Action and Phase Jump Analysis for LHC Orbits

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Action and Phase

$$z(s) = \sqrt{2J_z\beta_z(s)}\sin(\psi_z(s) - \delta_z) \quad (1)$$

When a magnetic error θ_z is present at $s = s_\theta$, the trajectory of the particle can be described independently before and after the error:

- Before the error $(s < s_\theta)$

$$z(s) = \sqrt{2J_0\beta_z(s)}\sin(\psi_z(s) - \delta_0) \quad (2)$$

- After the error $(s > s_\theta)$

$$z(s) = \sqrt{2J_1\beta_z(s)}\sin(\psi_z(s) - \delta_1) \quad (3)$$

J_0 and δ_0 are the action and phase before de error
J_1 and δ_1 correspond to the action and phase after the error.

With this equations, using the Courant–Snyder parameters to propagate the particle trajectory through the error (and after some algebra and trigonometric identities...)

We obtain the **KICK magnitude** as

$$\theta_z = \sqrt{\frac{2J_1 + 2J_0 - 4\sqrt{J_1 J_0}\cos(\delta_1 - \delta_0)}{\beta(s_\theta)}} \tag{4}$$

The kick θ_z could be generated by any the multiple components in the magnetic field multipole expansion.

$$\theta_x = B_0 - B_1 x(s_\theta) + A_1 y(s_\theta) + 2A_2 x(s_\theta) y(s_\theta) + B_2[-x^2(s_\theta) + y^2(s_\theta)] + \cdots \tag{5}$$

$$\theta_y = A_0 + A_1 x(s_\theta) + B_1 y(s_\theta) + 2B_2 x(s_\theta) y(s_\theta) + A_2[x^2(s_\theta) - y^2(s_\theta)] + \cdots \tag{6}$$

with $A_n = B' l a_n / B\rho$ and $B_n = B' l b_n / B\rho$

Linear errors

For **dipolar errors**, no dependence with any of the transverse coordinates is expected.

$$B_0 = \theta_z \tag{7}$$

For **quadrupolar errors**, linear dependency with the transverse coordinates is expected.

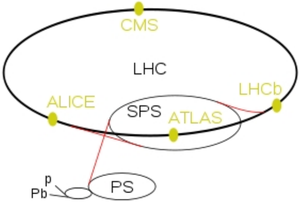
$$A_1 = \frac{\theta_x y(s_\theta) + \theta_y x(s_\theta)}{x^2(s_\theta) + y^2(s_\theta)} \tag{8}$$

$$B_1 = \frac{\theta_y y(s_\theta) - \theta_x x(s_\theta)}{x^2(s_\theta) + y^2(s_\theta)} \tag{9}$$

Requirements

- BPMs readings
  - Two before the error (to calculate (J_0, δ_0))
  - Two after the error (to calculate (J_1, δ_1))
- The lattice model
  - To obtain the Courant–Snyder parameters of the accelerator
- Multiturn runs
  - To increase the precision in the polinomial fitting (quadrupolar and sextupolar errors mainly), it is better if the multiturn run is made with high oscillations

The LHC



Images taken from [1] and [5]

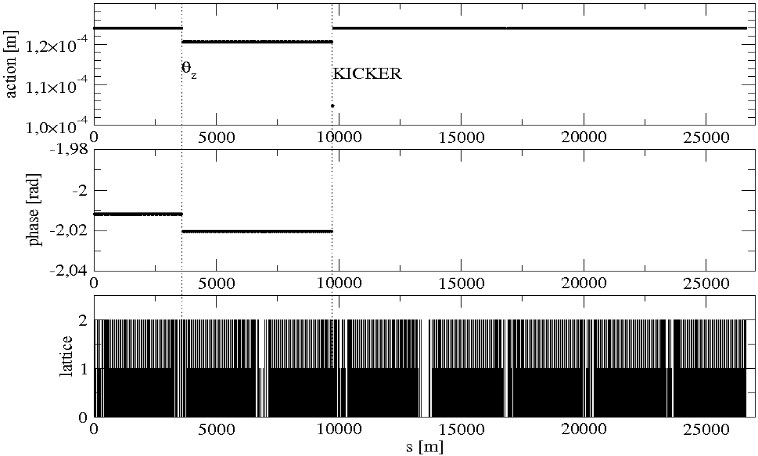
Error simulations

Dipolar and quadrupolar errors were included in the accelerator.

