AN AMPLIFIER FOR OPTICAL STOCHASTIC COOLING*

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1 INTRODUCTION

A method of Optical Stochastic Cooling (OSC) of protons/antiprotons and heavy ions in high energy colliders has been recently proposed [1], [2] to achieve much shorter damping time than in conventional microwave stochastic cooling. According to [2], (see also [3] for details on the method of stochastic cooling), stochastic cooling damping time can be written as:

$$\tau \simeq N_b \frac{\Pi}{\Delta f \sigma_z}, \tag{1}$$

where N_b is the number of particles in the bunch, Π is the circumference of the ring (we assume one cooling system), σ_z is the rms bunch length, and Δf is the bandwidth of the amplifier.[1] Clearly, for a fast cooling it is crucial to have a maximum possible Δf. Remarkably, currently available mediums for optical amplification possess bandwidths up to 10^{14} Hz [4], which is superior to any other amplifier.

Recall that cooling insertion into a storage ring consists of a pick-up undulator, bypass lattice, and kicker undulator. Relativistic beam particles radiate a light signal in a pick-up

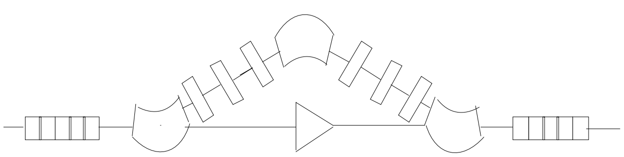

Figure 1: A schematic drawing of the cooling insertion into a storage ring for OSC.

undulator and proceed into the bypass which has a proper path length to provide the time delay needed for light amplification. Then, particles meet their own amplified radiation in the kicker undulator where they receive a correcting kick. The optical amplifier is located between the undulators. In practice, achieving a large difference (say, more than several tens of centimeters) between beam and light paths is difficult and time spent in the amplifier should be minimized. Therefore, we consider a single pass amplifier consisting of few high gain amplification stages.

The intent of this paper is to study specific features of application of the optical amplifier in stochastic cooling. We have found that this is convenient to do using practical examples, and for them we choose cooling of antiprotons in the TEVATRON collider[5] and cooling of electrons in the electron storage ring DELTA[6], although, there are no known for us plans to use OSC on these machines. Table (1) contains lists of parameters for both rings.

Table 1: Beam and ring parameters.

Storage ring	TEVATRON[5]	DELTA[6]
Type of particles	$\bar{p}$	e
Revolution frequency, f_0 [kHz]	49.5	2604
Number of bunches, m	36	1
Number of particles per bunch	1×10^{10}	1×10^{9}
Beam energy, E [GeV]	1000	0.3
Energy spread, σ_e	3×10^{-4}	8×10^{-4}
Bunch length, σ_z [cm]	45	2

2 AMPLIFIER FOR COOLING OF ANTIPROTONS

Damping time calculated for the TEVATRON using Eq.(1) is ~ 3 sec, but an amplifier with the average power $P \simeq 4 \times 10^5$ W would be required for such a fast damping. In this paper we consider much modest (and feasible with the current technology) amplifier with $P \simeq 2$ W. Then, instead of 3 sec, damping time becomes ∼23 min. (Damping time scales with the amplifier power as $P^{-1/2}$ [2]).

Table (2) contains optical properties of Ti:sapphire. We choose this material as the amplifier medium because of its wide bandwidth (one of the best among optical amplifiers) and because of its relatively long flourescense lifetime.

Table 2: Optical properties of Ti:sapphire at 300 K.

Property	Value	Ref.
Refractive index, n	1.76	[4]
Temp. coefficient $^{a)}$, dn/dT	1.3×10^{-5} K^{-1}	[7]
	1.4×10^{-6} K^{-1}	[8]
Saturation intensity, I_s	2.4×10^5 W/cm^2	[4]
Quantum efficiency, η	0.9	[4]
Flourescence peak, λ_f	780 nm	[4]
Flourescence lifetime, τ_f	3.2 μs	[4]
Bandwidth, FWHM, $\Delta\omega_a/2\pi$	10^{14} Hz	[4]

$^{a)}$ There is a discrepancy in the data cited in Ref. [7] and Ref. [8].

As a pump source for Ti:sapphire, we consider a cw argon-ion laser with 15 W of power operated at $\lambda_{Ar} = 514$ nm. This laser will produce ~ 10 W of the available power in Ti:sapphire at $\lambda_f = 780$ nm. (The remaining 5 W, which accounts for a difference between absorption photon energy and flourescence photon energy and quantum

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[1] In a general case Δf is the bandwidth of a whole system that includes pick-up, amplifier and kicker, but in OSC it is the amplifier that determines the overall bandwidth.

efficiency of Ti:sapphire, will go into heat.) From 10 W of available power in the last amplification stage we are going to use approximately 2 W for output signal. The remaining 8 W are continuously drained by the flourescence emission.

