

# MODELING SPACE CHARGE EFFECTS IN OPTICAL BUNCHERS\*

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## Abstract

This paper introduces a 1-D, self-consistent simulation of Inverse Free Electron Laser (IFEL) based optical bunchers. Starting with a review of the conventional (monochromatic) IFEL bunching, we consider slow amplitude variation and multiple harmonics of the fundamental radiation wave. We model two new bunching schemes – the adiabatic and harmonic microbuncher and compare the bunching mechanisms between the optical bunchers as well as space charge effects to the modified IFEL interaction. Here, we present a model approximating the space charge fields and verification of the simulation with the well-known Free Electron Laser (FEL) simulator, Genesis 1.3.

## INTRODUCTION

In recent years, the Inverse Free Electron Laser (IFEL) has gained popularity because of its accessibility as tabletop accelerator with high energy gradient and most of all – because of its ability to microbunch beams on orders of the radiation scale [1]. Microbunched beams finds various applications throughout accelerator and beam physics including its use as a seed for high-gain harmonic generation (HG) FEL, preinjectors for advanced accelerators, and production of high quality, high current beams [2, 3].

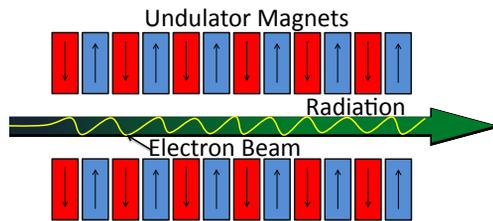


Figure 1: IFEL Interaction - Copropagating electron beam and laser within an undulator.

An IFEL consists of a laser and an electron beam copropagating within an undulator such that the electric field of the radiation is parallel to the wiggling of the electrons induced by the undulator (Figure 1). Efficient energy exchange between the beam and laser occurs when the resonant condition is satisfied – that is, when the electrons have energies such that the oscillation induced by the wiggler in its rest frame is in resonant with the electric field of the laser. This is satisfied when electrons has energy [1]:

$$\gamma_r^2 \simeq \frac{\lambda_\omega}{2\lambda} \left(1 + \frac{K^2}{2}\right) \quad (1)$$

where  $\gamma_r$  is the resonant electron energy,  $\lambda_\omega$  is the undulator wavelength,  $\lambda$  is the radiation wavelength, and  $K = \frac{eB}{mck_\omega}$  is the normalized undulator parameter.

The IFEL was introduced by Palmer in 1972, who proposed the possibility of 10 GeV electron acceleration using the IFEL interaction [4]. In recent years, various experiments have shown the success of IFEL to achieve high-energy gradients with good beam quality [2], with experiments being done in national laboratories such as ATF and Livermore [5, 6]. In addition, the use of the IFEL microbunched beams have been demonstrated to increase radiation gain of over seven orders of magnitude in the HG FEL [3].

Here we present a 1-D IFEL simulation with the capability of including slow amplitude variation and multiple harmonics of the fundamental radiation wave. Thus, we can model the three different types of IFEL based optical bunchers: the conventional (monochromatic), harmonic, and the adiabatic microbuncher. With the newly proposed harmonic and adiabatic buncher, the ability to modulate the electron beam density is increased significantly, with bunching factors larger than 90 % [2, 7]. Space charge effects in ultra-relativistic electron beams may no longer be neglected, and considerations of its effects on the resulting beam becomes more important. We first present a review of the IFEL based optical bunchers. Then, we introduce a model approximating the space charge fields and verification of the bunching simulation with Genesis 1.3.

## MICROBUNCHING THEORY

### Monochromatic Microbuncher

The basic IFEL interaction of microbunching can be described in terms of phase space variables,  $\gamma$  and  $\psi$ , as alternative representations of the momentum and position of the electrons. The equations of motion are [1]:

$$\frac{d\gamma_i}{dz} = \frac{1}{2} k K_l \frac{K J J}{\gamma_i} \sin \psi_i + \frac{e}{m_0 c^2} E_i^{sc} \quad (2)$$

$$\frac{d\psi_i}{dz} \simeq k_\omega - k \frac{1 + K^2/2}{2\gamma_i^2} \quad (3)$$

where the  $i$  denotes the  $i^{th}$  electron,  $k_\omega = \frac{2\pi}{\lambda}$  is the undulator wavenumber,  $k = \frac{2\pi}{\lambda}$  is the radiation wavenumber,  $K_l = \frac{eE_0}{m_0 c^2 k}$  is the normalized radiation potential, and  $JJ = J_0(G) - J_1(G)$  is the coupling factor due to the geometry of the undulator, where  $G \approx \frac{K^2}{4+2K^2}$ .

