INCREASE OF THE TRANSVERSE STRONG HEAD-TAIL STABILITY THRESHOLD BY AN ALTERNATING CHROMATICITY *

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Abstract

It has been shown that [1], the transverse head-tail instability can be suppressed by modulating the chromaticity over a synchrotron period. In this work, we demonstrate that the threshold of the strong head-tail instability can be significantly increased by the alternating chromaticity (AC). We present results of multi-particle simulation and a new criterion for the SHT instability.

1 INTRODUCTION

The transverse collective beam instability induced by the coupling impedances in a storage ring has two categories: head-tail (HT) and strong head-tail (SHT) instabilities [2]. The HT instability is generated by the chromaticity in a ring, and has no stability threshold. The transverse SHT instability (also known as the transverse microwave instability) occurs when the betatron tune shift is larger than the synchrotron tune, and limits the current carrying capacity in a storage ring. In a previous work [1], we analyzed a new method for suppression of the HT instability by means of variation of the chromaticity over a synchrotron period. Both analytical and numerical studies suggest that a threshold can be developed, and it can be increased to a value larger than the standard SHT stability threshold. The underlying physical mechanisms of the damping scheme are rotation of the head-tail phase (such that the chromatic effect causing the instability is cancelled out in a synchrotron period) and Landau damping (due to the incoherent betatron tune spread generated by the alternating chromaticity.) In this paper, we demonstrate that the AC scheme not only provides damping for the HT instability, but also increases the threshold of the SHT instability. Going beyond the discussions provided in Ref. [1], we provide more simulaton results, and a new approximate SHT stability criterion.

2 DAMPING MECHANISMS

The transverse chromaticity is defined as

$$\xi = \frac{\Delta\omega_\beta/\omega_{\beta 0}}{\delta}, \tag{1}$$

where $\omega_{\beta 0}$ is the betatron angular frequency of the on-momentum particle, and $\delta = \triangle p/p$ is the relative momentum difference. With the AC scheme, the chromaticity is assumed to vary as

$$\xi(s) = \xi_0 + \xi_1 \sin\phi(s), \tag{2}$$

which is a function of "time" s, where s measures the distance around the ring, $\phi(s) = \omega_s s/c$, ω_s is the synchrotron angular frequency, and ξ_0 is the constant (DC) chromaticity. It is well-known that [2], the DC part of chromaticity engenders the HT instability. The AC part is introduced to provide an incoherent tune spread that suppresses the coherent instability without causing additional instabilities [1]. This incoherent chromatic tune spread can be estimated as

$$\sigma_\nu \approx \nu_{\beta 0}\xi_1\sqrt{\langle\delta^2 \sin^2\phi\rangle} = \sqrt{\frac{3}{8}}\nu_{\beta 0}\xi_1\sigma_\delta = \sqrt{\frac{3}{8}}\nu_s\chi_1, \tag{3}$$

for a Gaussian beam, where $\nu_{\beta 0} = \omega_{\beta 0}/\omega_0$, ω_0 is the revolution angular frequency, $\sigma_\delta = (\omega_s/c\eta)\sigma_z$, σ_z is the rms bunch length,

$$z = r_z\cos\phi, \quad \delta = (\omega_s/c\eta)r_z\sin\phi, \tag{4}$$

(r_z, ϕ) are the action-angle variables in the longitudinal phase space, the bracket $\langle\rangle$ means a longitudinal phase-space average, and $\chi_{(0,1)} = \omega_{\beta 0}\xi_{(0,1)}\sigma_z/c\eta$ is the DC(AC) phase shift between head and tail of a bunch. The AC incoherent tune spread contributes to Landau damping and decoherence. Unlike its DC counterpart, the AC part of the chromaticity deos not lead to HT instabilities [1]. It is simply because of an otherwise accumulating chromatic effect during the synchrotron oscillation is cancelled out if the sign of the chromaticity is reversed within a synchrotron period. As the alternating chromaticity contributes to Landau damping without inducing instabilities, one may use an AC amplitude as large as possible (within the tolerance of dynamic aperture reduction) to push up the SHT stability threshold, so as to achieve a higher bunch current in a storage ring.

In short, the underlying mechanisms for the AC shceme are: Landau damping and rotation of the head-tail phase.

