METHOD OF STRONG FOCUSING

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Abstract

New methods for focusing and stabilization of the motion of charged and polarized neutral particles by electromagnetic fields are described. The idea is to rotate highly inhomogeneous fields in time so that the coupling of particles' orthogonal oscillations near the focal axis is substantially modulated. Unlike the field alternation without rotation which is used throughout for strong focusing in systems like beam transport, accelerators, and traps, the rotation in such devices opens up new ways for focusing and trapping not only in a point or an axis but also in an orbit or cylindrical surface. It can give much tighter confinement, helps overcome space charge and aberrations problems and provides flexible monitoring of several beams.

1 INTRODUCTION

The strong focusing, i.e. the average focusing effect of frequently alternating focusing and defocusing fields, gains new properties suitable for beam line applications by exploring the field rotation about the optic axis rather than familiar non-rotating field alternation. The proposals and discussion have been concentrated on the strong focusing with magnetic or electric static quadrupoles rotating helically in space, e.g. [1-11]. Recently but independently and for problems of magnetic traps for neutral particles, the author found new methods of tight trapping by means of highly non-uniform fields rotating in time rather than in space. We suggest that the strong focusing with the electric and magnetic fields rotating in time can be effectively explored also in beam line applications for charged particles, particularly in radio frequency quadrupole accelerators and systems, providing a more diverse and flexible control of beam optics. In this paper, we will focus on general specifics of the proposed strong focusing. Basically, there is a correlation between the strong focusing under the fields varying in time and that of in space and we will pay attention to some features not discussed in the literature on static helical structures. The specifics of magnetic focusing of neutral particles will be briefly elucidated at the end of the discussion.

2 BASIC EQUATIONS

The analysis will be based on a special though a rather representative model, a continuously rotating quadrupole system for which the analytical calculations become tractable. The equations of transversal motion of a charged particle near the optic axis will be

$$\begin{aligned} \ddot{x} &= kx\cos(2\Omega t) + ky\sin(2\Omega t) + ax + g\dot{y} + f_x, \\ \ddot{y} &= kx\sin(2\Omega t) - ky\cos(2\Omega t) + ay - g\dot{x} + f_y \quad (1) \end{aligned}$$

where k is related to the strength of the quadrupole rotating at frequency Ω and a to the strength of the (smoothed) rf defocusing, beam space-charge and other forces assumed axially symmetric and static; g is related to the gyro factor of longitudinal magnetic field, it allows us to see similarities and interlacing with the effect of rotating quadrupole; f are other forces, neglected unless specified otherwise. With t replaced by longitudinal coordinate z and with $f = g = 0$, this is a familiar type of equations considered in the literature devoted to strong focusing with helical quadrupole static structures. A combined space-time rotation of the quadrupole is governed by the equations of same type.

The rotating quadrupole, unlike the alternating, non-rotating, gives rise to the modulation of both the partial frequencies of the orthogonal oscillations about the optic axis and the coupling between these oscillations. This additional modulation enhances the strong focusing effect and enriches it with new, gyroscopic-like properties.

3 FIELD ARRANGEMENT

The time rotation, say, of an electric planar quadrupole field can be created by a.c. current of frequency 2Ω driving out of phase two identical quads of parallel (along z) electrodes with each quad creating a standard four-electrode planar quadrupole field geometry centered at the axis of symmetry. The two quads are turned about z by angle $\pi/4$ with respect to each other, i.e. in case of thin electrodes they form, in cross section, the apices of a rectilinear octagon. As a result, the lines of the a.c. field created by one quad are orthogonal to the lines of the other and shifted in time phase by $\pi/2$. The same idea of creating the rotating field with two conventional a.c. field configurations which are out of phase and ensemble with orthogonal field lines can be explored for the arranging of the rotation of a multi-pole field. For the field of multipole-number N rotating at frequency Ω, the frequency of a.c. current should be $N\Omega$.

The doubling of the field poles will impose additional tolerance limits on the mechanical arrangement of the system, being the price for more effective and versatile functions elucidated below.

4 STABILITY CONDITIONS

In the rotating frame $\mathbf{R} = (X_1, X_2, z)$ with the coordinates X_1, X_2 in the directions coinciding with the principal axes

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of the rotating quadrupole, the equations of particle motion (1) reduce to the form

$$\begin{aligned}\ddot{X}_1 &= (a+k)X_1 + 2(\Omega+g)\dot{X}_2 + \Omega^2 X_1 + F_1,\\ \ddot{X}_2 &= (a-k)X_2 - 2(\Omega+g)\dot{X}_1 + \Omega^2 X_2 + F_2.\end{aligned} \quad (2)$$

