1 INTRODUCTION

A method of Optical Stochastic Cooling (OSC) of protons/antiprotons and heavy ions in high energy colliders has been recently proposed [1], [2] to achieve much shorter damping time than in conventional microwave stochastic cooling. According to [2], (see also [3] for details on the method of stochastic cooling), stochastic cooling damping time can be written as:

\[ \tau \approx \frac{N_b}{f_0 \Delta f \sigma_z}, \]  

(1)

where \( N_b \) is the number of particles in the bunch, \( \Delta f \) is the bandwidth of the amplifier.\(^1\) Clearly, for a fast cooling it is crucial to have a maximum possible \( \Delta f \). Remarkably, currently available mediums for optical amplification possess bandwidths up to \( 10^{14} \) Hz [4], which is superior to any other amplifier.

Recall that cooling insertion into a storage ring consists of a pick-up undulator, bypass lattice, and kicker undulator. Relativistic beam particles radiate a light signal in a pick-up undulator and proceed into the bypass which has a proper path length to provide the time delay needed for light amplification. Then, particles meet their own amplified radiation in the kicker undulator where they receive a correcting kick. The optical amplifier is located between the undulators. In practice, achieving a large difference (say, more than several tens of centimeters) between beam and light paths is difficult and time spent in the amplifier should be minimized. Therefore, we consider a single pass amplifier consisting of few high gain amplification stages.

The intent of this paper is to study specific features of application of the optical amplifier in stochastic cooling. We have found that this is convenient to do using practical examples, and for them we choose cooling of antiprotons in the TEVATRON collider[5] and cooling of electrons in the electron storage ring DELTA[6], although, there are no known for us plans to use OSC on these machines. Table (1) contains lists of parameters for both rings.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Type of particles</td>
<td>( \bar{p} )</td>
<td>e</td>
</tr>
<tr>
<td>Revolution frequency, ( f_0 ) [kHz]</td>
<td>49.5</td>
<td>2604</td>
</tr>
<tr>
<td>Number of bunches, ( m )</td>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>Number of particles per bunch</td>
<td>( 1 \times 10^{10} )</td>
<td>( 1 \times 10^9 )</td>
</tr>
<tr>
<td>Beam energy, ( E ) [GeV]</td>
<td>1000</td>
<td>0.3</td>
</tr>
<tr>
<td>Energy spread, ( \sigma_e )</td>
<td>( 3 \times 10^{-4} )</td>
<td>( 8 \times 10^{-4} )</td>
</tr>
<tr>
<td>Bunch length, ( \sigma_z ) [cm]</td>
<td>45</td>
<td>2</td>
</tr>
</tbody>
</table>

2 AMPLIFIER FOR COOLING OF ANTI PROTONS

Damping time calculated for the TEVATRON using Eq.(1) is \( \sim 3 \) sec, but an amplifier with the average power \( P \approx 4 \times 10^5 \) W would be required for such a fast damping. In this paper we consider much modest (and feasible with the current technology) amplifier with \( P \approx 2 \) W. Then, instead of 3 sec, damping time becomes \( \sim 23 \) min. (Damping time scales with the amplifier power as \( P^{-1/2} \) [2]).

Table (2) contains optical properties of Ti:sapphire. We choose this material as the amplifier medium because of its wide bandwidth (one of the best among optical amplifiers) and because of its relatively long fluorescence lifetime.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refractive index, ( n )</td>
<td>1.76</td>
<td>[4]</td>
</tr>
<tr>
<td>Temp. coefficient (^a), ( d\eta/dT )</td>
<td>( 1.3 \times 10^{-5} ) K(^{-1} )</td>
<td>[7]</td>
</tr>
<tr>
<td></td>
<td>( 1.4 \times 10^{-6} ) K(^{-1} )</td>
<td>[8]</td>
</tr>
<tr>
<td>Saturation intensity, ( I_s )</td>
<td>( 2.4 \times 10^5 ) W/cm(^2)</td>
<td>[4]</td>
</tr>
<tr>
<td>Quantum efficiency, ( \eta )</td>
<td>0.9</td>
<td>[4]</td>
</tr>
<tr>
<td>Fluorescence peak, ( \lambda_f )</td>
<td>780 nm</td>
<td>[4]</td>
</tr>
<tr>
<td>Fluorescence lifetime, ( \tau_f )</td>
<td>3.2 ( \mu )s</td>
<td>[4]</td>
</tr>
<tr>
<td>Bandwidth, FWHM, ( \Delta \omega_0 / 2\pi )</td>
<td>( 10^{14} ) Hz</td>
<td>[4]</td>
</tr>
</tbody>
</table>

\(^a\) There is a discrepancy in the data cited in Ref. [7] and Ref. [8].