3 TOLERANCE

Here, we need to note that, it is known [3] that the chromaticity (DC part) could prevent the transverse modes from

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coupling (which the mode-coupling results in the SHT instability) until a higher current limit is reached, if ξ_0 is large enough. However, the HT instability occurs when $\xi_0 \neq 0$, and the the emittance growth could be unacceptable. By using the AC scheme, there is no HT instability. The only foreseeable limitaion would be the reduction of dynamic aperture due to resonances. As the damping of the HT instability (when $\chi_0 \neq 0$) by the AC scheme has been well studied [1], we focus on the effect of a pure AC scheme, i.e., $\chi_0 = 0$, in the following study.

According to Eq. (1), the normalized amplitude of variation of the incoherent betatron tune due to ξ_1 is

$$\left|\frac{\Delta\nu_\beta}{\nu_{\beta 0}}\right| = |\xi_1\delta| \approx \xi_1\sigma_\delta = \frac{\nu_s}{\nu_{\beta 0}}\chi_1. \tag{5}$$

In most applications, the synchrotron tune is much smaller than the betatron tune. Typically, $\nu_s/\nu_{\beta 0} \approx 10^{-4} \sim 10^{-3}$. Assuming a lattice can tolerate 0.2% variation of $\Delta\nu_\beta/\nu_{\beta 0}$, and if $\nu_s/\nu_{\beta 0} = 10^{-4}$, then the maximum AC amplitude one can employ is $\chi_1 = 20$. A larger tolerance of dynamic aperture reduction on $|\Delta\nu_\beta/\nu_{\beta 0}|$ and a smaller value of $\nu_s/\nu_{\beta 0}$ would allow a larger χ_1, which would in turn allow a higher SHT stability limit (as will be shown.)

4 NEW APPROXIMATE SHT STABILITY CRITERION

Let's now define a dimensionless parameter

$$\Upsilon = \Delta\nu/\nu_s, \tag{6}$$

which measures the ratio of the coherent betatron tune shift (due to coupling impedances) to the synchrotron tune. This parameter can be expressed in terms of accelerator parameters as

$$\Upsilon = \pi N r_0 \langle W_\perp\rangle c^2/8\gamma C\omega_{\beta 0}\omega_s, \tag{7}$$

where $r_0 = e^2/m_0c^2$, $N = \int dz'\rho(z')$ is the number of particles in a bunch, $\langle W_\perp\rangle = (1/N)\int_{-\infty}^{\infty} dz'\rho(z')W(z-z')$ is the averaged bunch wake, C is the circumference, γ is the relativistic factor. It can easily be shown that [2], by a two-particle model, the SHT stability limit is

$$\Upsilon \lesssim 1, \tag{8}$$

for a uniform wake function. For a realistic wake function, which is not uniform, the criterion is still valid, except that the bunch wake $\langle W_\perp\rangle$ is associated with a geometric factor. This stability threshold is consistent with the transverse Boussard criterion [4]. Note that, here $\Delta\nu$ is different with the chromatic tune shift, $\Delta\nu_\beta$, shown in Eq. (5). The imaginary part of the modes appears, when the real part of transverse modes couple at where the coherent betatron tune shifts approximately in the amount of synchrotron tune. Instabilities occurs when the mode frequencies are complex. This qualitative description is clearly manifested by Eqs. (6) and (8). Conversely, one can approximate the incoherent tune spread generated from the synchrotron oscillation as, $\sigma_\nu \sim \nu_s$, the stability limit reads

$$\Delta\nu \lesssim \sigma_\nu. \tag{9}$$

Now, with the AC, the incoherent tune spread including the AC [cf. Eq. (3)], is

$$\sigma_\nu \sim \nu_s(1 + \sqrt{3/8}\chi_1). \tag{10}$$

The SHT stability threshold can then be approximately increased from $\Upsilon_{SHT} \simeq 1$ to

$$\Upsilon_{SHT} \simeq 1 + \sqrt{3/8}\chi_1. \tag{11}$$

Of course, Eq. (11) is only an estimate. A rigorous evaluation of a new SHT instability threshold, for the AC scheme, should be obtained from the complete dispersion relation, in which Landau damping is included by the method of singular eigenfunction expansion. In this way, the eigenvalues of the azimuthal mode are exactly computed, and the new threshold Υ_{SHT} is where the modes couple. Detailed formulation can be found in Ref. [1]. Exact calculation of the azimuthal mode coupling is underway.

