BEAM DYNAMICS STUDIES ON THE ISAC RFQ AT TRIUMF

S. Koscielniak, R.E. Laxdal, R. Lee, L. Root
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada

Abstract
An 8 m long split-ring RFQ linac is being constructed at TRIUMF as part of the ISAC radioactive beam project. The RFQ is designed to accelerate unstable nuclei with \( q/A \geq 1/30 \) from 2 keV/u to 150 keV/u. Several unusual beam dynamics studies have been pursued during the latter stages of the design. To furnish engineering alignment tolerances, particle tracking in the 10-term potential was performed to estimate the RFQ performance with respect to vane displacement errors; the results are compared with a simple analytical model. In order to explore fringe field effects, particle tracking studies through computed three-dimensional fields at the RFQ entrance and exit were made to optimize the radial matching section (RMS) to the LEBT optics, and to appraise the value of a transition cell[9]. In this paper we describe the methods and results of the beam dynamics simulations and summarize final specifications for the RFQ.

1 INTRODUCTION
The ISAC project conceptual design[1] consists of an ISOL and two c.w. linear accelerators: a 35 MHz RFQ and a 105 MHz drift tube linac (DTL). The Radio Frequency Quadrupole (RFQ) beam dynamics design[2] has stabilized with the following values: focusing parameter \( B = 3.5 \), characteristic radius to pole tip \( r_0 \) and vane transverse radius of curvature \( \rho = r_0 = 0.741 \) cm, modulation index \( 1.1 \leq m \leq 2.6 \), local bore radius \( 0.37 \leq a(\text{cm}) \leq 0.71 \) and inter-vane voltage \( V = 74 \) kV. An unusual feature of the design is the constant synchronous phase of \( \phi_s = -25^\circ \) as a consequence of an external multi-harmonic buncher[5] operating at 11.66 MHz fundamental frequency. The buncher introduces an 86 ns pulse spacing into the beam that is useful in some physics experiments. The tapering down of the focusing parameter, \( B \), toward the RFQ exit, in an earlier design, has been eliminated in favour of exit 'transition cells'. The RMS has been customized to minimize beam size and convergence at the RFQ entrance. The RMS is immediately followed by accelerating cells with \( m = 1.1 \), and a transition cell has been introduced to avoid the discontinuity.

2 RADIAL MATCHING SECTION
The longitudinal profile of the vane tips is obtained from an analytic expression[4] for the RMS potential. Since the vane cross-sections are semi-circles of radius \( \rho \) with straight vane extensions and differ from those assumed in the analytic study the RMS region of the RFQ has been modelled using RELAX3D[13, 6].

The transverse emittance orientation required at the RFQ entrance is determined using TRACK[11] to trace the matched ellipses at the exit of the RMS backwards through the RELAX3D potentials and then back to the last matching quadrupole located 9 cm in front of the tank entrance. The emittance growth due to aberrations in the 4Q matching section at the RFQ entrance is found to decrease as the distance from quadrupole to RMS decreased and as the number of cells in the RMS increased. A quadrupole to RMS spacing of 17.8 cm and a 10 cell RMS section, producing 0.7 % emittance growth for a 50\( \pi \mu \text{m} \) emittance beam, is used in the final design.

Figure 1: The fields in the RMS calculated using RELAX3D.

A transition cell with a sinusoidal half-period longitudinal vane shape is used to join the unmodulated RMS vanes to the first modulated cell. To make room for this, the RMS is shortened to nine cells, and the tenth cell is the transition cell. This section of the RFQ along with the first three accelerating cells is modelled using RELAX3D, and the calculated electric fields are shown in Fig. 1. Particles are tracked through the calculated RMS potential grids and are then fed into PARMTEQ and accelerated through the RFQ. For comparison, particles are also simulated using PARMTEQ only. The results show that the transition cell has no significant effect on the RFQ’s acceptances or output emittances and so acts as a successful bridge through this non-adiabatic region.

