SHIELDED TRANSIENT SELF-INTERACTION OF A BUNCH ENTERING A CIRCLE FROM A STRAIGHT PATH

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Abstract

When a short (mm-length) bunch with high (nC-regime) charge is transported through a magnetic bending system, self-interaction via coherent synchrotron radiation (CSR) and space charge may alter the bunch dynamics significantly. We consider a Gaussian rigid-line-charge bunch following a straight-path trajectory into a circle, with the trajectory centered between two infinite, parallel, perfectly conducting plates. Transients associated with CSR and space charge generated from source particles both on the straight path and the circle are calculated, and their net effect on the radiated power is contrasted with that of shielded steady-state CSR.

1 INTRODUCTION

When short (mm-length), high-charge (nC-regime) bunches are injected into magnetic bending systems, coherent synchrotron radiation (CSR) and space charge may cause serious degradation of beam quality. This possibility is a serious concern for various transport-lattice designs associated with, for example, free-electron lasers (FELs), including bunch-compressor chicanes preceding wigglers and recirculation loops associated with energy recovery. Almost all previous theoretical work on CSR has concerned its steady-state properties. Examples concerning steady-state CSR in free space include the frequency-domain [1] and time-domain analyses [2]. Examples concerning steady-state CSR with shielding, i.e., in the presence of conducting walls, also include frequency-domain [3, 4, 5] and time-domain [6] analyses. Only recently have transients in finite-length magnetic bends begun to be considered, the principal example being a time-domain analysis [7], concerning the transient interaction of a bunch with itself as it passes from a straight path into a circle in free space. These investigators showed that both space-charge forces originating from the straight path and CSR forces originating from the circle make important contributions to the transient self-interaction.

In this paper, we generalize the theory of transient self-interaction in a magnetic bend by incorporating conducting walls to introduce shielding of CSR. Working in the time domain, we consider an electron bunch with a rigid-line-charge Gaussian distribution orbiting in the center plane between two infinite, parallel conducting plates. The bunch moves from a straight path to a circular orbit and begins radiating. Transient forces arising from source particles on the straight path (space charge) and on the circle (space charge and CSR) are calculated, and their net effect is obtained. Parallel plates are incorporated by including forces originating from image charges.

2 ANALYSIS

The Hamiltonian for an electron with charge e is:

$$H = c\sqrt{(\mathbf{P} - e\mathbf{A}/c)^2 + m^2c^2} + e\Phi, \tag{1}$$

where $\mathbf{P} - e\mathbf{A}/c = \gamma m\mathbf{v}$ is the kinetic momentum for the electron, in which $\mathbf{v}$ is its velocity, γ is the Lorentz factor; Φ and $\mathbf{A}$ are the scalar and vector electromagnetic potential on the electron, respectively, arising from the interaction of an external field and the rest of the charge distribution. Given a rigid-line-charge bunch entering a circle from a straight path, the rate of change of the kinetic energy for an "observer" electron S located on the bunch at the space-time coordinate $(\mathbf{r}, t)$ can be derived from the above Hamiltonian in terms of the potentials Φ_0 and $\mathbf{A}_0$ on S generated by a single "source" electron S':

$$\begin{aligned}
&mc^2\frac{d\gamma}{dt} = \beta c F_\theta, \quad F_\theta = \int_{-\infty}^{\infty} ds' F_{\theta 0}(\mathbf{r}, t, s')\, n(s'), \\
&F_{\theta 0}(\mathbf{r}, t, s') = \frac{e}{\beta c}\left[-\frac{d\Phi_0}{dt} + \frac{\partial}{\partial t}(\Phi_0 - \boldsymbol{\beta}\cdot\mathbf{A}_0)\right], \\
&(\Phi_0, \mathbf{A}_0) = e\left[\frac{(1, \boldsymbol{\beta})}{(1 - \boldsymbol{\beta}\cdot\mathbf{n})R}\right]_{\text{ret}},
\end{aligned} \tag{2}$$

