# A HIGH-PERFORMANCE RECTILINEAR FOFO CHANNEL FOR MUON COOLING*

D. Stratakis[#], J. S. Berg, R. B. Palmer, Brookhaven National Laboratory, Upton NY, USA

V. I. Balbekov, Fermi National Laboratory, Batavia IL, USA

## Abstract

An ionization cooling channel is a tightly spaced lattice containing absorbers for reducing the momentum of the muon beam, rf cavities for restoring the longitudinal momentum and solenoids for focusing the beam. Such a lattice is an essential step for a Muon Collider. Here, we explore a new scheme for designing ionization cooling channels for muon related applications. In this scheme, emittance reduction is achieved within a rectilinear channel with wedge absorbers for cooling and tilted solenoids for emittance exchange. Such configuration addresses several of the engineering challenges of a conventional helical channel. We numerically examine the performance of our proposed channel and compare it against conventional designs. We also review the conductor current densities requirements for all of the simulated scenarios.

## INTRODUCTION

A key technical challenge in the development of a Muon Collider is that the phase space of the beam that comes from pion decay greatly exceeds the acceptance of downstream accelerators system. Therefore, a cooling channel is required. Given the short life time of a muon particle, ionization cooling is the only practical method that can be realized.

Over the past decade several progresses have been made in the design and simulation of 6D RFOFO cooling rings [1] based on emittance exchange. This is generally accomplished by using a wedge shaped absorber in a region with dispersion. Particles with higher energies will pass through more material than particles with lower energy as a result of dispersion, eventually leading to reduction of longitudinal emittance. Such rings have been shown to provide an impressive to two order magnitude reduction of the normalized 6D phase-space volume with a transmission well above 50%. In order to avoid the complexity arising from the required injection and extraction systems, those rings involved without loss in performance to a RFOFO helical channel [2], commonly known as the Guggenheim. A chief advantage of the Guggenheim is that avoids any issues related to absorber overheating.

Although a helical lattice can be advantageous in paper, still many questions remain. For instance, the beam can be intercepted by the fringe fields from the nearby solenoids. In addition, engineering constraints may arise at the last stages as the radius of curvature becomes less than 1.5 m. To further relax the aforementioned constraints we design and simulate here a rectilinear FOFO snake channel which has been proposed recently

by one of the authors [3] and show that it can offer the same performance as the Guggenheim. In particular, we show that by applying a novel tapering scheme in which numerous parameters of the structure change from stage to stage based on the emittance reduction rate and transmission, we can achieve a good cooling efficiency that remains flat through the channel. This is because the beam emittance is always above the equilibrium emittance.

## DESIGN PARAMETERS

The main issue with a channel of a fixed radius, cell length, and frequency is its gradual loss of cooling efficiency. Therefore, to keep the cooling process going we examine a tapered channel in which various parameters change in order to avoid reaching equilibrium. In this scheme, parameters such as cell length and radius of curvature progressively change from stage to stage based on the emittance reduction rate and transmission. Recent studies have shown [4] that such a concept improves performance substantially. In addition, with this approach the same emittance can be obtained in much shorter scale.
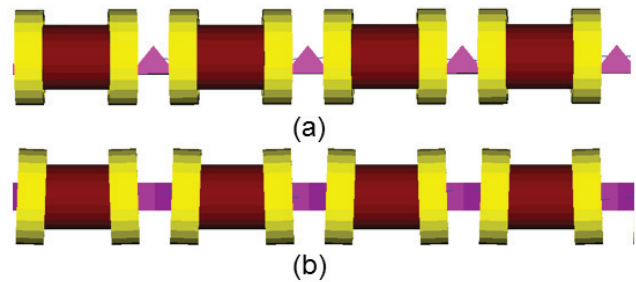


Figure 1: Conceptual design of our proposed rectilinear cooling channel: (a) Top view; (b) Side view. Note that the coils (yellow) are slightly tilted to generate dispersion.

At the first stage of the tapered channel the focusing will be relatively weak to avoid excessive angular divergence that can arise from the large transverse emittance of the initial muon beam. However, the weak focusing implies that the beta function and thus the equilibrium emittance are also relatively large, so the transverse cooling weakens as the limit is approached. To avoid this, this stage is terminated and we couple into the next stage that has a lower beta. This is achieved by simultaneously scaling down the cell dimensions and raising the strength of the on-axis solenoidal field. As a result this will produce a piecewise multi-stage shaped channel where each stage will consist of a series of