As a pump source for Ti:sapphire, we consider a cw argon-ion laser with 15 W of power operated at \( \lambda_{Ar} = 514 \) nm. This laser will produce \( \sim 10 \) W of the available power in Ti:sapphire at \( \lambda_f = 780 \) nm. (The remaining 5 W, which accounts for a difference between absorption photon energy and fluorescence photon energy and quantum

\(^1\) In a general case \( \Delta f \) is the bandwidth of a whole system that includes pick-up, amplifier and kicker, but in OSC it is the amplifier that determines the overall bandwidth.

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efficiency of Ti:sapphire, will go into heat.) From 10 W of available power in the last amplification stage we are going to use approximately 2 W for output signal. The remaining 8 W are continuously drained by the fluorescence emission.

A schematic of an amplifier based on Ti:sapphire is shown in Figure (2). The amplifier has four identical stages with a total gain of $G_{dB} = 44 \text{ dB}$. The corresponding bandwidth of the amplifier, defined as the distance between two frequency points at which the amplitude gain has fallen to half the peak, is calculated [9]:

$$
\Delta f = \frac{3}{2\pi} \sqrt{\frac{G_{dB}}{2} - 3} \approx 4 \times 10^{13} \text{Hz}. \quad (2)
$$

A small signal gain of the amplifier stage is equal [9]:

$$
G_0 = \exp \left\{ \frac{I_p \lambda A_I}{I_s \lambda_f} \right\}, \quad (3)
$$

where $I_p$ is the pump laser intensity. Using $I_s = 2.4 \times 10^5 \text{ W/cm}^2$, $\lambda_f / \lambda_A \approx 1.5$, $\eta = 0.9$ and $G_0 = 12.5$, we get $I_p \approx 10^6 \text{ W/cm}^2$. This intensity can be achieved by focusing a 15 W argon-ion laser into an area of $A = 1.5 \times 10^{-5} \text{ cm}^2$. Assuming a Gaussian beam, we find $\sigma_w \approx 1.5 \times 10^{-3} \text{ cm}$ in a waist and $Z_p = 2.5 \text{ mm}$ for the Rayleigh length. Therefore, a Ti:sapphire crystal with an absorption length of approximately $l = 5 \text{ mm}$ is required. According to [10], this can be made from a sapphire with a doping of $N_{Ti} \approx 5 \times 10^{18} \text{ Ti}^{3+} \text{ ions/cm}^3$ (crystal orientation: c-axis normal to the rod axis).

The value of $I_p$, calculated above, is $\sim 15$ times higher than that in commercial Ti:sapphire lasers, which operate not too far from the damage threshold for a Ti:sapphire crystal due to the effect of thermal lensing [11]:

$$
\frac{1}{F} \approx \frac{1}{2D \lambda F} \frac{dn}{dT} = A. \quad (4)
$$

Here $F$ is the focal length of the thermo lens, $P_T$ is the heating power, and $\lambda_T$ is the thermal conductivity (for sapphire $\lambda_T = 0.33 \text{ W/(cm K)}$ at 300 K, and $\lambda_T = 10 \text{ W/(cm K)}$ at 77 K [12]). One solution of the thermo lensing problem is cooling the Ti:sapphire crystal to the liquid nitrogen temperature [7]. Then, a sharp rise in the thermo conductivity will overpass the increase of $I_p$.

Let us consider now how the pulse of beam radiation in the undulator is distorted by the amplification medium.

There could be the phase and the amplitude distortions, but only the phase distortions possess a threat for cooling, since the amplitude distortions average out during the damping time which consists of many thousands beam passes through a cooling system.