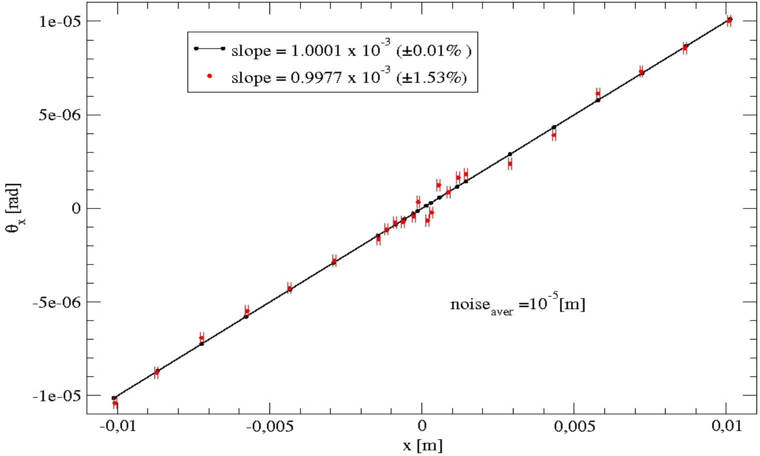
Orbits with the errors were obtained by simulations

- MAD-X V4.01
  - Experiments off
  - Period lhcb1
  - Lattice model V6.5
  - Energy 450[GeV] (injection)
  - Particle PROTON