The terms proportional to Ω represent Coriolis forces and to Ω^2 centrifugal; $F_{1,2}$ are related to $f_{1,2}$ via the rotation transformation,

$$F_1 + iF_2 = \exp(i\Omega t)(f_x + if_y).$$

Let us analyse the system (2) for $F_{1,2} = 0$. The equilibrium state is at $X_{1,2} = 0$ and the normal oscillations near the z axis are of the form $\exp(\pm i\Omega_\pm)$ where

$$\Omega_\pm^2 = \check{\Omega}^2 - \check{a} \pm \sqrt{k^2 - 4\check{a}\check{\Omega}^2} \quad (3)$$

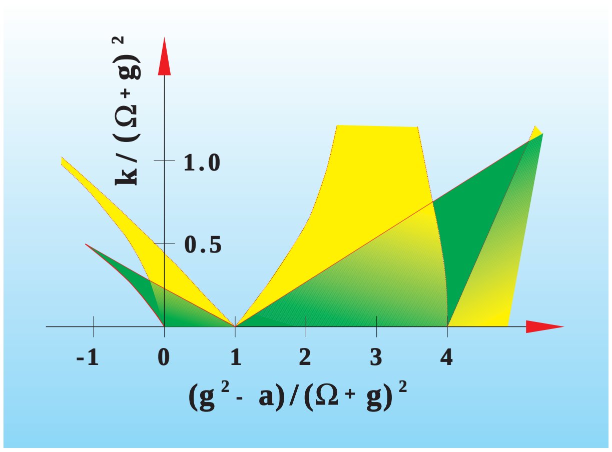
and

$$\check{\Omega} = \Omega + g, \quad \check{a} = a - g^2.$$

The regime of stable focusing requires the conditions of $\Omega_\pm^2 > 0$, otherwise defocusing occurs via an instability which is parametric in the picture of fixed coordinate frame. The stability condition on k and Ω is given by

$$\check{a} < \check{\Omega}^2, \quad 4\check{a}\check{\Omega}^2 < k^2 < \left(\check{a} + \check{\Omega}^2\right)^2 \quad (4)$$

and is represented by the green shaded region in the figure. The yellow shaded region is a familiar shape in conventional strong focusing systems, it corresponds to the model (1) with the x-y coupling terms $\sim \sin(2\Omega t)$ omitted and $\check{a} = a$. As seen from the region at $g^2 - a < 0$, the rotating quadrupole gives stable focusing with considerably smaller gradients k's. The rotation also resists the influence of short-range field inhomogeneities, since no parametric resonance instabilities arise near $(g^2 - a) = n^2(\Omega + g)^2$ with $n = 2, 3, ...$



5 FOCUSING STRENGTH AND INDUCED GYROSCOPY

In the fixed frame, the transversal particle motion is a superposition of four harmonics of frequencies $\Omega \pm \Omega_\pm$ of elliptical polarization. These four frequencies are displaced asymmetrically with respect to zero, pointing to a specific gyroscopic modulation of the motion which may be 100 % depending on initial conditions. The case of fast rotation, $k \ll \Omega^2$, gives an idea of characteristic trends. Then both kinds of parametric influence in Eq. (1), $\sim \cos(2\Omega t)$ and $\sim \cos(2\Omega t)$, contribute equally to the average focusing force. This force is radial and its gradient

$$k_o = \frac{k^2}{4\Omega^2} \quad (5)$$

is twice as powerful as that of the alternating, non-rotating quadrupole of same strength k. As k/Ω^2 increases, the focusing becomes stronger than twice, since the modulated coupling $\sin(2\Omega t)$ gives rise to both the net radial forces and the net gyro forces similar to the terms $\sim g$ in (1). Within leading approximation, the corresponding gyroscopic factor

$$g_o = \frac{k^2}{8\Omega^3}. \quad (6)$$

The g_o gives the scale of the modulation frequency. In next order approximation in k/Ω^2, the averaged dynamics is modified by this gyro force similarly to the effect of the terms $\sim g$ in (1), renorming a to $a - g_o^2$. This enhances the net focusing force with k/Ω^2 by about 20% in a certain interval inside the stability region. The focusing strength can be considerably more than doubled by the use of rotating multi-pole fields.

6 FOCUSING WITH ROTATING MULTI-POLE FIELDS

The fact that the rotation of non-uniform fields with multipole numbers higher than quadrupole can be rather effective for the focusing is evident from the following. The higher the number of a multi-pole component, the weaker its contribution to the overall focusing action in the vicinity of rotation axis. The contribution, however, rapidly increases with the distance from the axis. The action of synchronously rotating multi-pole fields results in the appearing in Eq.. (2) of forces $F_{1,2}$ with polynomial dependence on X_1 and X_2 and independent of t. The equilibrium states are given by the algebraic equations in $X_{1,2}$

$$X_s = -F_s/(\Omega^2 + a + k_s) \quad (7)$$

with $s = 1, 2$, $k_s = \pm k$ respectively. The roots of (7) represent a number of orbiting states of frequency Ω of different orbital radius and different azimuth. The oscillations in close vicinity of such states obey the linearized equations of the same type discussed above and, hence, the same analysis of stability and other properties can be applied. The strength of focusing, being proportional to the square of gradients $F_{1,2}$, increases sharply with the multi-pole number and the radius of equilibrium orbit and this considerably favours the confinement. In addition to enlarging the area of strong focusing, new schemes can be arranged for flexible monitoring of several beams, overcoming space charge and aberrations problems.

