EMITTANCE STUDY OF COMBINED FUNCTION TBA LATTICE AT SRRC

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Abstract

In order to provide more flexible light source to the users, the emittance of the SRRC TBA lattice was studied. Several ways to reduce or enlarge the emittance of the lattice had been studied. The smaller emittance of the ring had been approached by releasing the dispersion free constraints in the long straight section. The results of the emittance study are described in this paper.

1 INTRODUCTION

As a third generation light source, a major goal of SRRC is to achieve higher photon brightness. In addition to the insertion devices installed in long straight sections to enhance the photon flux, the improvement of ring lattice design to minimize the beam emittance has also been studied. Several ways to reduce or enlarge the emittance of the lattice had been studied.

The natural beam emittance of an electron storage ring is determined by the equilibrium of radiation damping with quantum fluctuation[1]. For a non isomagnet magnet the natural beam emittance is:

$$\varepsilon_x = \frac{\sigma_{x\beta}^2(s)}{\beta(s)} = \frac{C_q\gamma_0^2 < \frac{H}{\rho^3} >}{J_x < \frac{1}{\rho^2} >}. \qquad (1)$$

The natural beam emittance of an isomagnet magnet is given as:

$$\varepsilon_x = \frac{C_q\gamma_0^2 < H >_{\rm mag}}{J_x\rho}. \qquad (2)$$

Where $C_q = 3.84 \times 10^{-13} m$ is the quantum coefficient, γ_0 the electron energy over the electron rest mass energy, ρ the bending radius of the dipole magnet and J_x is damping partition number of the bending plane. The H function is defined as: $H = \gamma D^2 + 2\alpha DD' + \beta D'^2$, where α, β, γ are the Courant and Synder (twiss) parameters[2]. D is the dispersion function and D' is the derivative of the dispersion function with respect to the path length. $< H >_{\rm mag}$ means averaging over the bending magnet. The minimum beam emittance of an isomagnet ring is[3]

$$\varepsilon_{me} = \frac{1}{4\sqrt{15}}\frac{C_q\gamma_0^2\theta^3}{J_x}. \qquad (3)$$

Here θ is the bending angle of the dipole. However as is proved by S.Y. Lee[4] that it is impossible for an isomagnet TBA or QBA with equal dipole length to achieve the above minimum beam emittance. The length of the inner dipole should be $3^{1/3}$ times the length of the outer dipole. The achievable equilibrium beam emittance of the isomagnetic sector equal dipole length TBA lattice at small angle approximation is

$$\varepsilon^*_{\rm METBA} = 1.1064\frac{C_q\gamma_0^2\theta^3}{4\sqrt{15}J_x}. \qquad (4)$$

In this report the minimum emittance of SRRC TBA lattice is derived. The study of the adjustment of the ring lattice to achieve different beam emittance is reported. An feasible operation path of the ring to achieve the lower emittance lattice is also given.

2 THE STUDY OF THE BEAM EMITTANCE OF SRRC COMBINED FUNCTION TBA LATTICE

The SRRC is a six-fold symmetry TBA lattice with a mirror symmetry from the middle of the second bend in the arc[5]. There are four pairs of quadrupoles in each superperiod, three(Q1, Q2 and Q3) in the long straight section and one(Q4) in the achromat. The combined function dipole is rectangular of length 1.22 m, bending radius 3.495 m and gradient 0.37.

Using similar procedures as in reference[4], the H function averaged through a combined sector magnet is given by:

$$\begin{aligned}
< H >_{\rm mag} &= H_0 + (\alpha_0 D_0 + \beta_0 D_0')\theta E(q) - \frac{1}{3}(\gamma_0 D_0 \\
&\quad + \alpha_0 D_0')\rho\theta^2 F(q) + \frac{\beta_0}{3}\theta^2 A(q) \\
&\quad - \frac{\alpha_0}{4}\rho\theta^3 B(q) + \frac{\gamma_0}{20}\rho^2\theta^4 C(q)
\end{aligned}$$

$$\begin{aligned}
H_0 &= \gamma_0 D_0^2 + 2\alpha_0 D_0 D_0' + \beta_0 D_0'^2 \\
A(q) &= -\frac{(6q - 3\sinh 2q)}{4q^3} \\
B(q) &= \frac{6q - 8\cosh q + 2\sinh 2q}{q^4} \\
C(q) &= \frac{30q - 40\sinh q + 5\sinh 2q}{q^5} \\
E(q) &= -\frac{2(1 - \cosh q)}{q^2} \\
F(q) &= \frac{6(q - \sinh q)}{q^3}
\end{aligned}$$

