

BEAM BREAKUP IN A MICROTRON*

Byung C. Yunn and Lia Merminga
 Thomas Jefferson National Accelerator Facility
 12000 Jefferson Avenue, Newport News, VA 23606, USA

Abstract

In a microtron, the path length change from pass to pass is a fixed multiple of the RF wavelength, and the accelerating system can be reasonably well approximated as a single cavity. Under such circumstances it is possible to derive an analytical formula for the multipass beam breakup threshold current. The threshold current determined by numerical simulations agrees very well with the formula for a machine with a small number of passes. The analytic formula can serve as a useful guide in examining optics designs to improve the BBU threshold.

1 INTRODUCTION

Multipass beam breakup (BBU) is an important performance-limiting factor in microtrons. Extensive computer simulations are generally unavoidable to determine BBU thresholds since the tens of recirculations involved make analysis very difficult. When the betatron phase advance through the accelerating cavities is small, a good approximation is to replace the whole accelerating system by a single cavity. Furthermore, the total kick received passing through the cavity can be treated as if received by a point kick located at the middle of the cavity in describing beam motion. The resulting simplification allows an analytic treatment of the multipass BBU phenomenon without losing too much information.

We have developed an analytic formula for the BBU threshold current in this single cavity approximation. A benchmarking of the formula using a 25-pass racetrack microtron as an example has shown its potential as a design tool. Time-consuming computer simulations could be minimized at the early stage of design.

2 A MODEL OF BBU

We start by defining our model of BBU in a microtron under the assumption that a single cavity approximation can be made. For the convenience of readers, we will closely follow the notations of references [1] and [2], which represent previous study of an analytical model of multipass BBU in recirculating linacs.

Consider a bunch (say, the N th one) entering a cavity at its p th pass through the linac with its motion represented by a two component column matrix, $U_p(N)$ of x and p_x . While traversing the cavity, the bunch will get a momentum kick due to transverse wakes excited in the cavity by preceding bunches in addition to gaining nominal energy.

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The bunch then recirculates and enters the cavity for the $(p + 1)$ th time. A dipole mode of frequency ω with a quality factor Q is assumed to be generated at the cavity for the following discussions. The equation of transverse bunch motion described in this physical picture is

$$U_{p+1}(N) = T^{p+1,p} U_p(N) + IT^{p+1,p} G \sum_{q=1}^{n_p} \sum_{M=1}^{N+S(p,q)} U_q(L) s_{NM}^{pq}(\omega, \tau), \quad (1)$$

where

$$s_{NM}^{pq}(\omega, \tau) = \left(\frac{e}{2}\right) \tau_{rf} \left(\frac{\omega}{c}\right)^2 L \left(\frac{R_{\perp}}{Q}\right) e^{-\frac{\omega}{2Q}((N-M)\tau + \tau_p - \tau_q)} \times \sin \omega((N - M)\tau + \tau_p - \tau_q),$$

and

$$S(p, q) = \begin{cases} \text{int.}(\frac{\tau_p - \tau_q}{\tau}), & \text{if } p \geq q \\ \text{int.}(\frac{\tau_p - \tau_q}{\tau}) - 1, & \text{otherwise} \end{cases}$$

$T^{p,q}$ is the transfer matrix from the cavity at the q th pass to the cavity at the p th pass. I is the average current and G is a 2×2 matrix with all elements equal to zero except $G_{21} = 1$. n_p is the number of passes and R_{\perp} is the transverse shunt impedance per unit length of the higher order mode ω excited in the cavity, which is assumed to have an effective length of L . We also note that R_{\perp} is related to other commonly used transverse shunt impedance Z'' as $R_{\perp} = \frac{c^2 Z''}{\omega^2}$. τ_p is the summation over recirculation times up to the p th pass, and $\tau_1 = 0$. In general, τ_p has to be an integer multiple of the RF period, τ_{rf} , if recirculations are all in the same direction. However, it is not necessary to be an integer multiple of the bunch spacing τ , when the beam is subharmonically bunched.