3 CPACK CALCULATIONS
Eight term potentials calculated by CPACK[10] are used in PARMTEQ to study beam dynamics of the accelerating section of the RFQ. In order to maintain synchronism and optimum transverse focussing, the analytic two-term potential values for the local aperture \( a \) and the modulation \( m \) are adjusted using a procedure similar to that described in references[3, 2], so that with the revised cell parameters, the two lowest order, CPACK calculated, coefficients are
equal to their two term potential values. After the adjustments to $a$ and $m$, PARMTEQ calculations using the 8-term CPACK potentials give results nearly identical to those produced using the ideal two term potential.

4 EXIT TRANSITION CELL

Crandall[9] suggested terminating the high energy end of an RFQ with a transition cell which would connect the exit of the last modulated cell to a section of unmodulated vane having a constant radius to pole tip $r_0$, hence zero on-axis potential. This makes the final energy easier to calculate analytically and by adjusting the length of the unmodulated section, the orientation of the output transverse emittances may be changed. In our case we calculate with RELAX3D the potentials at the exit and so can accurately predict the final energy, however we have found that the transition cell allows us to exert more control on the output energy and RFQ length. The ISAC RFQ, as designed, uses a simplified version of Crandall’s transition cell.

The ISAC RFQ must have an output energy of 150 keV/u, and, from rf design considerations, a length of $\approx 760$ cm while GENRFQ/PARMTEQ originally produced a design of 759.38 cm and 150.5 keV/u. To lengthen the design and to decrease the energy, the last cell was converted into a transition cell with a sinusoidal half period to bring the vanes to quadrupole symmetry. A 0.62 cm long unmodulated vane section follows the transition cell to bring the RFQ vanes to the correct length. A RELAX3D model of the exit region is shown in Fig. 2. Ray tracing through the RELAX3D calculated potentials indicates that this combination will produce a beam with an energy of 150.08 keV/u and with emittances which the 4Q optics section downstream of the RFQ can easily handle.

![RELAX3D model of the exit region showing three regular cells, a transition cell and a short unmodulated end section.](image)

Figure 2: A RELAX3D model of the exit region showing three regular cells, a transition cell and a short unmodulated end section.

Additional RELAX3D simulations using differing lengths $L_u$ of the unmodulated section are summarized in Figure 3, and show how changing $L_u$ can alter the transverse Twiss parameters. Given some freedom on the final length of the vanes, the transition cell and parallel vane drift can be effectively used to tailor the transverse characteristics of the output beam.

![Output emittances for end cells composed of a transition cell and an unmodulated vane of length $L_u$.](image)

Figure 3: Output emittances for end cells composed of a transition cell and an unmodulated vane of length $L_u$.

5 CALCULATED RFQ PERFORMANCE

The final RFQ performance simulations are done using an initial particle ensemble with matched transverse emittances of $\epsilon = 50\pi \mu m$ and the 11.66 MHz pre-bunched longitudinal distribution shown in Figure 4(a). Particles are tracked through the RMS RELAX3D entrance grid, through the RFQ’s acceleration section using PARMTEQ and the the 8-term CPACK potentials, and finally through the transition cell RELAX3D exit grid. 81 % of the beam is captured into the central RFQ bucket, 3.5 % of the beam is captured into satellite buckets located between the central buckets, and 15 % of the beam is transported to the RFQ exit but not accelerated. Transversely the emittance growth is $\sim 3 \%$. An ellipse in longitudinal phase space of area $0.22\pi$ keV/u-nsec encloses 98 % of the particles at the RFQ exit as shown in Figure 4. The beam quality is adequate for injection into the DTL[12] which will follow the RFQ.

![Input and output longitudinal emittance.](image)

Figure 4: Input and output longitudinal emittance.

6 SENSITIVITY ANALYSIS

An essential part of a design study is to perform a sensitivity analysis of the performance (transmission and emittance) with respect to injection errors of the beam and mechanical errors of the accelerating/focusing structure. Further details and numerical values appear in Reference [2].