5 MULTI-PARTICLE SIMULATION

A simulation code has been developed, which follows the motion of macro-particles that are initially loaded with a bi-Gaussian distribution in both longitudinal and transverse phase spaces. The transverse (for either vertical or horizontal) equation of motion for a particle in a bunch is

$$y''(z,s) + \frac{\omega_\beta^2(\delta)}{c^2}\,y(z,s) = -\frac{r_0}{\gamma C}\int_z^\infty dz'\rho(z')W_\perp(z-z')y(z',s), \tag{12}$$

where $y\,(z)$ is the transverse (longitudinal) oscillation amplitude with respect to the bunch center, and $' = d/ds$. The longitudinal motion is prescribed by Eq. (4). Eqs. (4) and (12) are transformed into a 4-D map for particle's longitudinal and transverse motions.

Specifically, the code simulates a bunched beam traversing a ring with a transverse impedance. The momentum P_y is changed by the kick of the transverse wake force, where $P_y = (c/\omega_{\beta 0})y'$. Particle's betatron oscillation is carried out by a rotation matrix, where $\omega_\beta(\delta) = \omega_{\beta 0}(1+\xi_1\delta\sin\phi)$ is used for the betatron angular frequency. A uniform transverse wake function is used. No longitudinal wake force is included. Results are numerically converged when the number of macro-particles simulated is larger than 400. Numerical values of accelerator parameters used in the simulations can be found in Ref. 1. In general, only two parameters are important to the dynamics studied in this work: Υ and χ_1, which provide scaling laws for accelerator parameters.

The curve of $\langle y\rangle$ presented in this paper is the bunch centroid motion averaged over a synchrotron period. It is defined as

$$\langle y\rangle(\tau_n) = \left[\frac{1}{2N_s+1}\sum_{i=\tau_n-N_s}^{\tau_n+N_s}\bar{y}^2(i)\right]^{1/2}, \tag{13}$$

$$\bar{y}(i) = \frac{1}{N_m}\sum_{m=1}^{N_m} y_m(i), \tag{14}$$

where N_m is the number of macro-particles used in the simulations, τ_n is the number of turn, and N_s is the integer part of $1/\nu_s$. The curve of $\varepsilon_{\mathrm{rms}}$ presented in Fig. 1(b) is the rms emittance of the phase space P_y vs. y.

In Figs. 1, we show the simulation results for the stabilization of the SHT instability by a large enough χ_1, when $\Upsilon = 1.65$ and $\chi_0 = 0$. In Fig. 2, simulation results of the excitation of $\langle y \rangle$ and $\varepsilon_{\mathrm{rms}}$ at the 8000th turn are shown in the χ_1 vs. Υ space. It can be clearly seen that, the excitations of the bunch centroid and emittance, due to the SHT instability, are significantly suppressed when the AC scheme is implemented. Large χ_1 gives large reduction of the instability growth. Simulation results of the bunch centroid motion agree with the approximate SHT stability threshold [cf. Eq. (11)]. A noteworthy implication of Fig. 2 is that, although the bunch centroid motion is stabilized when χ_1 exceeds the threshold value estimated, the emittance growth requires a much larger χ_1 such that the level of growth is acceptable. As large requirement of χ_1 involves tolerance of dynamic aperture reduction, tolerance of emittance growth when operating above the standard SHT threshold will thus rely on tolerance of dynamic aperture reduction. These tolerance depend on practical applications, and are left for future studies.

6 CONCLUSION

We have shown that, by using the AC scheme, the SHT stability threshold can be increased by the AC scheme, and the bunch excitations of the centroid motion and the emittance growth due to the SHT instability are significantly suppressed. The underlying mechanisms are Landau damping and rotation of the head-tail phase. The tolerance of dynamic aperture reduction is discussed in terms of the AC part of the head-tail phase χ_1. Simulations for the bunch centroid motion agrees with the approximation of the new SHT stability threshold.