In the special case where there is no net energy exchange between the radiation and electrons, the resonant energy is constant,  $d\gamma/dz = 0$ , and modulation of the density of

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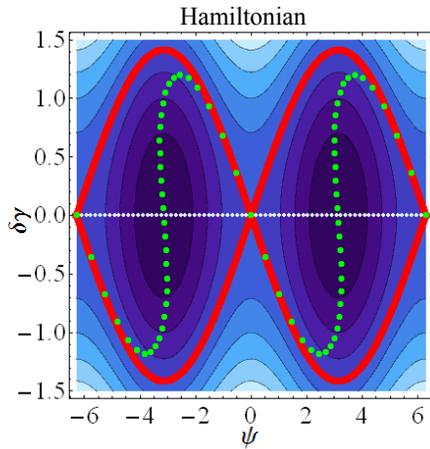


Figure 2: IFEL Hamiltonian for small oscillations and electron trajectories near resonant particle.

the electron beam occurs (microbunching). Figure 2 illustrates a contour of the Hamiltonian as well as the separatrix (bucket), denoted by the solid red line, and its reduction to a cosine-like potential for small oscillations about the resonant particle. In addition, Figure 2 shows the phase space evolution of electrons with energies close to the resonant energy (within the separatrix) reveal rotation within phase space, indicating the modulation of electron densities.

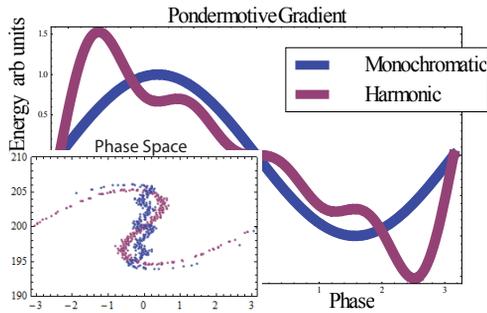


Figure 3: IFEL pondermotive gradients. The harmonic potential includes the first 4 harmonics of the fundamental radiation. The longitudinal phase space (lower left) illustrates the phase space of each gradient.

### Harmonic Microbuncher

For the harmonic microbuncher, the novel change is the addition of multiple harmonics of the fundamental radiation to copropagate with the electron beam [8]. The pondermotive potential is no longer limited by the nonlinearities experienced by the monochromatic buncher, but becomes more like an ideal simple harmonic oscillator with the addition of more harmonics (Figure 3) [2]. In theory, a combination of infinite harmonics can produce a perfectly linear restoring force and thus perfect bunching and infinite current, limited only by space charge [2].

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### Adiabatic Buncher

The adiabatic buncher is another proposed microbuncher that eliminates the nonlinearities of the pondermotive potential [7]. An appreciable quality of the adiabatic microbuncher over the other two is its ability to preserve longitudinal beam emittance [7]. In this interaction, the radiation power is slowly increased along the undulator to fulfill the adiabatic condition – to change the amplitude of the pondermotive gradient on time scales much longer than the synchrotron period in order to preserve beam quality. In practice, the adiabatic condition is challenging to implement but can be approximated by utilizing the diffraction dominated regime of a gaussian wave and a very long undulator [7]. Figure 4 illustrates the validity of the approximation and the output longitudinal phase space.

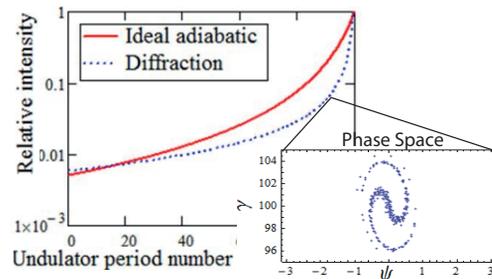


Figure 4: Fulfillment of the adiabatic condition and the approximation of the diffraction effects [7].

## SIMULATION

### Space Charge Approximation

The space charge fields are modeled by approximating each macroparticle as charged finite radial disks and summing the effects due all other macroparticles. In solving the coupled differential equations (2) and (3), we focus only on one bucket, with the assumption that all other buckets behave similarly. To evaluate the space charge effects, we must also account for the neighboring buckets and their contribution to the primary bucket.