3 THRESHOLD CURRENT

As noted in [2], we must deal with an $(n_p - 1)$ -dimensional eigenvalue problem for the threshold current for multipass BBU in the most general circumstances even for a single cavity case. Deferring an analysis of general subharmonic bunching schemes until a later time, we restrict our attention to the case when every bucket of the accelerating mode of the cavity (i.e. $\tau = \tau_{rf}$) is filled. The model simplifies significantly in this situation and the eigenvalue problem reduces to

$$\frac{1}{I} = \left(\frac{e}{2}\right) \tau \left(\frac{\omega}{c}\right)^2 L \left(\frac{R_{\perp}}{Q}\right) h(\Omega) \sum_{p=2}^{n_p} \sum_{q=1}^{p-1} T_{12}^{p,q} e^{-i\Omega(\tau_p - \tau_q)}, \quad (2)$$

where

$$h(\Omega) = \frac{H(\Omega) \sin \omega \tau}{1 + H^2(\Omega) - 2H(\Omega) \cos \omega \tau},$$

and

$$H(\Omega) = e^{-\frac{\omega \tau}{2Q}} e^{-i\Omega \tau}.$$

This still is a complicated nonlinear equation for Ω which requires a numerical approach to solution in general. Note that the imaginary part of Ω changes sign from plus to minus as we cross threshold current. Solving Eq.(2) to first order in current, we find the following expression for the BBU threshold current:

$$I_{th} = -\frac{2c^2}{e\omega L R_{\perp} M}, \quad (3)$$

where M is

$$M = \sum_{p=2}^{n_p} \sum_{q=1}^{p-1} T_{12}^{p,q} e^{\frac{\omega}{2Q}(\tau_p - \tau_q)} \sin \omega(\tau_p - \tau_q).$$

This is our central result. The important parameters determining the BBU threshold current are clearly spelled out.

4 APPLICATIONS

We have selected a 25-pass microtron essentially identical to the NIST RTM (racetrack microtron)[3] as our example to test the validity of Eq.(3) and have performed extensive simulations of the microtron BBU problem using the computer code TDBBU[4] developed at Jefferson Lab originally for the 5-pass nuclear physics CEBAF accelerator. Figure 1 summarizes results of a TDBBU threshold current scan of the microtron when limited to 1 to 3 recirculations. The parameters for simulation and analytic study are:

Injection Energy = 5 MeV

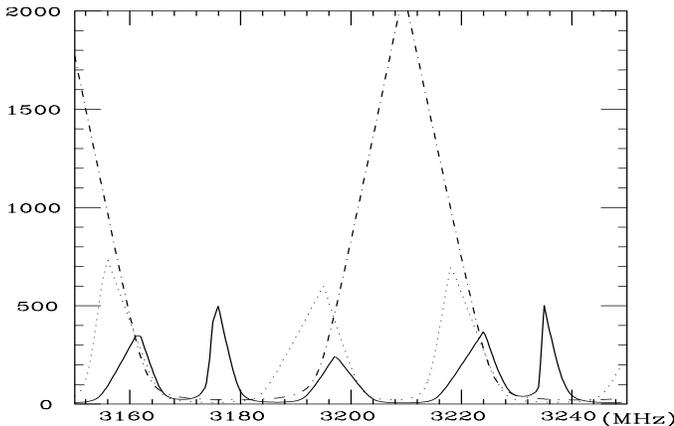


Figure 1: TDBBU scan of threshold current (mA) vs. HOM frequency (MHz). The dot-dashed line is for a two-pass machine, the dotted line for a 3-pass, and the solid line is for a 4-pass system.

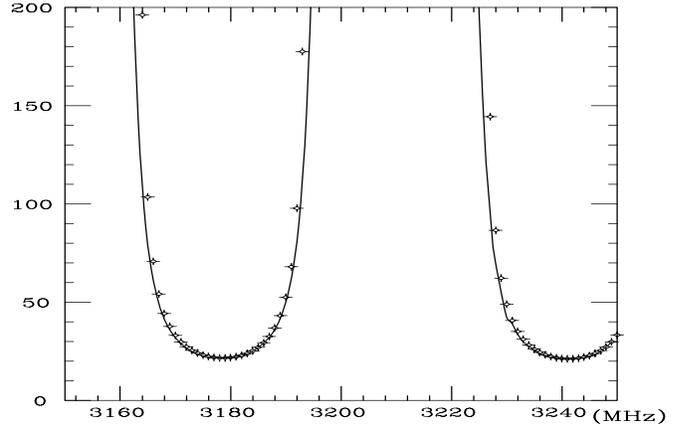


Figure 2: I_{th} (mA) vs. HOM frequency (MHz): Comparison of simulation results with I_{th} predicted with Eq.(4) for a 2-pass system.

