LONGITUDINAL RADIATION EXCITATION OF QUASI-ISOCHRONUS RING 'New SUBARU'

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Abstract

The longitudinal radiation excitation is a diffusion process in time axis which came from the stochastic fluctuation of where the photo-emission takes place. Usually this radiation excitation hides behind the well-known radiation excitation of energy spread. However in a quasi-isochronous ring, it dominates over the bunch stretch owing to the radiation excitation of energy spread. We give an analytical expression for the equilibrium state within linear optics including this excitation. Some consequencies of this excitation is calculated using parameters of New SUBARU.

1 INTRODUCTION

A quasi-isochronus storage ring has an advantage in forming ultra short bunch, which has many applications. It is expected to offer higher luminosity in a collider, a very short pulsed synchrotron light and hopefully a higher peak current of an electron beam. The idea of utilizing a quasi-isochronous ring to shorten the bunch length comes from a well-known expression that the equilibrium bunch length is proportional to the square root of the linear momentum compaction factor $\sqrt{\alpha}$ [1]. The shortening of the equilibrium bunch length by means of reducing α is demonstrated at several synchrotron light source rings [2, 3, 4, 5, 6, 7, 8]. Their studies showed that an intensity effct, a potential well distortion or instabilities, restricts the bunch to shorten when non-linear part of the momentum compaction factor is corrected. The limit to the bunch shortening with low beam current was not clarified until we introduced an idea of *longitudinal radiation excitation* [9].

The idea of the longitudinal excitation is a heating by the stochastic fluctuation of where the photo-emission takes place in a ring. If the photo-emission takes place at the first bending dipole from the RF, the electron goes around most part of the ring with lower energy compared with the starting one. On the other hand, if the photo-emission happens at the last bending dipole, the electron goes around most part of the ring with starting energy and finishes the revolution with the same energy as the previous case. When α is positive, the revolution time of the former case is shorter than that of the latter. This stochastic variation produces a heating in time axis. In usual storage rings the longitudinal excitation is buried in the energy spread excitation and hardly can be observed. On the other hand, in a quasi-isochronous storage ring the longitudinal excitation becomes remarkable, since one can reduce α to zero but can not do the longitudinal excitation. In this report the quantity of this effect is calculated using the New SUBARU storage ring.

Table 1: Parameters of New SUBARU ring.

Electron energy	$E_0 =$	1.5 GeV
Circumference	$L_0 =$	118.716 m
Revolution period	$T_0 =$	396 ns
Momentum compaction factor	$\alpha =$	-0.0012
		$\sim$ +0.0011
Natural energy spread	$\sigma_{EN} =$	0.072 %
Longitudinal damping time	$1/\kappa_s =$	3.42 ms
RF voltage gradient	$V' =$	550 kV/ns
Bending angle of normal B	$\theta =$	34 degrees
Bending angle of invert B	$\theta_c =$	-8 degrees
Curvature of radius	$\rho_0 =$	3.22 m

New SUBARU [10] is a name of the project to construct an 1.5 GeV synchrotron light source ring in the SPring-8 site using the 1.0 GeV Linac as an injector. Laboratory of Advanced Science and Technology for Industry (LASTI) of Himeji Institute of Technology is in charge of the construction collaborating with SPring-8. The storage ring is of a race track type with two fold symmetry. Table1 summarizes main storage ring parameters. It has 6 quasi-isochronus and achromatic bending cells and 6 dispersion free straight sections. A type of the bending cell is a quasi-isochronus TBA using a small invert dipole as a middle bend in order to control α.

2 LONGITUDINAL RADIATION EXCITATION

2.1 Analytical expression of the longitudinal excitations

To derive analytical expressions for the longitudinal excitation, we assume the followings; (a) circulating electrons or positrons are ultra-relativistic, (b) the ring has only one RF gap, (c) the absolute value of curvature radius in bending dipoles is constant.

We assume that the RF gap is located at $s = 0$ or $s = L_0$, where s is the azimuthal coordinate and L_0 is the circumference of a ring. At the lowest order of perturbation, the change of path length in one turn caused by N photo-emissions is expressed by

$$\Delta L = \sum_{j}^{N} \int_{s_j}^{L_0} \frac{\eta(s)}{\rho(s)} \frac{u_j}{E_0} ds \,. \tag{1}$$

where $\eta(s)$, $\rho(s)$ and E_0 are respectively the horizontal dispersion function, the radius of curvature and the synchronous energy. Symbols u_j is the energy loss produced by the jth photo-emission at the azimuthal location s_j. In

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this paper Δ stands for the change by the photo-emissions in one turn, while δ stands for a displacement from the barycenter.

