THEORETICAL INVESTIGATION ON MULTIBUNCH INSTABILITIES IN ELECTRON STORAGE RINGS AND LINEAR ACCELERATORS

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Abstract

In this paper we establish simple analytical criteria for the loaded quality factors of the dipole modes in the accelerating rf structures to prevent the multibunch instabilities in electron storage rings and linear accelerators.

1 INTRODUCTION

In the modern circular and future linear colliders, the option of multibunch operation mode has to be adopted to guarantee the required luminosity. Due to the long range transverse wake potentials, the transverse motion of a bunch in a bunch train can be influenced by the precedent bunches. If the long range transverse wake potentials are not properly controlled multibunch instabilities can occur and the luminosity will be degraded. The classical treatment of the multibunch instabilities can be found in ref. 1 for example. In this paper we try to treat the problem in a different way. We assume that each bunch is represented by a point charge and the detailed discussion about the single bunch longitudinal and transversal instabilities in electron storage rings can be found in refs. 2 and 3.

A charged particle executes betatron oscillation in circular and linear accelerators can be regarded as an independent damped linear oscillator if there is no long range transverse wake potentials permitting the particle "talking" to its neighbours. The mechanisms of damping come from the synchrotron radiation in electron storage rings and the adiabatic acceleration in linear accelerators, respectively. The quality factor of this oscillator is related to the damping time and the betatron oscillation frequency. When the long range wake potentials are strong enough the particles in the bunch train will begin to be coupled from one to another and the independent oscillators become a chain of coupled oscillators with losses, and the betatron oscillation energies of the particles upstream can be transmitted to those of the particles downstream, known as multibunch instabilities. The physical picture described above is similar to that of a coupled rf cavity chain. Now, let's look at a chain of coupled rf cavities with losses which has been studied in detail in ref. 4, one finds that to prevent the coupling between cavities the criterion $K_cQ < 2$ should be satisfied, where K_c is the coupling coefficient in the dispersion curve and Q is the quality factor of the corresponding mode. By analogy, one can find the criteria under which the multibunch instabilities can be prevented in storage rings and linear accelerators.

2 MULTIBUNCH INSTABILITIES IN ELECTRON STORAGE RINGS

Particles in a storage ring execute betatron oscillations. If we neglect the effect of synchrotron radiation excitation and the long range wake potentials, the betatron motion of each bunch can be simplified as a damped oscillator expressed as

$$y = A\cos\left(\omega_y \frac{s}{c}\right)\exp\left(-\frac{\omega_y}{2Q_{y,r}}\left(\frac{s}{c}\right)\right) \tag{1}$$

where y denotes the transverse deviation in horizontal plane x or vertical plane z from the design orbit, ω_y is the angular betatron frequency, and $Q_{y,r}$ (the subscript r denotes the storage ring case) is the quality factor of the oscillator expressed

$$Q_{y,r} = \frac{\omega_y E_0}{<\mathcal{P}_0> J_y} \tag{2}$$

where $<\mathcal{P}_0>$ is the average synchrotron radiation power for one turn, E_0 is the particle energy, J_y is the radiation damping partition number with $J_{y=x} = 1 - \mathcal{D}$ and $J_{y=z} = 1$ ($-2 < \mathcal{D} < 1$). In reality, however, charged particles interact with the environment and produce long range wake potentials which make the independent oscillating oscillator become a coupled oscillator chain. The coupling coefficient $K_{c,r}$ between the two successive bunches can be calculated from the coherent frequency change due to the long range wake potential similar to the single bunch case [3]

$$\left|\frac{\Delta\nu_{y,c}}{\nu_y}\right| = \frac{e^2 N_e W'_\perp(s_b)\overline{\beta_{y,c}}}{\nu_y E_0} \tag{3}$$

where $W'_\perp(s_b)$ (V/C/m) is the long range dipole wake potential of one turn and of unit transverse displacement, s_b is the distance between two successive bunches, N_e is the particle population in the bunch, $\overline{\beta_{y,c}}$ is the average beta function at the position of the rf cavities, and ν_y is the tune number. By analogy with a coupled rf cavity chain, one finds the coupling coefficient expressed as follows

$$K_{c,r} = 2\left|\frac{\Delta\nu_{y,c}}{\nu_y}\right| \tag{4}$$

According ref. 4, one knows that under the condition

$$K_{c,r}Q_{y,r} < 2 \tag{5}$$

there will be no coupling between two successive oscillators. From eqs. 2 and 4, one finds

$$W'_\perp < \frac{<\mathcal{P}_0> J_y}{2\pi f_0 e^2 N_e \overline{\beta_{y,c}}} \tag{6}$$

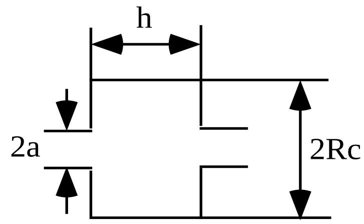

Figure 1: A single rf cavity.

