Longitudinal polarization at the 8 o'clock interaction point in RHIC. Snakes and spin rotators on.

### 3.3 Scanning Tune Space

The two previous examples were run at the nominal RHIC betatron tune. When the fractional value of the tune combines with the supersymmetry of RHIC lattice, one finds resonance strength enhanced. A typical value is the fraction  $1/6$  (or  $5/6$ ). Tune space has been previously explored [6]. We repeated the scanning of the tune line  $1/6$  by tracking a certain number of particles through a strong resonance  $G\gamma = 411 - \nu_y$ . The results, for fractional tunes close to the offending one, are shown in Figure 3. Systematic scan allowed us to determine the strength and the profile of the tune resonance.

To move the tune, it would be very cumbersome to run *Mad* and read its output for every tune in the range. To accomplish this, in *Spink* each quadrupole in the lattice is padded with two thin lenses of variable strength  $\delta q$ .

## 4 WORK IN PROGRESS

Many tracking runs have been executed, some on a large number of particles on a supercomputer at the Riken Institute, and the results averaged. Results in general agree with the conclusions drawn above from the tracking of one particle, but their discussion is beyond the limits of the present report.

## 5 ACKNOWLEDGEMENTS

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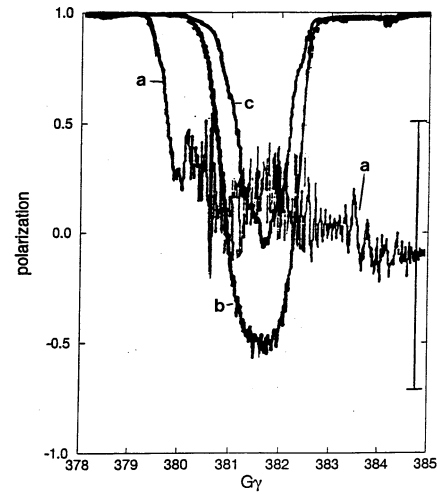


Figure 3: Crossing the  $G\gamma = 381.82$  resonance with snakes on and varying the vertical fractional betatron tune in proximity of  $1/16$ . (a) Spin tracking at  $\Delta\nu = 0.16691$ . (b)  $= 0.16608$ . (c)  $= 0.16758$ .

where the computing is done. In particular we acknowledge valuable comments and criticism of Drs. Ernest Courant, Thomas Roser and Michael Syphers.

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