The displacement in relative energy by N photo-emissions and that of the arriving time from the reference are

$$\frac{\Delta E}{E_0} = \sum_j^N \frac{u_j}{E_0}, \tag{2}$$

$$\Delta\tau = \frac{T_0}{L_0}\sum_j^N \int_{s_j}^{L_0} \frac{\eta(s)}{\rho(s)}\frac{u_j}{E_0}ds, \tag{3}$$

where T_0 is a revolution period. Introducing the incomplete momentum compaction factor $\tilde{\alpha}(s_j)$ defined by

$$\tilde{\alpha}(s_j) = \frac{1}{L_0}\int_{s_j}^{L_0}\frac{\eta(s)}{\rho(s)}ds, \tag{4}$$

Eq. (3) is simply rewritten by

$$\Delta\tau = T_0\sum_j^N \frac{u_j}{E_0}\tilde{\alpha}(s_j). \tag{5}$$

For the sake of a convenience we write the variance of $\tilde{\alpha}$ as I_α, which is calculated only from the geometrical parameters of a ring:

$$I_\alpha = \frac{1}{L_0^2}\left\langle\left[\int_{s_j}^{L_0}\frac{\eta(s)}{\rho(s)}ds - \langle\tilde{\alpha}\rangle L_0\right]^2\right\rangle. \tag{6}$$

The variances of $\Delta E/E_0$ and $\Delta\tau$ are calculated as

$$\left\langle\!\left\langle\left(\frac{\Delta E}{E_0} - \left\langle\!\left\langle\frac{\Delta E}{E_0}\right\rangle\!\right\rangle\right)^2\right\rangle\!\right\rangle = \langle N\rangle\left\langle\frac{u^2}{E_0^2}\right\rangle, \tag{7}$$

$$\left\langle\!\left\langle\left\langle(\Delta\tau - \langle\langle\langle\Delta\tau\rangle\rangle\rangle)^2\right\rangle\!\right\rangle\!\right\rangle = T_0^2 I_\alpha\langle N\rangle\left\langle\frac{u^2}{E_0^2}\right\rangle, \tag{8}$$

where the brackets $\langle\ \ \rangle$ represent an average over N, over photon energy or over s_j only at the bending dipoles.

2.2 Equilibrium energy spread and bunch length

To calculate the equilibrium bunch length, we start with a matrix equation of the linearized synchrotron oscillation in a storage ring. A single turn transfer of a synchrotron oscillation is described by

$$X_n = AX_{n-1} + D_n. \tag{9}$$

Here the vectors X_n and D_n denote respectively the state of a particle in the synchrotron oscillation phase space and the radiation excitation in the nth turn and A is a transfer matrix including radiation damping. The state vector X_n is defined at just before the RF gap ($s = L_0$) by

$$X_n = \begin{pmatrix} \left.\frac{\delta E}{E_0}\right|_n \\ \left.\delta\tau\right|_n \end{pmatrix}, \tag{10}$$

where $(\delta E/E_0)|_n$ and $\delta\tau|_n$ are the relative energy displacement and the arriving time displacement from the barycenter at just before the RF gap in the nth turn. The one turn transfer matrix A is written as

$$A = \begin{pmatrix} 1-2\kappa_s T_0 & \frac{eV'}{E_0}(1-2\kappa_s T_0) \\ -\alpha T_0 & 1-\alpha T_0\frac{eV'}{E_0} \end{pmatrix}. \tag{11}$$

where V' and κ_s are the gradient of the RF acceleration voltage and the radiation damping coefficient. The vector D_n is described by

$$D_n = \begin{pmatrix} \left.\frac{\Delta E}{E_0}\right|_n - \left\langle\!\left\langle\frac{\Delta E}{E_0}\right\rangle\!\right\rangle \\ \left.\Delta\tau\right|_n - \langle\langle\langle\Delta\tau\rangle\rangle\rangle \end{pmatrix}. \tag{12}$$

The suffix n in the averaging brackets is omitted because the averaged values are independent of the turn number n. When $\alpha eV' > 0$ the equibrium state X_∞ exists whether the eigen values of A are complex or real. That is

$$X_\infty = \sum_{m=0}^{\infty} A^m D_m. \tag{13}$$

Although the energy spread is constant over a circumference, the equilibrium bunch length varies along s. The state vector after infinite turns at any place s in the ring $X_\infty(s)$ is given by a transformation from $X_\infty = X_\infty(L_0)$ as follows

$$\begin{pmatrix} 1 & 0 \\ -\tilde{\alpha}(s)T_0 & 1 \end{pmatrix} X_\infty(s) = X_\infty. \tag{14}$$

The equilibrium energy spread is given by the variance of the energy displacement of $X_\infty(s)$, that is

$$\sigma_E^2 = \frac{1+(\Omega^* T_0)^2 I_\alpha/\alpha^2}{1-\left(\frac{1}{2}\Omega^* T_0\right)^2}\sigma_{EN}^2 \tag{15}$$

with

$$(\Omega^* T_0)^2 = \frac{\alpha eV'}{E_0}T_0. \tag{16}$$

Here σ_{EN}^2 is the natural energy spread given by

$$\sigma_{EN}^2 = \frac{1}{4\kappa_s T_0}\langle N\rangle\left\langle\frac{u^2}{E_0^2}\right\rangle. \tag{17}$$