Energy Gain per pass = 1.404 MeV

$\tau^{-1} = 2380$ MHz

$(\frac{R_{\perp}}{Q})L = 250 \Omega$

$\tau_2 = 38\tau$

and each subsequent path length is increased by one RF wavelength per pass. The HOM frequency is allowed to vary between 3150 MHz and 3250 MHz and Q of the mode is set to 10000.

Let us start with a 2-pass system. Since $n_p = 2$, we get

$$I_{th} = -\frac{2c^2}{e\omega L R_{\perp} T_{12}^{2,1} e^{\frac{\omega \tau_2}{2Q}} \sin \omega \tau_2}. \quad (4)$$

This is a threshold formula studied in detail by Sereno[5]. Clearly, the '12' element of transfer matrix and phase advance are the two most important parameters in designing a system like this as far as the BBU threshold is concerned. Regions of high threshold correspond to regions where $\sin \omega \tau_2$ term is positive. I_{th} is infinity at least in the first order approximation we make. As one can see from Figure 1, it is finite even though very high. We have compared I_{th} given by Eq.(4) with the TDBBU scan (dot-dashed line of Figure 1) in Figure 2. The minimal threshold current of this 2-pass machine is 21 mA.

For a 3-pass system, I_{th} is obtained from Eq.(3) with M replaced by

$$M = T_{12}^{2,1} e^{\frac{\omega \tau_2}{2Q}} \sin \omega \tau_2 + T_{12}^{3,1} e^{\frac{\omega \tau_3}{2Q}} \sin \omega \tau_3 + T_{12}^{3,2} e^{\frac{\omega}{2Q}(\tau_3 - \tau_2)} \sin \omega(\tau_3 - \tau_2). \quad (5)$$

Noting that $\tau_3 = 77\tau$ we find that the 38th, 39th and 77th subharmonics of the fundamental accelerating frequency appear in Eq.(5). As a result, the pattern of threshold current scan should be quasi-periodic with a period of approximately 62 ($\simeq \frac{2380}{38.5}$) MHz with two minima in the period. The minimum at 3175 MHz is due to all $T^{i,j}$ terms contributing coherently to their maximum values at the frequency. The second minima occurs when phase terms are such that $T^{3,1}$ and $T^{3,2}$ terms are maximally

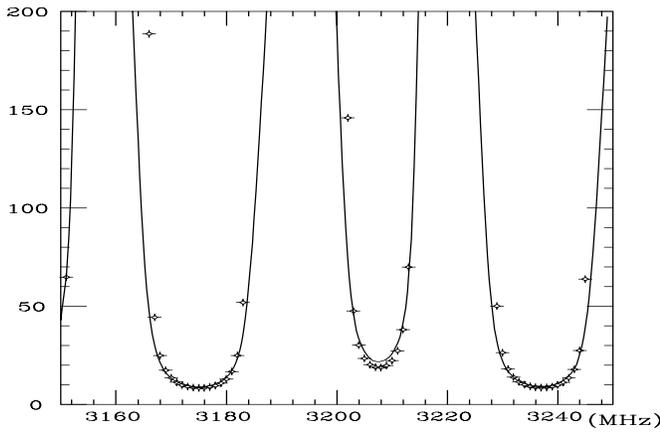


Figure 3: I_{th} (mA) vs. HOM frequency (MHz): Comparison of simulation results with I_{th} predicted with Eq.(5) for a 3-pass system.

adding with the $T^{2,1}$ term being 180 deg apart in phase. We also notice that the graph is truly periodic with the period of 2380 MHz, if the small perturbation in amplitudes due to exponential terms is neglected. We have compared analytic I_{th} with the TDBBU scan(dotted line of Figure 1) in Figure 3. The agreement is excellent and the lowest threshold current we found is 9 mA in this 3-pass operation.