Where $q = \sqrt{|K|}l$, l is the length of the dipole, $K = -(k - \frac{1}{\rho^2})$, k is the gradient of the combined function magnet and ρ is the bending radius. $\beta_0,\ \alpha_0,\ \gamma_0,\ D_0,\ D'_0$ are the value of the twiss parameters and dispersion function at the beginning of the dipole. By setting $D_0 = D'_0 = 0$ and minimize the $<H>_{\rm mag}$ with respect to initial twiss function, the minimum $<H>_{\rm mag}$ for the outer dipoles is obtained

$$<H>_{\rm outer} = \frac{\sqrt{16AC - 15B^2}}{4\sqrt{15}} \rho\theta^3.$$

The $<H>_{\rm mag}$ for the center dipole minimized with respect to initial dispersion function is given by:

$$<H>_{\rm center} = \rho\theta^3 \frac{1}{12}(\frac{\beta_0}{l}\bar{A} - \alpha_0\bar{B} + \frac{4}{15}l\gamma_0\bar{C})$$

$$\bar{A} = 4A - 3E^2,\ \ \bar{B} = 3B - 2EF,\ \ \bar{C} = \frac{9}{4}C - \frac{5}{4}F^2$$

By applying the symmetry condition of betatron function at the center dipole and the matching of dispertion function between center dipole and outer dipoles, the value of the twiss parameter at the beginning of the center dipole and the minimum $<H>_{\rm center}$ can be determined. The initial values of $\beta_0,\ \alpha_0$ and γ_0 are $1.25388l,\ 0.49743,\ \frac{0.99487}{l}$. The same results can also be applied to rectangular dipole magnet. For SRRC q is 0.654875. The minimum emittance of the combined rectangular magnet of SRRC is given by:

$$\begin{aligned} \varepsilon &= \frac{C_q\gamma_0^2}{J_x\rho_0}(\frac{2}{3} <H>_{\rm outer} + \frac{1}{3} <H>_{\rm center}) \\ &= 1.1797\frac{C_q\gamma_0^2\theta^3}{4\sqrt{15}J_x} \end{aligned} \quad (5)$$

For small angle limit, this is reduced to equation (5). The minimum emittance of SRRC is 5.97×10^{-9}m.rad

The operational emittance of SRRC is 2.115×10^{-8}m.rad which is 2.5 times larger than the value of possible minimum beam emittance. An attempt to achieve a feasible operation with lower beam emittance is tried. The quadrupole strength of the designed lattice had been modified to try to fit the matching condition of the minimum beam emittance. However the real components are not free to move in space. With this constraint the original design of SRRC lattice is near a local minimum of beam emittance. Without changing hardware components and still keeping achromatic condition, the reduction of beam emittance by adjusting the strength of quadrupoles is very little. In order to further reduce the beam emittance a few systematic searches had been tried in the study. The first is to change the strength of Q2 stepwisely while keeping the strength of the other three quadrupoles the same. The change of the strength of Q2 does not reduce the beam emittance much. The adjustable range of Q2 strength to obtain a stable solution is from $2.78\ m^{-2}$ to $3.18\ m^{-2}$. Another study is moving the position of Q4 slightly and varying the strength of Q4 to preserve the achromatic condition. From the study we can have a lower emittance lattice which is 70% lower of the nominal value. But in real operation the movement of the quadrupole is not feasible. Without moving the quadrupole position and still keeping the achromatic condition, it is hard to reduce the beam emittance. Therefore we release the achromatic condition in the long straight section. The tune of lattice is matched to the operational value by adjusting the strengths of Q1, Q2 and Q3. The results of Q4 strength and tune matching is shown in Table 1.

Table 1: The natural beam emittance with different strength of Q4.

Q4	ϵ_0	ν_x	ν_y	$\frac{\sigma_p}{p}$
m^{-2}	10^{-8} (m.rad)			10^{-4}
2.51	5.11	7.247	4.097	6.91
2.61	3.39	7.247	4.097	6.64
2.71	2.12	7.247	4.097	6.47
2.81	1.45	7.247	4.097	6.37
2.857	1.39	7.247	4.097	6.35
2.91	1.49	7.247	4.097	6.33
3.01	2.28	7.247	4.097	6.34
3.11	3.75	7.247	4.097	6.40
3.21	5.74	7.247	4.097	6.53

From Table 1 it shows that the minimum emittance 1.39×10^{-8}m.rad, occurs at Q4 strength of 2.857 m^{-2}. By varying the strength of Q4, different value of emittance is derived. The reason to match the tune close to the operational tune will be discussed below. In the next section we take the minimum emittance derived in this case as an example to apply to the storage ring. The transversal beam size σ_x and σ_y of the nominal lattice and the lower emittance lattice is shown in Figure 1.