where $F_{\theta 0}$ is the longitudinal electric force exerted by S' on S; $n(s')$ is the line-density of the bunch, with s' denoting the distance of electron S' from the bunch center in the bunch rest frame. The subscript "ret" in the single-electron potentials incorporates the retardation relation for a photon emitted by S' at $(\mathbf{r}', t')$ to reach S at $(\mathbf{r}, t)$: $c(t - t') = |\mathbf{R}|$, with $\mathbf{R} \equiv \mathbf{r}(t) - \mathbf{r}'(t')$. In addition, we have in Eq. (2) $\mathbf{n} = \mathbf{R}/|\mathbf{R}|$ and $\boldsymbol{\beta}_{\text{ret}} = \mathbf{v}_{\text{ret}}/c$. The force exerted on single particle by the whole bunch calculated by way of Eq. (2) forms the basis of our analysis. In what follows, we shall use the indices "(a)" and "(b)" to denote the case that at retarded times t' the source particle S' is located on the straight path and on the circle, respectively. To take into account of the interaction on S from image charges due to the presence of the parallel plates, the source particle S' is allowed to have an offset z' perpendicular to the plane of the orbit. This will correspondingly affect the retardation times associated with image charges.

2.1 *Case (a): S' on straight path at t', S on circle at t*

Fig. 1 depicts an observer electron S at angle θ on the circle of radius ρ at time t experiencing a force generated from a source electron S' (which, in Fig. 1, is an image charge) at coordinate $\mathbf{r}' = (-x', 0, z')$ at time t' ($x' \geq 0$). With $t = 0$

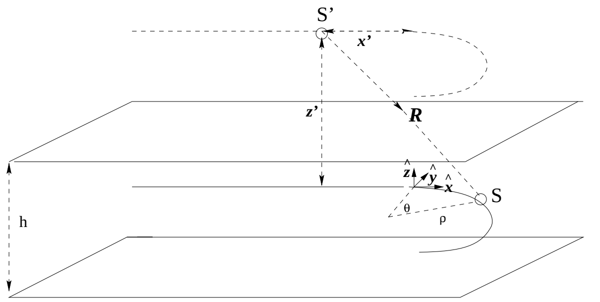

Figure 1: Interaction of S' on S, with S' on the straight path prior to the bend at the retarded time, and S on the circular orbit.

being the moment when the bunch center enters the circle, the trajectories of S and S' are respectively described by

$$S:\rho\theta = s+\beta ct, \qquad S':-x' = s'+\beta ct'. \qquad (3)$$

In the coordinate system $(\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}})$, depicted in Fig. 1, the vector $\mathbf{R}$ from S' to S is $(R_x,R_y,R_z) = (\rho\sin\theta + x', \rho\cos\theta-\rho, z')$. According to Eq. (2), the longitudinal electric force exerted by S' on S can be obtained from

$$\begin{aligned}
&F_{\theta 0}^{(a)} = -e\partial V^{(a)}/\partial\Delta s,\ \ V^{(a)} = V_0^{(a)} + (\Phi_0 - \boldsymbol{\beta}\cdot\mathbf{A}_0)^{(a)},\\
&V_0^{(a)} = \mathcal{V}_0^{(a)}(\theta,\infty) - \mathcal{V}_0^{(a)}(\theta,\Delta s),\\
&\mathcal{V}_0^{(a)}(\theta,\Delta s) = e[(1-\cos\theta) - R_y R_x'\sin\theta/R_\perp^2]/R_1,\\
&(\Phi_0 - \boldsymbol{\beta}\cdot\mathbf{A}_0)^{(a)} = e(1-\beta^2\cos\theta)/R_1.
\end{aligned} \qquad (4)$$

with $\partial V_0^{(a)}/\partial\Delta s = d\Phi_0^{(a)}/\beta c\,dt$. Here $R_1^2 = R_x'^2 + R_\perp^2/\gamma^2$, $R_\perp^2 = R_y^2 + R_z^2$, and $R_x' \equiv R_x - \beta R = \rho(\Delta\phi + \sin\theta - \theta)$ is the distance from S_p' to S projected on the $\hat{\mathbf{x}}$-direction, with S_p' denoting the position of S' at time t were it to continue executing uniform linear motion at all retarded times $t' \le t$. We are letting $\Delta s \equiv s - s'$ denote the distance between S' and S in the rest frame of the bunch, and we define $\Delta\phi = \Delta s/\rho$. We will show that when $R_x' = 0$, the straight path introduces transient space-charge forces on the bunch comparable to transient CSR forces from the circle.