Obtaining the electric field of one macroparticle on another and applying periodic boundary conditions, the electric field can be evaluated by:

$$E_0^{sc} = \epsilon \sum_x \sum_{b=-\infty}^{\infty} \text{sgn}(\Delta_0 - b) \left( 1 - \frac{1}{\sqrt{1 + \left(\frac{\Lambda}{\Delta_0 - b}\right)^2}} \right)$$

where  $\epsilon = \frac{1}{2\epsilon_0} \frac{I_p \lambda}{Nc} \frac{1}{\pi \sigma_x^2}$ ,  $\Delta_0 = \frac{\psi_x - \psi_0}{2\pi}$ ,  $\Lambda = \frac{\sigma_x}{\lambda}$ ,  $I_p$  is the input peak current,  $N$  is the total number of macroparticles,  $b$  is the bucket, and  $\sigma_x$  is the transverse beam size.

When examining the sum over the buckets, we can see that contributions due to the buckets located to the left can be manipulated such that the buckets need only to be summed from  $b = 0$  to  $b = \infty$ . In addition, because the

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sign function is defined to be 0 for  $\text{sgn}(0) = 0$ , this avoids double counting of the particle in the bucket of interest. Then, the contribution of the electric field on the  $0^{th}$  particle due to the  $x^{th}$  particle and its mirroring buckets can be written as:

$$E_0^{sc}(\psi_x) = \epsilon \text{sgn} \Delta_0 \left( 1 - \frac{1}{\sqrt{1 + \left(\frac{\lambda}{\Delta_0}\right)^2}} \right) + \sum_{b=1}^{\infty} \frac{1}{\sqrt{1 + \left(\frac{\lambda}{\Delta_0 - b}\right)^2}} - \frac{1}{\sqrt{1 + \left(\frac{\lambda}{\Delta_0 + b}\right)^2}}$$

It is then possible to evaluate this sum for various phase positions,  $\Delta_0$ , and use a  $n^{th}$  order fit function to replace this sum. This heavily simplifies the summation process by replacing an infinite sum to a simple polynomial function and reduces the space charge calculations to evaluating one sum that includes periodic boundary conditions.

Table 1: IFEL Monochromatic Bunching Parameters

<b>Electron Beam</b>	
Input Energy	100 MeV
Emittance	2 mm mrad
Peak Current	20000 Amps
<b>Laser</b>	
Power	50 TW
Wavelength	800 nm
Spot size ( $1/e^2$ ) at focus	3.56 mm
Rayleigh Range	50m
<b>Undulator</b>	
Undulator Length	0.96m
Undulator Period	.0639m
Undulator Parameter	.056

### Verification of Simulation

The simulation, executed in Mathematica 8.0, was verified with the well-known FEL simulator, Genesis 1.3, developed by Sven Reiche [9]. This simulation is capable of modeling the conventional IFEL buncher, as well as the harmonic and adiabatic buncher.

In order to properly compare the models and the correctness of the space charge effects, we simulated in a particular case with parameters shown in Table 1. In this regime, the laser waist is much larger than the transverse size of the beam, reducing transverse effects and inducing a regime where 1-D simulations are justified. Additionally, in this regime, the gain length is much greater than the plasma length – that is  $\frac{L_g}{L_p} \gg 1$ . This ensures that there is no radiation gain, which would otherwise counteract the space charge effects. Thus, we can clearly compare the effects of the pondermotive potential as well as space charge effects between the simulation and the Genesis 1.3 simulation without erroneous contributions existing in other particle simulators.

We found that the solutions of the equations of motion as well as the space charge calculations are consistent in the Mathematica simulation in comparison with Genesis 1.3. The bunching factors for space charge on and off are shown in Figure 5, showing the consistency between the models.

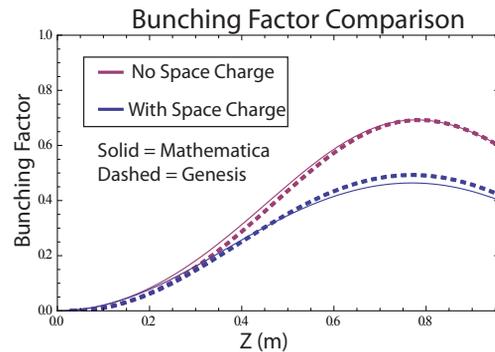


Figure 5: Comparison of Bunching Factor of monochromatic microbuncher between the simulations.

## CONCLUSION

Verification with Genesis 1.3 and isolation the space charge effects validates the use of this simulation to adequately compare the different type of bunchers as well as the space charge effects.

Future plans include comparing the benefits and drawbacks of each – their experimental feasibility, bunching capabilities, and effects on the beam’s final emittance, energy spread, peak current. Because space charge effects become much more dominant in the harmonic and adiabatic buncher, considerations of the extent of its affects on beam quality will also be analyzed.

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