7 REFERENCES

[1] W.-H. Cheng, A. M. Sessler, and J. S. Wurtele, "Damping of the Transverse Head-Tail Instability by Periodic Modulation of the Chromaticity", to be published in Physical Review Letters; Lawrence Berkeley National Laboratory Report, LBNL-39642 (1996); W.-H. Cheng, A. M. Sessler, and J. S. Wurtele, "Varying Chromaticity: A Damping Mechanism for the Transverse Head-Tail Instability", submitted to Physical Review E; Lawrence Berkeley National Laboratory Report, LBNL-39717 (1996).

[2] A. W. Chao, *Physics of Collective Instabilities in High Energy Accelerators*, (John Wiley & Sons, New York, 1993).

[3] M.-P. Level, "The Cure of Transverse Mode Coupling Instability in Super-Aco", Proc. 4th Adv. ICFA Beam Dynamics Workshop on Collective Effects in Short Bunches, KEK, 1990, p. 101.

[4] D. Boussard, CERN Lab II/RF/Int 75-2 (1975).

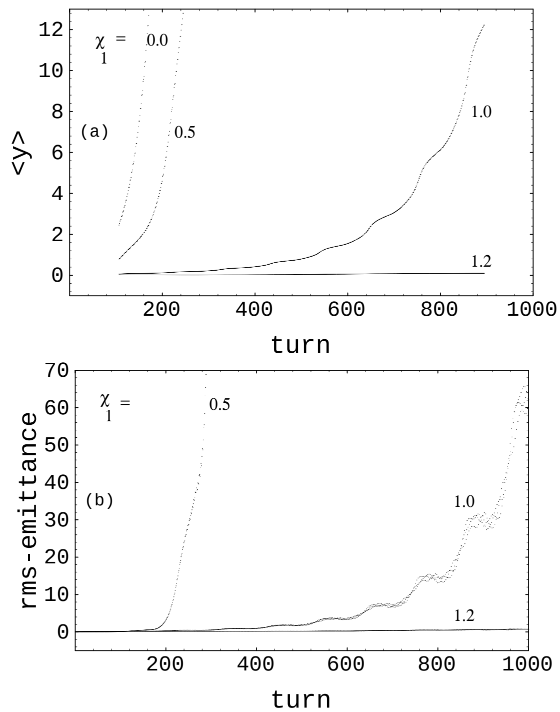

Figure 1: Multi-particle simulation results showing stabilization of the SHT motions of (a) the centroid, and (b) the rms-emittance of a Gaussian beam by χ_1, where the standard SHT stability limit is $\Upsilon_{SHT} = 1$ (when $\chi_1 = 0$.) In these figures, $\chi_0 = 0$, $\Upsilon = 1.65$. According to Eq. (11), when $\chi_1 = 1.2$, $\Upsilon_{SHT} = 1.73$.

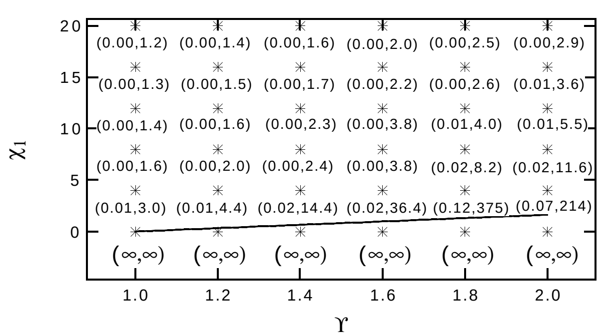

Figure 2: Simulation results of the bunch exciations when Υ is above the SHT stability limit. The numerical values attached beneath the points (*) are $(\langle y \rangle, \Delta\varepsilon_{\mathrm{rms}})$, where $\langle y \rangle$ is the averaged centroid motion at the 8000th turn, and $\Delta\varepsilon_{\mathrm{rms}} = \varepsilon_{\mathrm{rms}}(8000)/\varepsilon_{\mathrm{rms}}(0)$. The approximate new stable limit (the solid line) are plotted according to the criterion estimated in Eq. (11). The region above the solid line is stable for the bunch's centroid motion. When $\chi_1 = 0$, the standard SHT stability limit is $\Upsilon_{SHT} = 1$.