2.3 Intrinsic Bunch Shortening Limit

The variance of the equilibrium bunch length $\sigma_\tau^2(s)$ depends on $\tilde{\alpha}(s)$, therefore a function of position in the ring. If we can choose any value as $\tilde{\alpha}(s)$, $\sigma_\tau^2(s)$ takes a minimum

$$\sigma_{\tau,\min}^2 = \left[1+(\Omega^* T_0)^2\frac{I_\alpha}{\alpha^2} + \frac{4(\kappa_s T_0)^2 I_\alpha}{\alpha^2+(\Omega^* T_0)^2 I_\alpha}\right] \times \left(\frac{\alpha}{\Omega^*}\right)^2\sigma_{EN}^2 \tag{18}$$

at

$$\tilde{\alpha}\left(s\right) = \frac{\alpha}{2}\left[1 - \frac{4\kappa_s T_0 I_\alpha}{\alpha^2 + \left(\Omega^* T_0\right)^2 I_\alpha}\right]. \qquad (19)$$

In the case of a sufficiently small α, Eq. (18) is approximated as

$$\sigma^2_{\tau,\,\mathrm{min}} \approx T_0^2\left[I_\alpha + 4\left(\frac{\kappa_s E_0}{eV'}\right)^2\right]\sigma^2_{EN}\,. \qquad (20)$$

One can easily confirm that I_α approaches to a nonzero constant as $\alpha \to 0$ and that $\sigma_{\tau,\,\mathrm{min}}$ has a lower limit. Because $4[(\kappa_s E_0)/(eV')]^2$ is always positive, $\sigma_{\tau,\,\mathrm{min}}$ cannot be smaller than

$$\sigma^2_{\tau,\,0} = T_0^2 I_\alpha \sigma^2_{EN}\,. \qquad (21)$$

This is an intrinsic limit of the bunch length determined by the ring geometrical parameters I_α, T_0 and σ^2_{EN}.

2.4 *Variance of the incomplete momentum compaction factor I_α*

In a conventional ring the continuous increase of $\tilde{\alpha}(s)$ around the ring is very much larger than a oscillating part of $\tilde{\alpha}(s)$, so

$$I_\alpha = \frac{1}{12}\alpha^2$$

is a good approximation. However in a quasi-isochronus ring, sections with positive and negative $\tilde{\alpha}$ appear alternatively. In a quasi-isochronus ring using TBA cell with invert dipole,

$$\begin{aligned} I_\alpha &= \left(\frac{\rho_0}{L_0}\right)^2\left(\frac{\theta^6}{252}\right) \\ &\times \left[1 + \frac{2\theta_c}{3\theta}\left(1 + \frac{\theta_c^3}{20\theta^3}\right)\left(1 - \frac{\theta_c}{2\theta} + \frac{\theta_c^2}{4\theta^2}\right)\right] \end{aligned} \qquad (22)$$

at the isochronus limit. Here θ and θ_c are bending angles of the normal the invert bend, respectively. We have assumed that both θ and θ_c are much less than unity.

3 CONSEQUENCES OF LONGITUDINAL RADIATION EXCITATION

The most important consequence of the longitudinal radiation excitation is the existance of the intrinsic bunch shortening limit. Now we know the final goal of the study on the quasi-isochronus operation. We also predict the energy shift from the bunch head to the tail, represented by Eq. (19).

The other consequence is a restriction to the isochronus ring FEL (ISRFEL) [11]. Its basic idea is that in isochronus or quasi-isochronus storage ring the electrons remain trapped in the optical potential wells formed by the laser fields. Such a design eliminates the heating by the reconstruction of microbunches taking the advantage of radiation damping in a storage ring. However, according to our calculations any shorter bunch than $\sigma_{\tau,0}$ cannot exist stationary in any linear potential. Consequently ISRFEL with shorter wave length than several times $\sigma_{\tau,0}$ is forbidden. In New SUBARU $\sigma_{\tau,0}$ was calculated to be about 0.12ps (=0.035mm). therefore ISRFEL has a possibility only in over a few tenth milli meter wave region.

The other consequence is the supression of collective beam instability of ultra-high frequency. The longitudinal radiation excitation reduces any fine time structure in a beam, such as density modulation, energy modulation, and transversal oscillation. The diffusion constant in time axis is

$$\begin{aligned} \kappa_t &= \left\langle\left\langle\left\langle\left(\Delta\tau - \langle\langle\langle\Delta\tau\rangle\rangle\rangle\right)^2\right\rangle\right\rangle\right\rangle / T_0 \\ &= 4\kappa_s T_0^2 I_\alpha \sigma^2_{EN}\,. \end{aligned} \qquad (23)$$

This would reduce the modulations of harmonic number n with damping coefficient of $\left(2\pi n/T_0\right)^2\kappa_t$.

In New SUBARU, we compare this damping coefficient with that of the Landau damping of longitudinal microwave instability, using coasting beam approximation. We used $n = 10T_0/\sigma_\tau$. The result says at smaller α than 2×10^{-6} the diffusion by the longitudinal radiation excitation dominates over the Landau damping in New SUBARU.

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