SYNCHROTRON RADIATION NEAR FIELD IN 3D*

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Abstract

The Coherent Synchrotron Radiation (CSR) effect plays an important role in particle accelerators where high current electron beams are required, e.g., in X-ray Free Electron Laser. These electron beams are typically compressed to kA current in magnetic bends therefore they are subject to CSR effect. The widely used 1D CSR model relies on particle's radiation field along the circular trajectory near its present position, i.e., Green's function in 1D. We augment our previous 2D numerical CSR model by extending the synchrotron radiation near field calculation in the vertical direction. Our 3D calculation includes the dependence of the field on the relativistic beam energy through the scaled spatial variables and can be used to construct an efficient 3D CSR model due to the self-similarity of the field pattern in these variables.

INTRODUCTION

Coherent synchrotron radiation occurs in accelerators when high current beam bunches move along a circular beam path. It is a collective effect due to the coherence of the synchrotron radiation emitted by individual charge particle in the beam. As the radiation field on a particle is enhanced by the number of nearby particles if they emit coherently, the beam can undergo emittance growth or microbunching instabilities from interacting with its own coherent radiation. CSR is a major adverse effect on the performance of the FEL and has been investigated extensively in 1D and 2D [1-8]. As accurate understanding of the CSR effect requires modeling that accounts for the realistic beam shape and parameters, a 3D model will be essential and valuable. Although various CSR models have been developed so far, the most efficient and widely used models are based on a convolution approach using the time-independent longitudinal synchrotron radiation near-field, i.e., the "Green's function", of a single particle. We note that this near-field Green's function is relative to the present position of the radiating particle and in fact consists of both the near field and far field contributions from the particle at various retarded positions. Extending such CSR model into 3D requires an accurate description of the said Green's function in 3D, which is described in detail in the next two sections.

APPROXIMATE GREEN'S FUNCTION FOR RADIATION FIELD IN 2D

In this section we first review the 2D CSR model developed in Ref. [9]. This will serves as the premise of the 3D model discussed in the next section.

A diagram of the geometry is shown in Fig. 1. It is assumed that an electron is moving along a prescribed circular trajectory of radius R at constant angular velocity in the bending plane perpendicular to the magnetic field. The present and retarded positions of the particle are denoted as P and P', the corresponding velocities denoted as $\vec{\beta}$ and $\vec{\beta}'$, respectively. The field point A is represented by its coordinates (x, α) in this 2D geometry where *x* is the radial displacement of A relative to P and α is the angular difference between these two points.

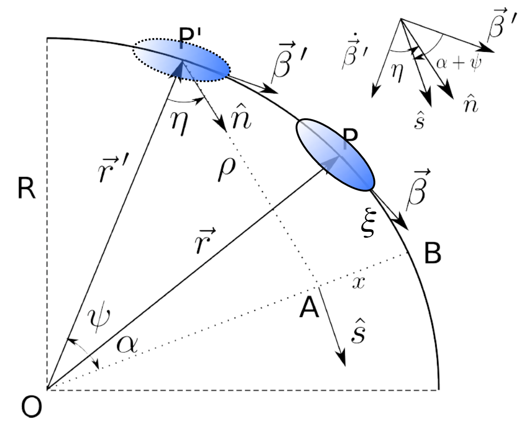

Figure 1: Geometry of the 2D model for calculating the longitudinal electric field from the synchrotron radiation of a particle in uniform circular motion. P and P' are the present and retarded positions of the particle, respectively.

The geometric relations that relate the present and retarded position of the particle are,

$$1+(1+x)^2-2(1+x)\cos(\alpha+\psi)=\psi^2/\beta^2 \quad (1)$$

$$1+\psi^2/\beta^2-2(\psi/\beta)\cos\eta=(1+x)^2 \quad (2)$$

where *x* has been normalized to *R*.