6.1 Beam injection errors

The PARMTEQ program was modified so that the description of a field-corrected RFQ could be read from file. Based upon tracking an ensemble of 4400 particles (with the 8-term potential) with various initial injection errors it appears there is a quadratic dependence of the transverse
emittance growth on transverse injection errors, e.g. Figure 5.

![Figure 5: Emittance growth vs horizontal displacement.](image)

6.2 Vane position errors

The vane displacement study was performed with the TRIUMF versions of RFQCOEF[10] and PARMTEQ. The RFQ m, a, l description is fed into COEF so that vane displacements can be added and new potential function coefficients calculated.\(^1\) When vanes are moved, the 8-term potential is supplemented with dipole and sextupole components to give a 10-term potential. The dipole coefficients were calibrated against analytic[7] estimates. Let us number the vane displacement study was performed with the TRIUMF versions of RFQCOEF[10] and PARMTEQ. The RFQ m, a, l description is fed into COEF so that vane displacements can be added and new potential function coefficients calculated.\(^1\) When vanes are moved, the 8-term potential is supplemented with dipole and sextupole components to give a 10-term potential. The dipole coefficients were calibrated against analytic[7] estimates. Let us number the types of vane displacement are important: (i) where vanes counter-clockwise, and suppose that the pair of vanes 2&4 both move upward an amount \(\delta\), while vanes 1&3 remain fixed. The potential function is:

\[
\Phi = \left(\frac{V}{2}\right) \left\{ A_{10} (r^2 \cos 2\theta + 2\delta r \sin \theta) + A_{10} k z [I_0 (kr) + k\delta x I_1 (kr) \sin \theta]\right\}.
\]

Based upon the mechanical support and connections two types of vane displacement are important: (i) where vanes 1&3 and/or 2&4 move as pairs with radial errors (likely vibrational modes); and (ii) where all vanes twist (likely thermal deformation). As a means to ‘roll together’ the variety of cases it was decided to plot emittance growth and transmission versus the average modulus of the vane displacement. The transverse emittance growth dependence on vane error appears to be quartic, Figure 6. The longitudinal inverse ‘brilliance’ [where \(B_z^{-1} = \epsilon_z/(\text{survival fraction})\)] has a quadratic dependence on vane error.

6.3 Emittance growth

Non-linear terms in the equation of motion give rise to an amplitude dependence of the betatron tune and to amplitude increase; so that emittance grows. Let \(dX\) and \(dX'\) be alignment and angle errors of the beam centroid. Let \(\delta\) the beam half-width and \(\delta'\) the maximum divergence (i.e on the envelope). If filamentation (due to tune spread) is complete but there is no change in action, the emittance growth ratio can be estimated[8][7] (assuming \(\alpha \approx 0\)) as follows:

\[
\frac{\epsilon_{\text{new}}}{\epsilon_{\text{old}}} = 1 + 2 \left[ \frac{dX^2}{\delta^2} + \frac{(dX')^2}{(\delta')^2} \right].
\]

For the particular case of a constant displacement \(\delta\) of a pair of vanes, we substitute \(dX = \delta\), \(dX' = 0\) to obtain

\[
\frac{\epsilon_{\text{new}}}{\epsilon_{\text{old}}} = 1 + 2 \frac{\delta^2}{\delta^2} = 2.
\]

The theoretical curve of Fig. 6. For the ISAC, the emittance is \(\epsilon_{\text{old}} = 0.1\pi\) mm.mrad (normalized), \(\delta = 2\) mm and \(\delta' = 0.05\) mradian.

6.4 Conclusion

The emittance growth due to vane errors grew faster than the theoretical quadratic dependence, which implies the ‘action’ must be increasing. The emittance growth due to beam errors grew slower than the theoretical dependence, which implies that filamentation is not complete. Clearly, vane errors are more damaging than injection errors.

7 REFERENCES


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\(^1\) Even for zero displacements, there were small changes in the \(A_{01}\) and \(A_{10}\) coefficients, but this inaccuracy has since been corrected.