The expression for the I_{th} of a 4-pass system is already too long to fit on one line. It involves 6 interpass transfer matrices $T^{2,1}, T^{3,1}, T^{3,2}, T^{4,1}, T^{4,2}$, and $T^{4,3}$ with associated sinusoidal terms resulting in a characteristic 3 minima threshold current pattern. This arises as a result of superposing terms which contain roughly 3 frequencies, the 39th, 78th and 117th subharmonics of 2380 MHz. Comparison of the TDBBU scan(solid line of Figure 1) with the analytically obtained I_{th} exhibits a good agreement as shown in Figure 4. The lowest threshold current is 6.4 mA, occurring at the HOM frequency of 3210 MHz.

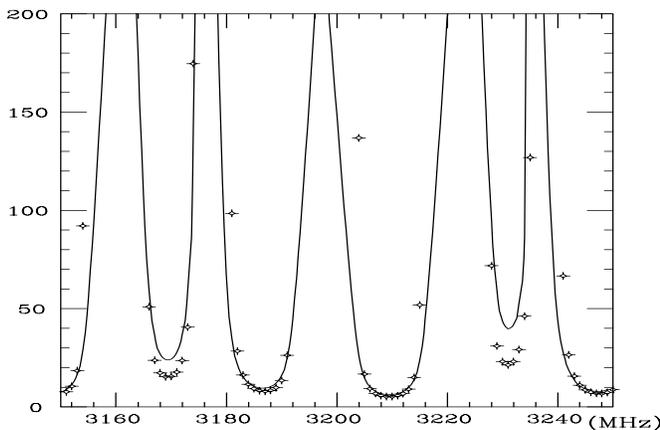


Figure 4: I_{th} (mA) vs. HOM frequency (MHz): Comparison of simulation results with analytic prediction of I_{th} for a 4-pass system.

Finally, in Figure 5 drawn in solid line we present a TDBBU scan for a of 25-pass microtron. The threshold current calculated from Eq.(3) is also plotted. It is encouraging to find that regions of low threshold current are accurately predicted.

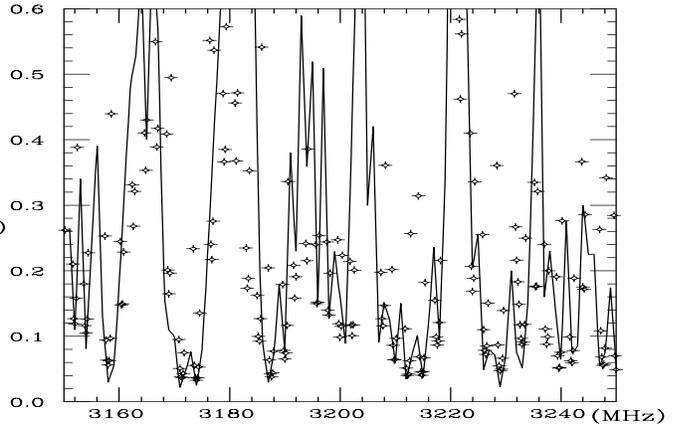


Figure 5: I_{th} (mA) vs. HOM frequency (MHz): Comparison of simulation results with I_{th} predicted with Eq.(3) for a 25-pass microtron.

5 CONCLUSION

The agreement between the approximate expression Eq.(3) and simulation is quite satisfactory even for a 25-pass machine which involves superposition and interference of 300 distinct frequencies and recirculation matrices. Our analytic formula is able to pinpoint regions of HOM frequency where one should look to find lower threshold current. Furthermore, it provides a qualitative understanding of the characteristic pattern of valleys and peaks in the threshold current scan of a typical multipass machine.

The analytic formula for BBU threshold current presented here in Eq.(3) makes various parameter interrelations clear and should serve as a useful guide in designing microtrons.

6 ACKNOWLEDGEMENTS

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