where f_0 is the revolution frequency. For an isomagnetic ring

$$W'_{\perp} < \frac{\gamma^4 J_y}{6\pi\epsilon_0 \rho N_e \overline{\beta_{y,c}}} \tag{7}$$

where ρ is the local bending radius and γ is the normalized particle energy. In a storage ring the accelerating rf cavities are the main components which produce long range wake potentials (narrow band impedance). In the following one considers only the TM_{110} mode in the accelerating rf cavities since $W'_{\perp}(s_b) \approx W'_{\perp,110}(s_b)$ for the long range wake potential. The TM_{110} mode wake potential can be expressed as

$$W'_{\perp,110} = N_c h \frac{2cK_1}{\omega_{rf,1} a^2} \sin(\omega_{rf,1}\frac{s_b}{c})\times$$

$$\exp\left(-\frac{\omega_{rf,1}}{2Q_{1,r}}\left(\frac{s_b}{c}\right)\right)\exp\left(-\frac{\omega_{rf,1}^2\sigma_z^2}{2c^2}\right) \tag{8}$$

where N_c is the number of the cavities in the ring, h is the inner length of the cavity, σ_z is the rms bunch length (σ_z is used to calculate the transverse wake potential and the point charge assumption is still valid), $\omega_{rf,1}$ and $Q_{1,r}$ are the angular frequency and the loaded quality factor of the dipole mode, respectively. According to ref. 5, K_1 in eq. 8 corresponding to a single cavity can be expressed analytically as follows

$$K_1 = \frac{J_1^2\left(\frac{u_{11}}{R_c}a\right)}{\epsilon_0 \pi R_c^2 J_2^2(u_{11})} S(x_1)^2 \tag{9}$$

$$S(x) = \frac{\sin x}{x} \tag{10}$$

$$x_1 = \frac{hu_{11}}{2R_c} \tag{11}$$

$$\omega_{rf,1} \approx \frac{cu_{11}}{R_c} \tag{12}$$

where R_c is the cavity radius, a is the iris radius as shown in Fig. 1, and $u_{11} = 3.832$ is the first root of the first order Bessel function. $\omega_{rf,1}$ in eq. 12 can be rather precisely determined by using the analytical formulae from perturbation methods [6][7]. Being pessimistic, we assume $\sin(\omega_{rf,1}\frac{s_b}{c}) = 1$ and find consequently from eq. 7 that

$$\exp\left(-\frac{\omega_{rf,1}}{2Q_{1,r}}\left(\frac{s_b}{c}\right)\right) <$$

$$\frac{\gamma^4 \omega_{rf,1} a^2 J_y}{12\pi c\epsilon_0 \rho N_c h N_e K_1 \overline{\beta_{y,c}} \exp\left(-\frac{\omega_{rf,1}^2\sigma_z^2}{2c^2}\right)} \tag{13}$$

and

$$Q_{1,r} < \frac{u_{11} s_b}{2R_c ln\left(\frac{12\pi\epsilon_0\rho R_c N_c h N_e K_1 \overline{\beta_{y,c}} \exp\left(-\frac{u_{11}^2\sigma_z^2}{2R^2}\right)}{\gamma^4 u_{11} a^2 J_y}\right)} \tag{14}$$

To reach the required loaded $Q_{1,r}$, waveguide type higher order mode couplers can be installed on the accelerating rf cavities and the dimensions of the coupling apertures can be determined analytically as shown in ref. 8. From eq. 14 one can find the condition under which the dipole mode need not to be damped. This condition is simply that $Q_{1,r} \to \infty$ (this condition is somewhat strong but very useful since it doesn't depend on the specific unloaded dipole mode quality factor) when N_e satisfies

$$N_e \leq N_e^* = \frac{\gamma^4 u_{11} a^2 J_y}{12\pi\epsilon_0 \rho R_c N_c h K_1 \overline{\beta_{y,c}} \exp\left(-\frac{u_{11}^2\sigma_z^2}{2R^2}\right)} \tag{15}$$

Taking Beijing Tau-Charm Factory (BTCF) parameters for example, from eq. 14 one gets $Q_{1,r} = 99$ with $s_b = 12$ m, $R_c = 0.224$ m, $h = 0.22$ m, $a = 0.044$ m, $K_1 = 1.4 \times 10^{11}$ (V/C/m), $N_c = 12$, $N_e = 1.5 \times 10^{11}$, $\overline{\beta_{y,c}} = 10$ m, $\sigma_z = 0.01$ m, $J_{y=z} = 1$, $\rho = 8.58$ m, and $E_0 = 2$ GeV. This result justifies what has been found in ref. 10.