2.2 Case (b): S' on circle at t', S on circle at t

The motions of S and S' are now described by

$$S:\rho\theta = s+\beta ct, \qquad S':\rho\theta' = s'+\beta ct'. \qquad (5)$$

Causality requires $\Delta\theta = \theta - \theta'$ to depend on $\Delta\phi = (s-s')/\rho$, the relative spacing of the two particles in the bunch rest frame, in the manner

$$\rho\Delta\theta = \rho\Delta\phi + \beta R, \qquad R = \sqrt{[2\rho\sin(\Delta\theta/2)]^2 + z'^2}. \qquad (6)$$

Here only the forward radiation is considered in that $\Delta\theta \ge 0$. In free space, for which $z' \equiv 0$, the causality condition is $\Delta\theta = 4\mathrm{sh}[(1/3)\mathrm{sh}^{-1}(3\gamma^3\Delta\phi/2)]/\gamma$. For image charges, with $z' \ne 0$, $\sigma_s \ll h/\rho$, and $\Delta\theta \gg \gamma^{-1}$, one can approximate Eq. (6) by $\Delta\theta^4 - (24\Delta\phi)\Delta\theta - 12(z'/\rho)^2 = 0$, which has the solution

$$\Delta\theta = \frac{\Delta\theta_0}{3^{1/4}}\left[\sqrt{\sqrt{\frac{\mathrm{sh}\eta}{\mathrm{sh}(\eta/3)}} - \mathrm{sh}\frac{\eta}{3}} + \frac{\Delta\phi}{|\Delta\phi|}\sqrt{\mathrm{sh}\frac{\eta}{3}}\right], \qquad (7)$$

where $\Delta\theta_0 \equiv (12)^{1/4}|z'/\rho|^{1/2}$ is the value of $\Delta\theta$ when $\Delta\phi = 0$, and $\eta \equiv \mathrm{sh}^{-1}[9\Delta\phi^2/2|z'/\rho|^3]$. Limiting cases include $\eta \ll 1$, for which $\Delta\theta \simeq \Delta\theta_0\left[1 + (3/4)^{1/4}\Delta\phi|z'/\rho|^{-3/2}\right]$, and $\eta \gg 1$, for which $\Delta\theta \simeq 2(3\Delta\phi)^{1/3}H(\Delta\phi)$, with $H(x)$ denoting the Heaviside step function.

It can be shown that $d\Phi_0^{(b)}/dt = 0$, and consequently

$$\begin{aligned}
&F_{\theta 0}^{(b)} = -e\partial V^{(b)}/\partial\Delta s, \qquad V^{(b)} = (\Phi_0 - \boldsymbol{\beta}\cdot\mathbf{A}_0)^{(b)},\\
&V^{(b)} = e\frac{\beta(1-\beta^2\cos\Delta\theta)}{\rho(\Delta\theta - \Delta\phi - \beta^2\sin\Delta\theta)},
\end{aligned} \qquad (8)$$

where causality determines $\Delta\theta(\Delta\phi)$ per Eq. (6).

2.3 Longitudinal Electric Force on S from Whole Bunch

To remove the singularity due to the rigid-line-charge model when S and S' overlap, and to isolate the consequences of the circular motion of S, we now calculate the residual longitudinal electric force exerted by the whole bunch on S: $\hat{F}_\theta \equiv F_\theta - F_s$, where F_s is the integral of F_{s0} over the charge distribution, with $F_{s0} \equiv -e\partial V_s/\partial\Delta s$ being the space-charge force obtained when the bunch moves on a straight path with constant velocity $\mathbf{v}$, i.e., $V_s = e\gamma^{-2}(\Delta s^2 + z'^2/\gamma^2)^{-1/2}$. The corresponding residual potentials are $\hat{V}^{(a,b)} = V^{(a,b)} - V_s$.

To calculate $\hat{F}_\theta$, we let $\Delta s = \Delta s_t(\theta,z')$ when S' is at the entry to the circle $\theta' = x' = 0$, with $\Delta s_t(\theta,z') = \rho\theta - \beta\sqrt{[2\rho\sin(\theta/2)]^2 + z'^2}$. In applying Eq. (2), $F_{\theta 0}^{(a)}$ is used for $F_{\theta 0}$ if $\Delta s > \Delta s_t$ and $F_{\theta 0}^{(b)}$ is used if $\Delta s_0 < \Delta s \le \Delta s_t$, with $\Delta s_0(z') = -\beta|z'|$ designating the transition point between forward and backward radiation occurring at $\Delta\theta = 0$. Upon applying Eqs. (4) and (8) and integrating Eq. (2) by parts, noting that $\hat{V}^{(a)}(\theta,\infty) = \hat{V}^{(b)}(\Delta s_0) = 0$, we obtain the residual longitudinal electric force on S arising from the whole bunch:

$$\begin{aligned}
&\hat{F}_\theta(\theta,s,z') = F_0^{(a)} + F^{(a)} + F^{(b)};\\
&F_0^{(a)} = eV_0^{(a)}(\theta,\Delta s_t)n(s-\Delta s_t),\\
&F^{(a)} = e\int_{\Delta s_t(\theta,z')}^{\infty} d\Delta s\hat{V}^{(a)}(\theta,\Delta s,z')\frac{dn(s-\Delta s)}{d\Delta s},\\
&F^{(b)} = e\int_{\Delta s_0(z')}^{\Delta s_t(\theta,z')} d\Delta s\hat{V}^{(b)}(\Delta s,z')\frac{dn(s-\Delta s)}{d\Delta s},
\end{aligned} \qquad (9)$$

where one has $V_0^{(a)}(\theta,\Delta s_t) = -e|R_y|/\rho(R+\rho\sin\theta)$ from Eq. (4). It turns out that $F_0^{(a)}$ is negligible compared to $F^{(a)}$ and $F^{(b)}$. Since the potentials are continuous at entry to the circle, $(\Phi_0 - \boldsymbol{\beta}\cdot\mathbf{A}_0)^{(a)}|_{\Delta s_t} = (\Phi_0 - \boldsymbol{\beta}\cdot\mathbf{A}_0)^{(b)}|_{\Delta s_t}$, a strong, energy-dependent, transient accelerating force arising from space charge generated on the straight path cancels with a strong, energy-dependent, transient decelerating force arising from CSR on the circle.

From Eq. (4) one can show that in the high energy limit, $\hat{V}^{(a)}$ behaves like a step function which cuts off at $\Delta s = \Delta s_c = \rho(\theta - \sin\theta)$: $\hat{V}^{(a)} \simeq U^{(a)}(\theta, z')H(-R'_x)$, with $U^{(a)}(\theta, z') = 2e|R_y|\sin\theta/R_\perp^2$. This result can be traced to the impulse-like behavior of the single particle space-charge force $F^{(a)}_{\theta 0}$ in Eq. (4) on S from S' when $R'_x = 0$. For $\gamma\theta > 1$, the cutoff occurs on the straight path, $\Delta s_c > \Delta s_t$, leading to $F^{(a)} \simeq eU^{(a)}(\theta, z')[n(s - \Delta s_c) - n(s - \Delta s_t)]$, which is an energy-independent transient force with peak value comparable in magnitude with $F^{(b)}$.

It is now straightforward to incorporate two infinite parallel plates with spacing h, with the bunch moving on the plane centered between the plates, by considering the array of image charges that comove with the bunch in the planes $z' = \pm nh$. The total shielded longitudinal force $\hat{F}^{\rm sh}_\theta$ on the electron S from all the image bunches is thus obtained from the unshielded force $\hat{F}_\theta$ in Eq. (9),

$$\hat{F}^{\rm sh}_\theta(\theta_0, s) = \sum_{n=-\infty}^{\infty} (-)^n \hat{F}_\theta(\theta = \theta_0 + s/\rho, s, nh), \qquad (10)$$

in which $\theta_0 = \beta ct/\rho$ is the angular coordinate of the bunch center. Eq. (10) constitutes the starting point for calculating bend-induced energy spread, which in turn causes degradation in transverse emittance.

3 POWER LOSS

To look at the amplitudes and duration of the transients, we turn to a calculation of the shielded power loss $\hat{P}^{\rm sh}(\theta_0)$ of the bunch induced by its self-interaction. This is obtained by integrating the rate of kinetic energy loss of a single electron over the portion of the bunch on the circle:

$$\hat{P}^{\rm sh}(\theta_0) = \hat{P}(\theta_0, 0) + 2\sum_{n=1}^{\infty}(-)^n \hat{P}(\theta_0, nh),$$
$$\hat{P}(\theta_0, nh) = -\beta c \int_{-\theta_0}^{\infty} ds \hat{F}_\theta(\theta = \theta_0 + s/\rho, s, nh)n(s), \qquad (11)$$