identical cells. A schematic of our proposed cooling system is shown in Fig.1. Note that the coils (yellow) are not evenly spaced; those on either side of the absorber are closer in order to increase the focusing at the wedge absorber (magenta). In order to produce dispersion, they are tilted by $1.1$-$1.3^0$. The absorber is wedge shaped so that higher momentum particles go through the thicker part. Each lattice cell contains the same number of rf cavities (dark red) which, depending on the stage, can be 4, 5, or 6. The rf frequencies are either 325 MHz or 650 MHz. Those frequencies are chosen in order to match with the initial linac of Project X.
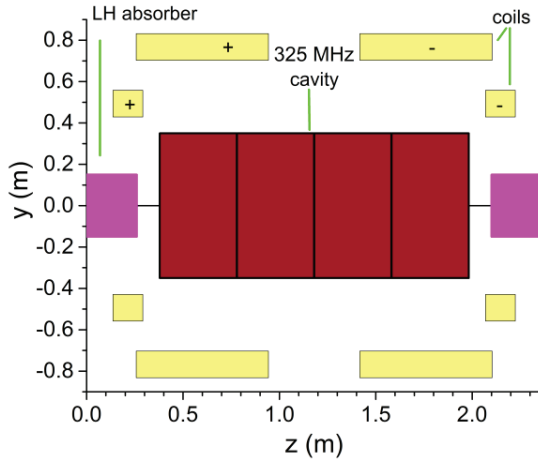


Figure 2: Close view of one cell at an early stage. Each cell has 4 coils and two of them are raped around the cavities. The tilt of the coils is not shown.

Figure 2 shows the side view of one cell at an early stage of the channel (Stage 2). This stage consists of a sequence of 14 identical 2.36 m cells, each containing four 0.4 m-long 325 MHz pillbox cavities, and a wedge-shaped liquid hydrogen absorber, with 28.6 cm central thickness and 100 deg. opening angle, to assure energy loss. Moreover, each cell contains four solenoid coils of opposite polarity, yielding an approximate sinusoidal variation of the magnetic field in the channel with a peak on axis value of 3.1 T and providing transverse focusing with a peak beta value of ~ 36 cm. In order to provide the required dispersion the coils are tilted by 1.1 deg vertically. The lattice transmits in a region from 155 to 248 MeV/c, with a central (reference) momentum of 204 MeV/c.
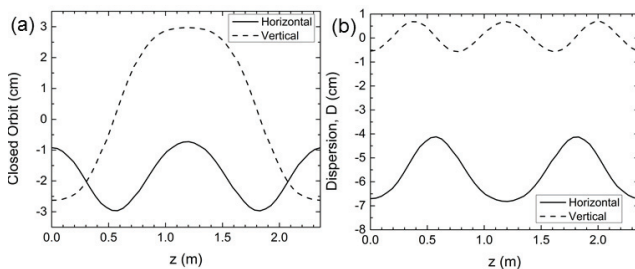


Figure 3: Closed orbit at the reference momentum of 204 MeV/c (left). Dispersion in the vertical and horizontal directions along the cell.

Deviation of the closed orbit along the cell for the reference momentum of 204 MeV/c is shown in Fig. 3(a). Notice that at the central momentum the vertical offset (dashed) from the center is ~ 2.6 cm at the entrance of the cell. Figure 3(b) displays the dispersion functions in x (solid) and y (dashed) along the cell. The dispersion at the absorber has the same sign in both directions and is mostly in the horizontal direction. Notice that the dispersion is maximum at the center of the absorber.

The beta function varies from 40.0 cm to 4.0 cm while the on-axis magnetic field increases progressively from 2.7 T to 13.1 T. It is important to emphasize that the maximum field on the coil at the last stage is 14.1 T with a current density of 196 A/mm$^2$. This value is below the critical published [5] limit for $Nb_3Sn$. The RF frequency is increased from 325 MHz to 650 MHz. Ideally, the cooling channel should be tapered with a continuously varying frequency. However, for practical implementation we keep the number of different frequencies as small as possible. As we will illustrate in the next section, 16 stages are enough to achieve the desired emittance for a Muon Collider. Detailed lattice parameters of our proposed channel can be found in Table 1.

## SIMULATION DETAILS

Simulations of the channel performance where done using the ICOOL code with 100,000 particles. The simulation is initiated after the bunch merger. We generated 3D cylindrical field maps for each of the stages by superimposing the fields from all solenoids in the cell and its neighbour cells. The rf cavities were modelled using cylindrical pillboxes running in the TM010 mode. The cavities are enclosed with 100 μm metallic Beryllium windows. The wedge material was liquid hydrogen. The absorbers are enclosed in Al windows ranging from 500 μm (Stage 1) to 10 μm (Stage 16).

The lattice cooling efficiency is defined as the rate of change of the 6D emittance divided by the change of muons. It is often used to quantify the lattice performance and it is displayed in Fig. 4. One can see that for a rectilinear channel it remains relatively flat along the channel with a maximum near 9.5. The helical lattice appears to have a slight higher performance. Note that a more detailed comparison between the two schemes can be found elsewhere [6].
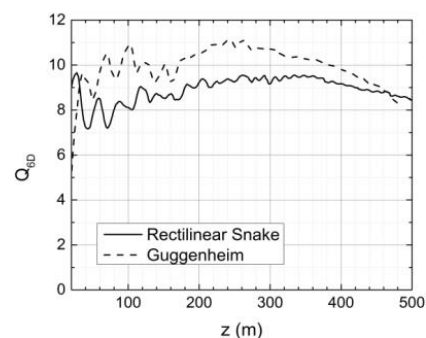


Figure 4: Cooling efficiency of the rectilinear channel (solid) and Guggenheim (dashed line).