Error Simulations

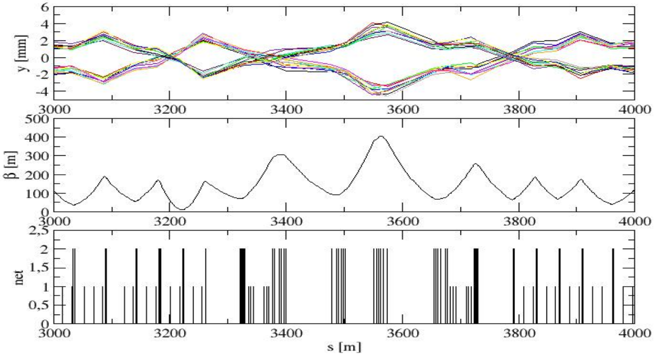


Including noise

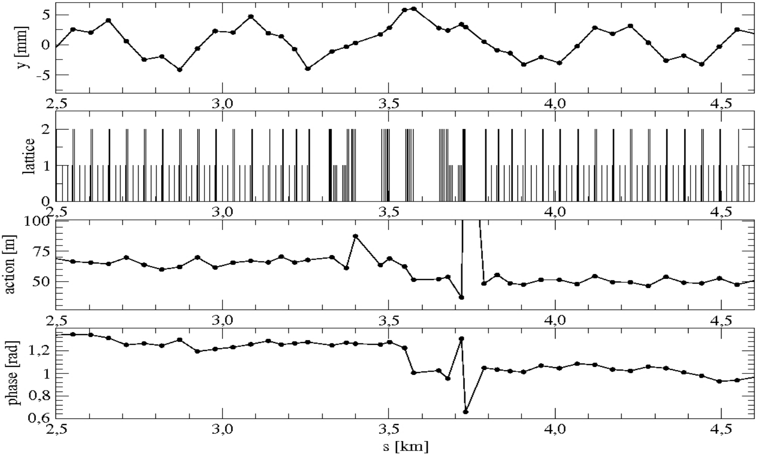


Phase range selection

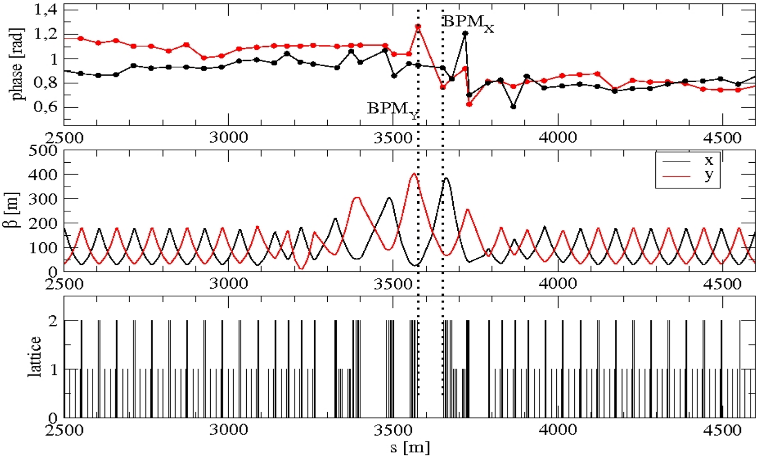
Phase average is calculated per orbit, then orbits with similar phase have aproximatelly same behaviour.



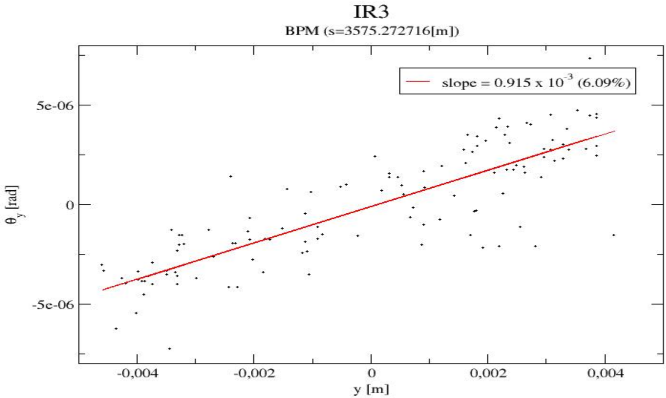
Analysing IR3



s coordinate selection



Error


IR3
BPM (s=3575.272716[m])
slope = 0.915 x 10^-3 (6.09%)
5e-06
0
-5e-06
θ_y [rad]
-0,004
-0,002
0
0,002
0,004
y [m]

Conclusions

- Simulations show that LHC linear magnetic errors could be recovered withing 1.53%
- Extension to sextupolar error has not been possible with this run
- Local error in IR3 has been identified
- Still some noise in the plots from the multiturn data. Noise could be reduced with data from multiple bunches
- IRs can be analysed independently
- Phase should be a continuum function (it might affect averages).

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- Direccion de Investigacion Sede Bogota, Universidad Nacional de Colombia (DIB, UNAL)

References

- Large Hadron Collider.
  http://en.wikipedia.org/wiki/File:LHC.svg
- Cardona J. et. al., Linear and non linear magnetic error measurements using action and phase jump analysis. PRST-AB 2009.
- Schmidt F., MAD-X User's guide.
  http://mad.web.cern.ch/mad/
- Schmidt F., A MAD-X primer.
  http://mad.web.cern.ch/mad/
- Design reports, beam parameters, lattice and optics from
  http://lhc.web.cern.ch/lhc/

Thank you.