3 MULTIBUNCH INSTABILITIES IN LINEAR ACCELERATORS

In a linear accelerator the physical picture is a little bit different from that in a storage ring. The betatron motion can still be written as that in eq. 1, the quality factor, however, is expressed as

$$Q_{y,L} = \frac{\omega_y E_0}{ceE_z} \tag{16}$$

where E_z is the accelerating gradient and the subscript L in this section denotes linear accelerator case. The damping effect is due to the fact that a particle is accelerated continuously and the betatron oscillation is adiabatically damped [9]. The relative coherent betatron oscillation frequency variation due to the long range transverse wakefield is

$$\left|\frac{\Delta\omega_{y,c}}{\omega_y}\right| = \frac{2\pi e^2 N_e c W'_{\perp,L}(s_b)\overline{\beta_y}}{\omega_y E_0} \tag{17}$$

where $W'_{\perp,L}(s_b)$ (V/C/m^2) is the long range transverse wakefield strength of unit transverse displacement and $\overline{\beta_y}$ is the average beta function value in the linac. In the following one considers only the TM_{110} mode in the accelerating rf structures since $W'_{\perp,L}(s_b) \approx W'_{\perp,L,1}(s_b)$ for the long range wakefield, where $W'_{\perp,L,110}(s_b)$ is the TM_{110} wakefield expressed as

$$W'_{\perp,L,110} = \frac{2cK_{1,L}}{\omega_{rf,1,L} a^2} \sin(\omega_{rf,1,L}\frac{s_b}{c})\times$$

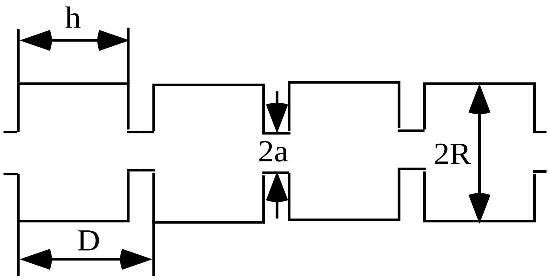

Figure 2: A disk-loaded accelerating structure.

$$\exp\left(-\frac{\omega_{rf,1,L}}{2Q_{1,L}}\left(\frac{s_b}{c}\right)\right)\exp\left(-\frac{\omega_{rf,1,L}^2\sigma_z^2}{2c^2}\right)F(s_b) \quad (18)$$

where $\omega_{rf,1,L}$ and $Q_{1,L}$ are the synchronous frequency and the loaded quality factor of the TM_{110} mode passband, respectively, and $F(s)$ is the wakefield reduction function comes from the detuning effect (for a constant impedance accelerating structure $F \equiv 1$). According to ref. 5, one knows that

$$K_{1,L} = \frac{hJ_1^2\left(\frac{u_{11}}{R}a\right)}{\epsilon_0 \pi D R^2 J_2^2(u_{11})}S(x_{1,L})^2 \quad (19)$$

$$x_{1,L} = \frac{hu_{11}}{2R} \quad (20)$$

$$\omega_{rf,1,L} \approx \frac{cu_{11}}{R} \quad (21)$$

where D is the period length of the accelerating structure, h is the inner cavity length, a and R are the iris and the cavity radius, respectively, as shown in Fig. 2. Knowing

$$K_{c,L} = 2\left|\frac{\Delta\omega_{y,c}}{\omega_y}\right| \quad (22)$$

and letting $Q_{y,L}K_{c,L} < 2$, one finds

$$W'_{\perp,L,1} < \frac{E_z}{2\pi e N_e \overline{\beta_y}} \quad (23)$$

Taking $\sin(\omega_{rf,1,L}\frac{s_b}{c}) = 1$ for the pessimistic case, one finds

$$\exp\left(-\frac{\omega_{rf,1,L}}{2Q_{1,L}}\left(\frac{s_b}{c}\right)\right) < \frac{E_z\omega_{rf,1,L}a^2}{4\pi c e N_e R K_{1,L}\overline{\beta_y}\exp\left(-\frac{\omega_{rf,1,L}^2\sigma_z^2}{2c^2}\right)F(s_b)} \quad (24)$$

and

$$Q_{1,L} < \frac{u_{11}s_b}{2Rln\left(\frac{4\pi e N_e R K_{1,L}\overline{\beta_y}\exp\left(-\frac{u_{11}^2\sigma_z^2}{2R^2}\right)F(s_b)}{E_z u_{11}a^2}\right)} \quad (25)$$

Similar to the storage ring case, one gets the condition under which no higher order mode coupler is needed

$$N_e \leq N_e^* = \frac{E_z u_{11}a^2}{4\pi e R K_{1,L}\overline{\beta_y}\exp\left(-\frac{u_{11}^2\sigma_z^2}{2R^2}\right)F(s_b)} \quad (26)$$

Here we give an example of an ideal detuned S-band linear accelerating structure. From eq. 25 one finds $Q_{1,L}$=2740 with $s_b = 5$ m, $R = 0.04$ m, $h = 0.0292$ m, $D = 0.035$ m, $a = 0.01$ m, $K_{1,L} = 10\times10^{12}$ (V/C/m), $N_e = 2\times10^{10}$, $\overline{\beta_y} = 85$ m, $\sigma_z = 0.005$ m, $F(s_b)$=0.0065, and $E_z = 17$ MV/m. If a constant impedance structure is used then $Q_{1,L}$=187 for the the same set of parameters.

4 CONCLUSION

In this paper we give simple criteria to determine the loaded quality factors of the dipole modes in the accelerating rf structures which are responsible for the multibunch instabilities in electron storage rings and linear accelerators. The relation between the beam and the machine parameters are well established, and the analytical criteria can be served as scaling laws to optimize the machine performance.

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