Table 1: Lattice Parameters of Our Proposed Tapered Rectilinear FOFO Channel

| Stage | Cell length [m] | RF freq. [MHz] | RF grad. [MV/m] | Beta [cm] | $B_o$ axis [T] | $B_{max}$ coil [T] | Coil tilt [deg.] | Dispersion [cm] |
|---|---|---|---|---|---|---|---|---|
| 1 | 2.75 | 325 | 17.5 | 40.0 | 2.7 | 7.1 | 1.1 | 6.6 |
| 2 | 2.36 | 325 | 17.5 | 35.8 | 3.1 | 5.5 | 1.1 | 6.8 |
| 3 | 2.02 | 325 | 18.5 | 28.6 | 3.6 | 7.8 | 1.1 | 7.5 |
| 4 | 1.73 | 325 | 17.5 | 25.1 | 4.1 | 5.6 | 1.2 | 3.4 |
| 5 | 1.49 | 325 | 19.5 | 21.1 | 4.9 | 7.9 | 1.3 | 5.0 |
| 6 | 1.38 | 325 | 18.0 | 17.4 | 5.4 | 8.5 | 1.1 | 3.8 |
| 7 | 1.27 | 325 | 20.5 | 14.6 | 6.0 | 9.4 | 1.2 | 5.4 |
| 8 | 1.15 | 325 | 20.5 | 13.5 | 6.5 | 10.2 | 1.1 | 4.4 |
| 9 | 0.995 | 650 | 24.0 | 10.8 | 7.7 | 10.4 | 1.1 | 2.9 |
| 10 | 0.806 | 650 | 27.0 | 8.7 | 9.5 | 12.9 | 1.1 | 2.3 |
| 11 | 0.806 | 650 | 26.0 | 7.1 | 10.3 | 12.7 | 1.1 | 1.8 |
| 12 | 0.806 | 650 | 27.0 | 6.1 | 11.0 | 13.0 | 1.1 | 1.7 |
| 13 | 0.806 | 650 | 27.0 | 5.2 | 11.6 | 13.5 | 1.1 | 1.6 |
| 14 | 0.806 | 650 | 26.0 | 4.3 | 12.6 | 14.1 | 1.1 | 1.4 |
| 15 | 0.806 | 650 | 26.0 | 4.0 | 13.1 | 14.2 | 0.5 | 0.6 |
| 16 | 0.806 | 650 | 26.0 | 4.0 | 13.1 | 14.2 | 0.5 | 0.6 |
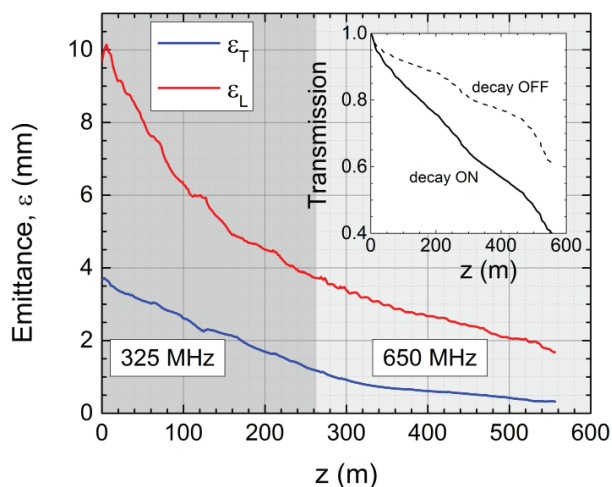


Figure 5: Longitudinal (red), transverse (blue) emittances and transmission as a function of distance along the channel. The final values of the emittances are also shown.

The transverse, longitudinal emittance and transmission including muon decays as a function of distance along stages 1 to 16 are shown in Fig. 5. The simulation produced a transverse emittance of 0.32 mm and a longitudinal emittance equal to 1.6 mm, which is close to the desired values for a Muon Collider. The transmission without decays is above 60%. If space-charge is included in the simulation an additional 10% of losses is predicted [7]. The quoted emittance values are normalized rms.

## SUMMARY

Cooling large emittance muon beams is an essential step for a Muon Collider. A rectilinear FOFO snake channel may reduce the engineering challenges compared to conventional helical structures (like in a Guggenheim helix). As this study demonstrated, by performing tapering the desired emittances for a muon collider can be obtained with a transmission above 60% without decays and 40% with muon decays. The cooling rate remains relatively flat along the channel with a peak value of 9.5. Furthermore, the maximum field on the coil is 14 T which is within the published critical limit for $Nb_3Sn$. The next step would be to improve the matching between individual stages, which will aid in boosting the performance. The authors like to thank P. Snopok and R. Ryne for many fruitful discussions.

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