An important novel property is the ability to provide focusing not only in the axis of rotation, but in a finite radius cylindrical surface. This can be arranged without higher multi-pole fields. The rotation of quadrupole field is then combined with the synchronous rotation about the same z-axis of transversal uniform field. For example, for the focusing with electric fields, the uniform component can be generated by two quads of electrodes that provide the rotating quadrupole. The driving frequency of the uniform component should be Ω, rather than 2Ω, and each of the two quads should be operated in the dipole mode (rather than quadrupole) so that now the orthogonal dipole fields are driven out of phase. The additional uniform field results in the particle dynamics being governed by the same equations (2) except that the equilibrium state of $X_{1,2}$ is shifted from the center of rotation, as given by Eq. (7), with $F_{1,2}$ equal to the instant components of the rotating uniform field at $t = 0$. In the fixed coordinate frame the shifted state is an orbital motion of frequency Ω and the rotating uniform field controls the azimuth and radius of the orbit. Note that though the motion equations for small displacements from this orbit coincide with (2), the confinement in this state differs from that of vanishing uniform field component since a harmonic of frequency Ω (synchronously rotating with the field) arises in addition to the four harmonics corresponding to the formal normal modes.

7 MAGNETIC TRAPPING OF NEUTRAL PARTICLES

The strong focusing with rotating highly non-uniform fields can be effective for focusing and confinement of both charged particles and neutral polarized particles. Let us briefly consider its application to trapping of neutral spin-polarized particles with magnetic fields rotating at a frequency Ω small compared to the frequency scale Ω_Z of Zeeman splitting of the spectrum of the trapped particles in the field. In such conditions the magnetic polarization μ of the particles follows the direction of instant field $\mathbf{B}(\mathbf{r}, t)$ adiabatically and the magnetic force field is potential. For the particles with vector μ antiparallel to $\mathbf{B}$, the magnetic force potential

$$V = \mu\, B$$

where $\mu = |\mu| = const > 0$ and $B = |\mathbf{B}|$. The potential is singular at $B = 0$ and is not harmonic, $\Delta V \neq 0$. We found that, in such conditions the strong focusing with rotating non-uniform fields has unique advantages for tighter, stable confinement of polarized neutral particles.

In particular, the 3D trapping of remarkable characteristics occurs when the field is of the form

$$\mathbf{B} = \mathbf{B}_{st}(\mathbf{r}) + \mathbf{b}(t, \mathbf{r}) \tag{8}$$

where $\mathbf{B}_{st}$ is a static quadrupole field of axial symmetry with the symmetry axis z in vertical direction and $\mathbf{b}$ is a sum of uniform and quadrupole planar fields synchronously rotating about axis z. In the rotating frame coordinates, the equilibrium state of the trapped particle is where the force potential

$$U(\mathbf{R}) = \mu B(\mathbf{R}) + Mgz - M\Omega^2(X_1^2 + X_2^2)/2$$

(Mgz is the gravity potential) is minimum. The equilibrium state $\mathbf{R} = \mathbf{R}_o$ represents a planar orbit and is most effective for confinement when the vertical component of the static field $\mathbf{B_{st}}(\mathbf{R_o})$ is small compared to the magnitude of the planar rotating field $\mathbf{b}(\mathbf{R_o})$. The dynamics of particle transversal oscillations near the equilibrium orbit is described approximately by the equations of structure (2). Details of the analysis are in [12]. It is important that the radius of equilibrium orbit can be reduced to zero by the variation of the rotating uniform field. This allows considerable decrease of current loads in the coils of the magnetic field and results in cardinal improvement of trapping characteristics. Not only the confinement, but also the polarization alignment of the trapped particles over their cloud is improved since the area of singularity of magnetic potential (the main contributor to the polarization misalignment) is removed.

In addition, the trapping with rotating fields of considered geometry opens up new, vast possibilities for magnetic separation and evaporative cooling of polarized neutral particles, since the equilibrium orbit radius critically depends on the gradient of rotating quadrupole field and on the rotation frequency Ω via a resonance denominator. The corresponding orbit resonance frequency differs from that of parametric resonance and both can be controlled independently in such a way as to enhance the differences in equilibrium states and stability of particles with different μ/M.

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