3 EXPERIMENT OF APPLYING THE LOWER EMITTANCE LATTICE TO THE STORAGE RING

Firstly, we tried to inject the beam with lower emittance lattice. However, the dispersion function is smaller at the sextupole position and the need of the strength of sextupole to correct the chromaticity is higher. The higher sextupole settings shrink the dynamic aperture and it was hard to inject beam. An alternative method is to switch to the lower emittance lattice after beam was stored. The advantage to keep the tune of the lower emittance lattice close to the operational one was with no beam loss during the lattice transition. If the tune of the two lattice is far away then the probability to cross the resonance and cause beam loss during the lattice transition is increased. However at the beginning of the experiment we still encountered beam loss during lattice transition. The problem was solved by finding a path to avoid the fifth difference resonance line in tune space which might be the cause of beam loss in the previous experiment. The path with and without beam loss

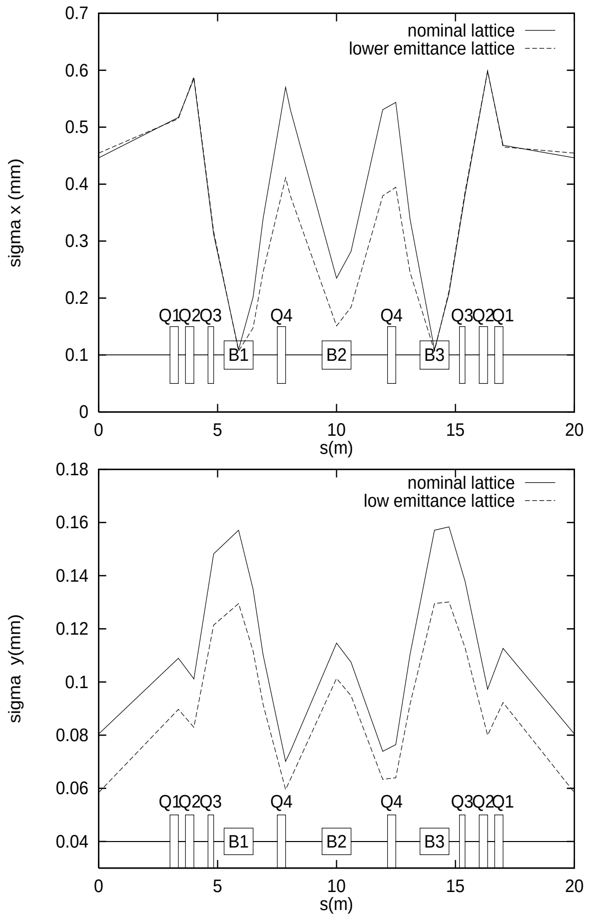

Figure 1: The transversal beam size of the operational lattice and the low emittance lattice.

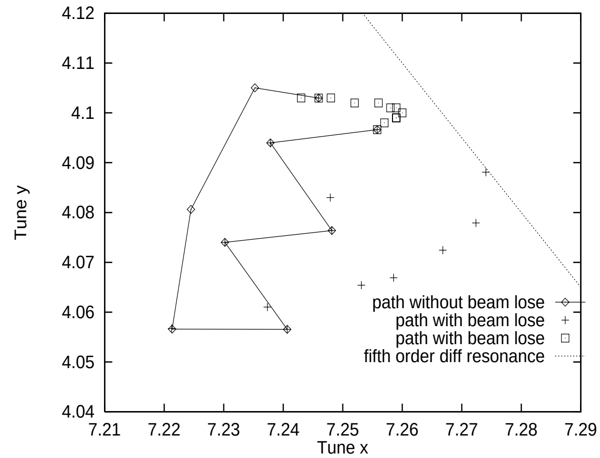

Figure 2: The transitional path from operational lattice to the low emittance lattice in the tune space.

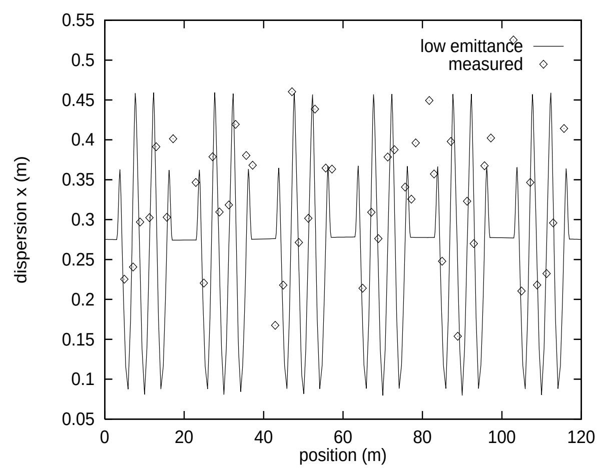

Figure 3: The measured dispersion function and modeled dispersion function of low emittance lattice.

in tune space is shown in Figure 2. The dispersion function which affected the beam emittance was measured and compared to the simulation value is shown in Figure 3.

4 DISCUSSION

A lattice with lower beam emittance derived from the original operation lattice is presented. The beam emittance is reduced by 35%. An operational path to change the ring lattice from the nominal lattice to a lower beam emittance lattice without beam loss was achieved. A table of quadrupole power supply setting is established for the control system to transit the lattice to lower emittance automatically. By using the same algorithm different value of beam emittance lattice is applicable to the SRRC storage ring.

5 REFERENCES

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