The phase distortions in the amplified signal appeared as a combination of the phase shifts caused by the depletion of the population inversion density in the medium, plus phase shifts caused by the beam energy spread. In the first case the leading part of the pulse changes the population inversion density of the medium, which will then act in a different manner at the trailing part. This means that different pulse parts will ‘see’ different optical path lengths, resulting in a time dependent phase change (chirp). To estimate this effect, let us calculate first the available energy in the Ti:sapphire crystal. It is $26 \mu\text{J}$ for the above parameters (8 W of spontaneous emission and 3.2 $\mu\text{s}$ of a flourescence lifetime). The energy consumed by the amplified pulse is $2W \times 0.55 \mu\text{s} = 1.1 \mu\text{J}$, where 0.55 $\mu\text{s}$ is a time interval between bunches in the TESVATRON ($1/m_f n_0$). Therefore, the change of the population inversion density in Ti:sapphire of the last amplification stage constitutes only 4.3% over the entire pulse length. A corresponding change in the refractive index is $\Delta n = 0.043 \beta N_{Ti}$, where the coefficient $\beta$ was measured to be $\sim 10^{-2} \text{cm}^3$ at 300 K [13]. Therefore, a change in the phase from the head to the tail of the pulse is $\Delta n k t \approx \pm 0.4 \text{ rad}$ ($k = 2\pi/\lambda_f$). A reduction in the cooling rate associated with this phase distortion is 8%.

The second effect (that is phase distortions induced by the beam energy spread) develops when a carrying frequency of the light pulse, $\omega_p$, is near the resonance frequency of atomic transitions in the medium. For an estimation of the magnitude of the phase shifts we approximate the complex gain coefficient $\alpha_m$ [9] with the Lorentzian:

$$
\alpha_m (\omega) = \alpha_m (\omega_0) \frac{1 - i(\omega - \omega_0)/\Delta\omega_0}{1 + (\omega - \omega_0)^2/\Delta\omega_0^2}, \quad (5)
$$

where $\omega_0$ is the central resonance frequency and $\Delta\omega_0$ is the resonance bandwidth. Assuming the input signal in the form:

$$
E_{in} = a(t) e^{i\omega_p t}, \quad a(t) = \begin{cases} a_0 & \text{if } -\tau_a/2 < t < \tau_a/2, \\ 0 & \text{otherwise}, \end{cases} \quad (6)
$$

where $a_0$ is the amplitude, $\tau_a = 2\pi/\omega_p$ is the duration of the pulse of the undulator radiation of a single particle and $p$ is the number of undulator periods, we get for the amplified signal:

$$
E_{out} = a_0 e^{i(\omega_p t + \phi)} \int_{-\infty}^{\infty} \sin \frac{\pi p}{\omega_p} \frac{\omega - \omega_p}{\omega_p} e^{i4\alpha_m (\omega_0) + i(\omega - \omega_p) t} d(\omega - \omega_p) \quad (7)
$$

The evaluation of the integral in Eq.(7) for different $\omega_p$ allows the finding of the phase shift as a function of $\omega_p$. A corresponding plot is shown in Figure (3). In this calculations we assumed $4l\alpha_m (\omega_0) \approx 5$, which gives a required $G_{dB}$. 

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2 There is indication that $dn/dT$ also drops with temperature [7].
with the number of photons coming from the signal:

\[ n_{ph} = \pi \alpha N_b \simeq 2 \times 10^8, \quad (9) \]

where \( \alpha = 1/137 \) is the fine structure constant.

### 3 AMPLIFIER FOR COOLING OF ELECTRONS

A demonstration experiment for OSC can be performed on electrons. As an example, we consider the storage ring DELTA [6]. Damping time given by Eq.(1) for this ring with beam parameters listed in Table (1) is 0.15 sec. At the same time, the synchrotron radiation damping at the energy of 300 MeV is 0.7 sec. Cooling would require the amplifier with \( G_{dB} = 27 \) and \( P = 1.6 \) mW. Such an amplifier is much simpler than the above-considered amplifier and can be made from two or three amplification stages, depending on the difficulty of the thermo lensing problem. Besides that, we do not expect to have any other problem with the amplifier for this machine.

### 4 CONCLUSION

By considering the optical amplifiers for OSC of antiprotons in the TEVATRON collider with the damping time of 23 min and cooling of electrons in the DELTA storage ring with the damping time of 0.15 sec, we come to the conclusion that, among the various effects known to us, related to optical amplification no one seems capable of having any noticeable impact on the cooling efficiency in OSC.

### 5 REFERENCES