where $\hat{P}(\theta_0, 0)$ is the power loss in free space, and $\hat{P}(\theta_0, nh)$ represents the bunch's power loss due to its interaction with the nth image bunch obtained from $\hat{F}_\theta$ given in Eq. (10). The integration spans $\theta \geq 0$, or $s \geq -\theta_0$. When the whole bunch is well into the bend, $\theta_0 \gg \sigma_s/\rho$, and the lower limit $-\theta_0$ effectively becomes $-\infty$. Denoting $\Delta s_t^{(n)}(\theta) = \Delta s_t(\theta_0, nh)$, we then have

$$\begin{aligned}
\hat{P}(\theta_0, nh) &\simeq \hat{P}^{(a)}_0(\theta_0, nh) + \hat{P}^{(a)}(\theta_0, nh) + \hat{P}^{(b)}(\theta_0, nh);\\
\hat{P}^{(a)}_0(\theta_0, nh) &= -\beta ce V^{(a)}_0[\theta_0, \Delta s_t^{(n)}(\theta_0), nh] f[\Delta s_t^{(n)}(\theta_0)],\\
\hat{P}^{(a)}(\theta_0, nh) &= -\beta ce U^{(a)}(\theta_0, nh)[f(\Delta s_c) - f(\Delta s_t^{(n)})]|_{\theta=\theta_0},\\
\hat{P}^{(b)}(\theta_0, nh) &= -\beta ce \int_{\Delta s_0(nh)}^{\Delta s_t^{(n)}(\theta_0)} d\Delta s\, \hat{V}^{(b)}(\Delta s, nh) g(\Delta s).
\end{aligned} \qquad (12)$$

where

$$f(\Delta s) \equiv \int_{-\infty}^{\infty} n(s)n(s - \Delta s)ds, \;\; g(\Delta s) \equiv df(\Delta s)/d\Delta s, \qquad (13)$$

In steady-state cases one has $\hat{P}(\infty, nh) = \hat{P}^{(b)}(\infty, nh)$. In particular, one can show that the result of free-space power loss $\hat{P}(\infty, 0)$ agrees with that of Schiff [1].

The power loss obtained from Eq. (11) using numerical integration for Gaussian bunch distribution, $n(s) = e^{-s^2/2\sigma_s^2}/\sqrt{2\pi}\sigma_s$, is displayed in Fig. 2 for parameters $\rho = 1$ m and $\sigma_s = 1$ mm, typical values in the recirculating accelerator that will drive Jefferson Lab's infrared FEL (the IR Demo) [8]. The dotted curve is the transient power loss of the bunch in free space, $\hat{P}(\theta_0, 0)/\hat{P}(\infty, 0)$, which rises from zero loss and saturates to steady state. This free-space result agrees with that given in Ref.[7]. The other curves in Fig. 2 pertain to the presence of parallel conducting plates. The solid curve corresponds to $h = 5$ cm, a typical pipe size in the IR Demo. The spacing is relatively large to suppress beam loss, and it provides little shielding of the self-interaction. Stronger shielding can be obtained for narrower gap size with fixed bunch length, as indicated by the dashed curve corresponding to $h = 2$ cm. In this case the steady-state power loss is 25% of the free-space value, in agreement with results obtained by power-spectrum analysis as reflected in Fig. 2 of Ref. [9]. Many features of the transient power loss can be derived analytically and expressed in closed form, as we plan to show in a future, more comprehensive paper.

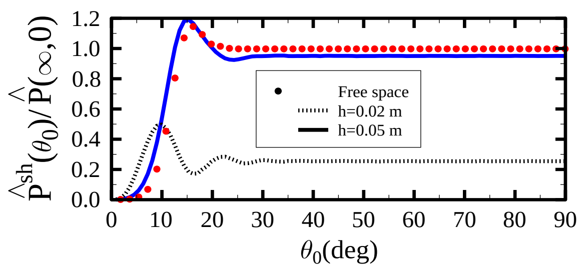

Figure 2: Transient power loss of a bunch, due to curvature-induced self-interaction in the presence of parallel plates, with $\rho = 1$ m, $\sigma_s = 1$ mm, $E = 40$MeV, and various plate spacings h. Here θ_0 is the angle of bunch center into the bend.

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