A schematic of an amplifier based on Ti:sapphire is shown in Figure (2). The amplifier has four identical stages with a total gain of $G_{dB} = 44$ db. The corresponding bandwidth of the amplifier, defined as the distance between two frequency points at which the amplitude gain has fallen to half the peak, is calculated [9]:

$$\Delta f = \frac{\Delta\omega_a}{2\pi}\sqrt{\frac{3}{G_{dB}/2-3}} \simeq 4\times10^{13}\text{Hz}. \qquad (2)$$

A small signal gain of the amplifier stage is equal [9]:

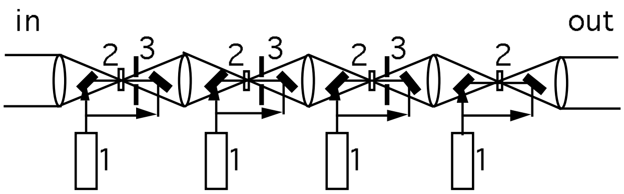

Figure 2: A schematic of the amplifier: (1) is the argon-ion laser, (2) is the Ti:sapphire crystal and (3) is the aperture. Arrows show pumping from two directions.

$$G_0 = \exp\left\{\eta\frac{I_p\lambda_{Ar}}{I_s\lambda_f}\right\}, \qquad (3)$$

where I_p is the pump laser intensity. Using $I_s = 2.4\times10^5$ W/cm^2, $\lambda_f/\lambda_{Ar} \simeq 1.5$, $\eta = 0.9$ and $G_0 = 12.5$, we get $I_p \simeq 10^6$ W/cm^2. This intensity can be achieved by focusing a 15 W argon-ion laser into an area of $A = 1.5\times10^{-5}$ cm^2. Assuming a Gaussian beam, we find $\sigma_w \simeq 1.5\times10^{-3}$ cm in a waist and $Z_R = 2.5$ mm for the Rayleigh length. Therefore, a Ti:sapphire crystal with an absorption length of approximately $\ell = 5$ mm is required. According to [10], this can be made from a sapphire with a dopping of $N_{Ti} \sim 5\times10^{19}$ Ti^{3+} ions/cm^3 (crystal orientation: c-axis normal to the rod axis).

The value of I_p, calculated above, is ~ 15 times higher than that in commercial Ti:sapphire lasers, which operate not too far from the damage threshold for a Ti:sapphire crystal due to the effect of thermal lensing [11]:

$$\frac{1}{F} \simeq \frac{1}{2\lambda_T}\frac{dn}{dT}\frac{P_T}{A}. \qquad (4)$$

Here F is the focal length of the thermo lens, P_T is the heating power, and λ_T is the thermal conductivity (for sapphire $\lambda_T = 0.33$W/(cm K) at 300 K, and $\lambda_T = 10$W/(cm K) at 77 K [12]). One solution of the thermo lensing problem is cooling the Ti:sapphire crystal to the liquid nitrogen temperature [7]. Then, a sharp rise in the thermo conductivity[2] will overpass the increase of I_p.

Let us consider now how the pulse of beam radiation in the undulator is distorted by the amplification medium. There could be the phase and the amplitude distortions, but only the phase distortions possess a threat for cooling, since the amplitude distortions average out during the damping time which consists of many thousands beam passes through a cooling system.

The phase distortions in the amplified signal appeared as a combination of the phase shifts caused by the depletion of the population inversion density in the medium, plus phase shifts caused by the beam energy spread. In the first case the leading part of the pulse changes the population inversion density of the medium, which will then act in a different manner at the trailing part. This means that different pulse parts will 'see' different optical path lengths, resulting in a time dependent phase change (chirp). To estimate this effect, let us calculate first the available energy in the Ti:sapphire crystal. It is 26 μJ for the above parameters (8W of spontaneous emission and 3.2 μs of a flourescence lifetime). The energy consumed by the amplified pulse is 2W$\times$0.55μs=1.1 μJ, where 0.55 μs is a time interval between bunches in the TEVATRON ($1/mf_0$). Therefore, the change of the population inversion density in Ti:sapphire of the last amplification stage constitutes only 4.3% over the entire pulse length. A corresponding change in the refractive index is $\Delta n = 0.043\beta N_{Ti}$, where the coefficient β was measured to be $\sim 10^{-24}$cm^3 at 300 K [13]. Therefore, a change in the phase from the head to the tail of the pulse is $\Delta nk\ell \simeq \pm0.4$ rad ($k = 2\pi/\lambda_f$). A reduction in the cooling rate associated with this phase distortion is 8%.

The second effect (that is phase distortions induced by the beam energy spread) develops when a carrying frequency of the light pulse, ω_p, is near the resonance frequency of atomic transitions in the medium. For an estimation of the magnitude of the phase shifts we approximate the complex gain coefficient α_m [9] with the Lorentzian:

$$\alpha_m(\omega) = \alpha_m(\omega_a)\frac{1 - i\frac{\omega-\omega_a}{\Delta\omega_a/2}}{1+\left(\frac{\omega-\omega_a}{\Delta\omega_a/2}\right)^2}, \qquad (5)$$

where ω_a is the central resonance frequency and $\Delta\omega_a$ is the resonance bandwidth. Assuming the input signal in the form:

$$E_{in} = a(t)\,e^{i\omega_p t},\ a(t) = \begin{cases} a_0 & \text{if } -\tau_u/2 < t < \tau_u/2, \\ 0 & \text{otherwise,} \end{cases} \qquad (6)$$

where a_0 is the amplitude, $\tau_u = 2\pi p/\omega_p$ is the duration of the pulse of the undulator radiation of a single particle and p is the number of undulator periods, we get for the amplified signal:

$$E_{out} = a_0 e^{i\omega_p t}\int_{-\infty}^{\infty}\frac{\sin\pi p\frac{\omega-\omega_p}{\omega_p}}{\pi p\frac{\omega-\omega_p}{\omega_p}}e^{4\ell\alpha_m(\omega)+i\left(\omega-\omega_p\right)t}d\left(\omega-\omega_p\right). \qquad (7)$$