Contact List

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- DIB (UNAL)
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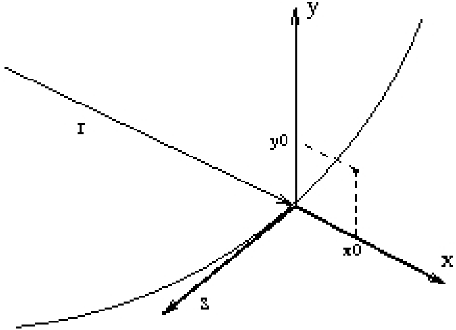
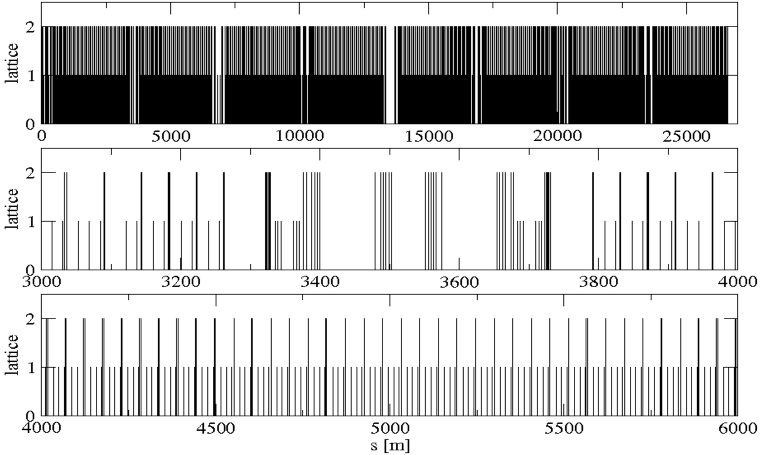
Coordinate system

The coordinate system is the usual (x, y, s), used for periodic accelerators

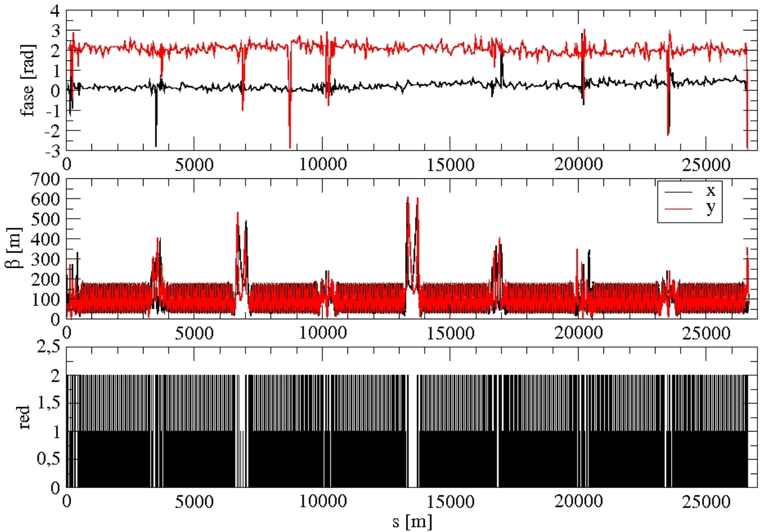


LHC lattice model

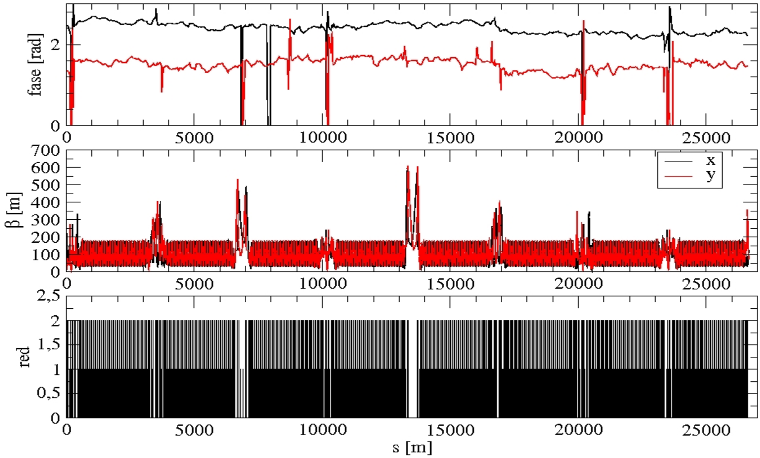


LHC lattice model

- LHC lattice model V6.5 (for MAD-X)
  - Monitors (BPMs), to extract simulated orbits
  - Sextupole correctors on arcs are turned off
  - All elements are used to extract β and ψ functions


fase [rad]
4
3
2
1
0
-1
-2
-3
0
5000
10000
15000
20000
25000
β [m]
700
600
500
400
300
200
100
0
x
y
red
2,5
2
1,5
1
0,5
s [m]

BPMs averages



Phase range selection