Using the Padé approximation

$$\cos(\zeta)\approx(1-5\zeta^2/12)/(1+\zeta^2/12)$$

and $|x|<<1$, $|\alpha|<<1$, $\psi^2<<12$, the transcendental equation Eq. (1) can be approximated as,

$$x^2+\alpha^2+2\alpha\psi+(x-\gamma^{-2}\beta^{-2})\psi^2-\psi^4/12\beta^2=0 \quad (3)$$

After adopting the scaled variables $\tilde{x}=x\gamma^2$, $\tilde{\alpha}=\alpha\gamma^3$ and $\tilde{\psi}=\psi\gamma$, dropping the second term α^2 on the left hand side of Eq. (3) which is only important at the opposition

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direction of the forward radiation cone and β^2, this equation can be written in general form,

$$\tilde{x}^2 + 2\tilde{\alpha}\tilde{\psi} + (\tilde{x}-1)\tilde{\psi}^2 - \tilde{\psi}^4/12 = 0 \quad (4)$$

Similarly, the longitudinal electric field from the particle's synchrotron radiation $E_s = -\partial(\phi - \beta A_s)/\partial\xi$, can be written into these scaled variables.

Since

$$(\phi - \beta A_s) = \frac{e\beta[1-\beta^2\cos(\alpha+\psi)]}{R\psi(1-\beta\sin\eta)} \quad (5)$$

one can obtain,

$$(\phi - \beta A_s) \approx \frac{e\gamma(1+\beta\sin\eta)}{R\tilde{\psi}[1+(\tilde{\psi}/2-\tilde{x}/\tilde{\psi})^2]} \quad (6)$$

It can be shown that $(1+\beta\sin\eta)$ weakly depends on γ in most regions at the $(\tilde{x},\ \tilde{\alpha})$ plane except for the vicinity of the negative $\tilde{\alpha}$ axis where radiation is negligible. Therefore,

$$E_s \approx \gamma^4 F(\tilde{x},\ \tilde{\alpha},\ \tilde{\psi})\ . \quad (7)$$

Here $F(\tilde{x},\ \tilde{\alpha},\ \tilde{\psi})$ is the functional form of the longitudinal electric field that only depends on $\tilde{x}$, $\tilde{\alpha}$ and $\tilde{\psi}$. We can then introduce the normalization of the electric field $\tilde{E}_s = E_s/(e\gamma^4/R^2)$ which reveals its amplitude scaling with the particle's energy.

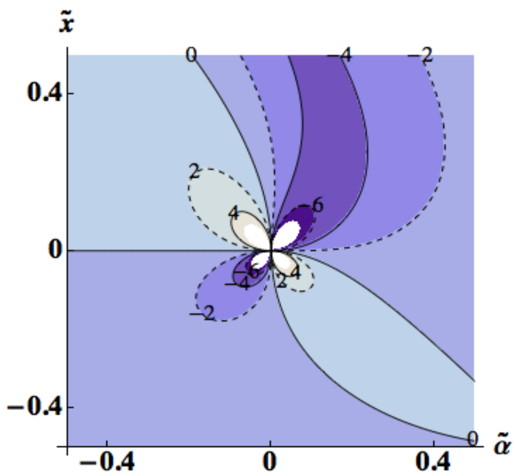

Figure 2: The general near field pattern of the longitudinal synchrotron radiation electric field in 2D for a particle in uniform circular motion.

Since $\tilde{\psi}$ implicitly depends on $\tilde{x}$, $\tilde{\alpha}$ from Eq. (4), Eq. (7) indicate that the longitudinal synchrotron radiation near field can be obtained, to a good approximation, from the invariant (with respect to γ) function $F(\tilde{x},\ \tilde{\alpha},\ \tilde{\psi}(\tilde{x},\ \tilde{\alpha}))$ with the proper scaling of the spatial variables and amplitude with the particle's energy. Figure 2 shows the longitudinal electric field $\tilde{E}_s = E_s/(e\gamma^4/R^2)$ as a function of $\tilde{x}$, $\tilde{\alpha}$. The field's amplitude is large along the four diagonal directions, forming a cloverleaf like pattern. While the fields quickly decreases along the other three diagonal directions, it extends to far greater distance in the top-right corner as shown in Fig. 2.