The evaluation of the integral in Eq.(7) for different ω_p allows the finding of the phase shift as a function of ω_p. A corresponding plot is shown in Figure (3). In this calculations we assumed $4\ell\alpha_m(\omega_a) \simeq 5$, which gives a required G_{db}.

[2]There is indication that dn/dT also drops with temperature [7].

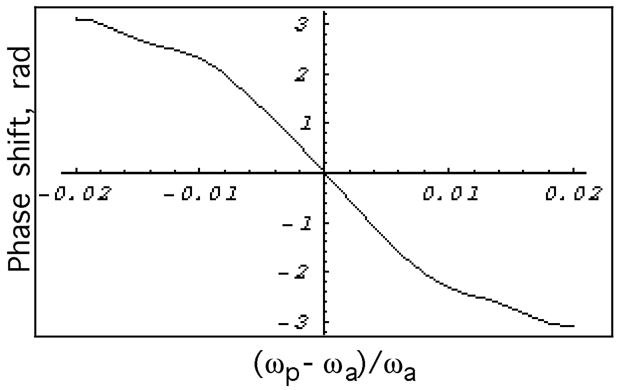

Figure 3: Phase shift of the amplifier output signal versus detuning from the resonance frequency of the atomic transition in Ti:sapphire crystal.

Recall, that the beam energy spread induces the spread of frequencies of the undulator radiation:

$$\omega_p - \omega_0 = 2\omega_0 \frac{\Delta\gamma}{\gamma}, \tag{8}$$

where ω_0 is the frequency of the undulator radiation of the equilibrium particle and $\Delta\gamma/\gamma$ is the relative energy deviation. If $\omega_0 = \omega_a$, i.e. the central frequency of the beam radiation in the undulator is matched to the resonance frequency of the medium, then there are no phase distortions in the amplified signals from the equilibrium particles, and phase distortions in the amplified signals from the non equilibrium particles are kept at a minimum. In this case, the phase shift in the signal for a particle with the energy deviation equal to the r.m.s. energy deviation in the TEVATRON is ~ 0.2 rad. Obviously, it has a negligible effect on the damping time.

Let us now consider two effects related to spontaneous emission. First of all, the spontaneous emission in one stage can reach the neighboring stages and get amplification there. Then, this amplified radiation reaches the next stage and so on. As a result, this process drains the inversion population of the medium. However, draining can be reduced to a minimum, specified below, by installing the aperture near the waist points of the light beam (see Figure 2). Then, only photons of the spontaneous emission emitted in the same mode as signal photons can propagate through the amplifier. The number of such photons is roughly equivalent to the one photon per second per Hz of the bandwidth[3] at the input of the amplifier [9]. It gives us $n_{sp} \simeq 3 \times 10^{13}$photons/sec at the input and $\sim 7 \times 10^{17}$photons/sec at the output, which is equivalent to 0.2 W of the power. Clearly, it is small compared with the 10 W of the available power.

The second effect, related to the spontaneous emission, is the noise that the amplifier adds to the signal. Here we have to compare $\sim 10^5$ photons at the amplifier input that we get by multiplying n_{sp} by the bunch length (~ 3.3 ns) with the number of photons coming from the signal:

$$n_{ph} = \pi\alpha N_b \simeq 2 \times 10^8, \tag{9}$$

where $\alpha = 1/137$ is the fine structure constant.

[3] Here it is the amplifier power gain bandwidth.

3 AMPLIFIER FOR COOLING OF ELECTRONS

A demonstration experiment for OSC can be performed on electrons. As an example, we consider the storage ring DELTA [6]. Damping time given by Eq.(1) for this ring with beam parameters listed in Table (1) is 0.15 sec. At the same time, the synchrotron radiation damping at the energy of 300 MeV is 0.7 sec. Cooling would require the amplifier with $G_{dB} = 27$ and $P = 1.6$ mW. Such an amplifier is much simpler than the above-considered amplifier and can be made from two or three amplification stages, depending on the difficulty of the thermo lensing problem. Besides that, we do not expect to have any other problem with the amplifier for this machine.

4 CONCLUSION

By considering the optical amplifiers for OSC of antiprotons in the TEVATRON collider with the damping time of 23 min and cooling of electrons in the DELTA storage ring with the damping time of 0.15 sec, we come to the conclusion that, among the various effects known to us, related to optical amplification no one seems capable of having any noticeable impact on the cooling efficiency in OSC.

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