APPROXIMATE GREEN'S FUNCTION FOR RADIATION FIELD IN 3D

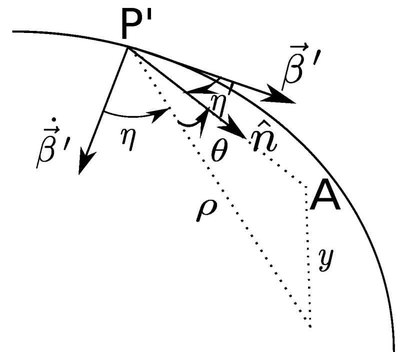

Figure 3: Schematic for the calculation in 3D.

We are now ready to extend the above result to 3D. In 3D, the position of field point A is denoted by its coordinates in the bending plane $(x,\ \alpha)$ and its vertical displacement y from this plane. Figure 3 shows the position of A and the angles it forms with respect to P'.

It is straightforward to show that in 3D Eq. (1) and (2) become,

$$1+(1+x)^2 - 2(1+x)\cos(\alpha+\psi) = \psi^2/\beta^2 - y^2, \quad (8)$$

$$1+(\psi^2/\beta^2 - y^2) - 2\sqrt{\psi^2/\beta^2 - y^2}\cos\eta = (1+x)^2, \quad (9)$$

where y is also normalized to R.

Since x^2 and y^2 appear together in Eq. (8) and (9), one can treat x and y similarly and define scaled variable $\tilde{y} = y\gamma^2$. With this scaled variable, Eq. (4) becomes,

$$\tilde{x}^2 + \tilde{y}^2 + 2\tilde{\alpha}\tilde{\psi} + (\tilde{x}-1)\tilde{\psi}^2 - \tilde{\psi}^4/12 = 0\,. \quad (10)$$

In 3D, the unit vector $\hat{n}$ is not necessarily in the bending plane, thus $\hat{n}\cdot\vec{\beta}' = \beta\cos\eta' = \beta\sin\eta\cos\theta$. Hence,

$$(\phi - \beta A_s) = \frac{e\beta[1-\beta^2\cos(\alpha+\psi)]}{R\psi(1-\beta\sin\eta\cos\theta)}\ . \quad (11)$$

Next we consider the $[1-\beta^2\cos(\alpha+\psi)]$ term and the $(1-\beta\sin\eta\cos\theta)^{-1}$ term in Eq. (11) separately. It is easy to show,

$$1-\beta^2\cos(\alpha+\psi)=\gamma^{-2}\left[1-\frac{(\tilde{x}^2+\tilde{y}^2)\beta^2\gamma^{-2}-\tilde{\psi}^2}{2(1+\tilde{x}\gamma^{-2})}\right]\approx\gamma^{-2}, \quad (12)$$

and

$$(1-\beta\sin\eta\cos\theta)^{-1}=(1+\beta\sin\eta\cos\theta)(1-\beta^2\sin^2\eta\cos^2\theta)^{-1} \quad (13)$$

The first term on the right hand side of Eq. (13) varies slowly with particle energy and is bounded, $0\le(1+\beta\sin\eta\cos\theta)\le 2$. The second term can be further simplified using the following relation,

$$\gamma\beta\cos\eta\cos\theta=\gamma\psi/2-\gamma\beta^2(2x+x^2+y^2)/2\psi\approx\tilde{\psi}/2-\tilde{x}/\tilde{\psi} \quad (14)$$

leading to,

$$1-\beta^2\sin^2\eta\cos^2\theta\approx\gamma^{-2}\left[1+\tilde{y}^2/\tilde{\psi}^2+(\tilde{\psi}/2-\tilde{x}/\tilde{\psi})^2\right]. \quad (15)$$

With the above simplification, Eq. (11) reduces to a form akin to Eq. (6),

$$(\phi-\beta A_s)\approx\frac{e\gamma(1+\beta\sin\eta\cos\theta)}{R\tilde{\psi}[1+\tilde{y}^2/\tilde{\psi}^2+(\tilde{\psi}/2-\tilde{x}/\tilde{\psi})^2]}. \quad (16)$$

Therefore, in 3D, the functional form of Eq. (7) becomes

$$E_s\approx\gamma^4 G(\tilde{x},\ \tilde{y},\ \tilde{\alpha},\ \tilde{\psi}). \quad (17)$$

From the Liénard-Wiechert formula, the longitudinal radiation field is given by,

$$E_s=\frac{e}{c\rho}\cdot\frac{(\hat{n}\cdot\dot{\vec{\beta}}')(\hat{n}\cdot\hat{s}-\vec{\beta}'\cdot\hat{s})-(\hat{s}\cdot\dot{\vec{\beta}}')(1-\hat{n}\cdot\vec{\beta}')}{(1-\hat{n}\cdot\vec{\beta}')^3}. \quad (18)$$

After the retarded angle $\tilde{\psi}$ is solved from Eq. (10), one can use the following identities,

$$\hat{n}\cdot\vec{\beta}'=\beta\sin\eta\cos\theta,\ \hat{n}\cdot\dot{\vec{\beta}}'=(\beta^2 c\cos\eta\cos\theta)/R, \quad (19)$$
$$\hat{n}\cdot\hat{s}=\sin(\alpha+\psi+\eta)\cos\theta$$

Eq. (18) becomes,

$$\tilde{E}_s=\frac{\beta^3}{\gamma^4\psi(1-\beta\sin\eta\cos\theta)^3}\cdot[(\sin\eta-\beta\cos\theta) \quad (20)$$
$$\cdot\cos(\alpha+\psi+\eta)-\sin^2\theta\sin(\alpha+\psi+\eta)\cos\eta]$$

Figure 4 shows $\tilde{E}_s$ as a function of $\tilde{y}$ and $\tilde{\alpha}$ for $\tilde{x}=0$. It should be noted that the radiation field is strongest in the forward direction within a ~ 45 degree cone in the scaled variables $\tilde{y}$ and $\tilde{\alpha}$, which is consistent with the well-known $1/\gamma$ radiation cone in the un-scaled spatial variables. This also implies that for a beam that is sufficiently large in the vertical y direction, both the 1D and 2D models in the ($\tilde{x},\tilde{\alpha}$) plane will overestimate the longitudinal CSR force.

Similar to our previous 2D CSR model [9], the calculation presented here can be done once and applied to a range of beam parameters. The key to such flexibility is the recognition of the self-similarity of the radiation field in three spatial dimensions through the scaled variables. This follows the pioneering work in one spatial dimension by several other authors [5,6]. However, due to the complex radiation field pattern in 3D, it is rather difficult to derive an analytic formula for such field in the identified scaled variables. Numerical calculation for the 3D radiation field of a single particle is generally necessary and possible, as demonstrated in this paper, whose result can then be used to construct a 3D CSR model through the convolution with the beam shape.

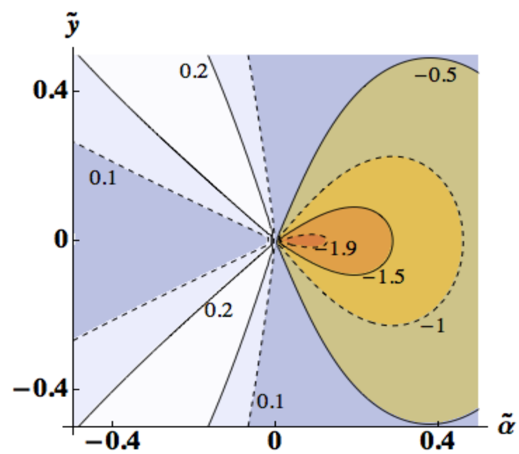

Figure 4: Longitudinal radiation field pattern for a highly relativistic in the vertical ($\tilde{y},\ \tilde{\alpha}$) plane at $\tilde{x}=0$.

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