High Fidelity Calculations of Wakefields for Short Bunches

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SLAC National Accelerator Laboratory

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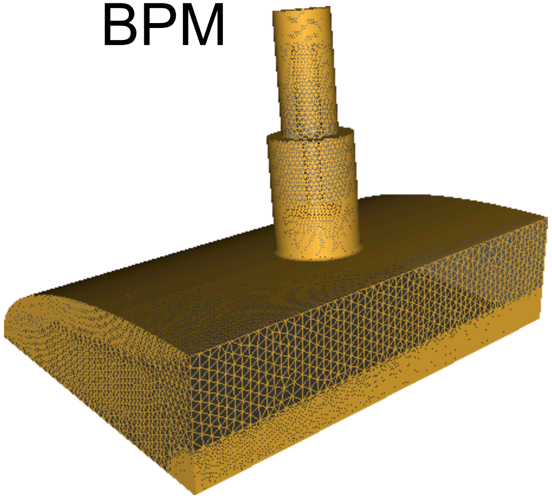
Wakefield Computation

- Wakefield computation involves
  - Evaluation of loss factors and kick factors of beamline components in storage rings and linacs
  - Determination of Green's functions for beam instability studies
- It requires substantial computing resources (memory and CPU) for short bunches
  - Fine resolution to resolve high frequency content
- Moving window techniques significantly reduce wakefield computing requirements by confining the computation around the bunch
  - Successfully used in finite difference time domain codes such as MAFIA, Microwave Studio, GdfidL, Echo
  - Its implementation in finite element time domain codes using unstructured grids such as T3P has recently been investigated

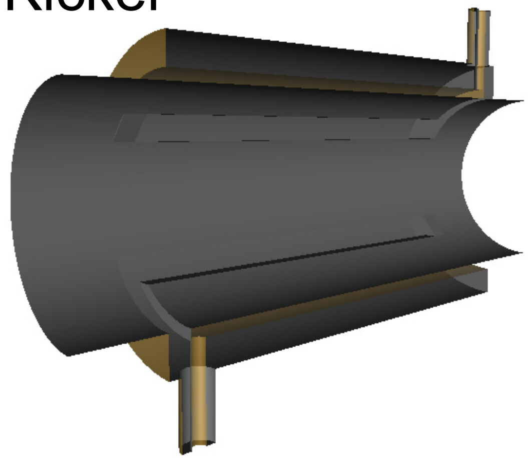
Ref: L.-Q. Lee et al., J. Comput. Phys. **229**, 9235 (2010)

Modeling of Accelerator Components

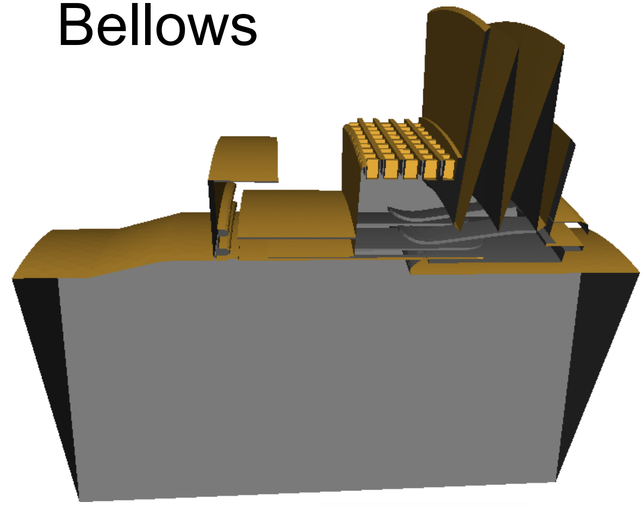
BPM

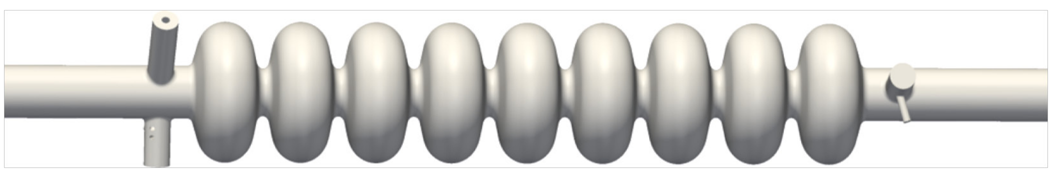


Kicker

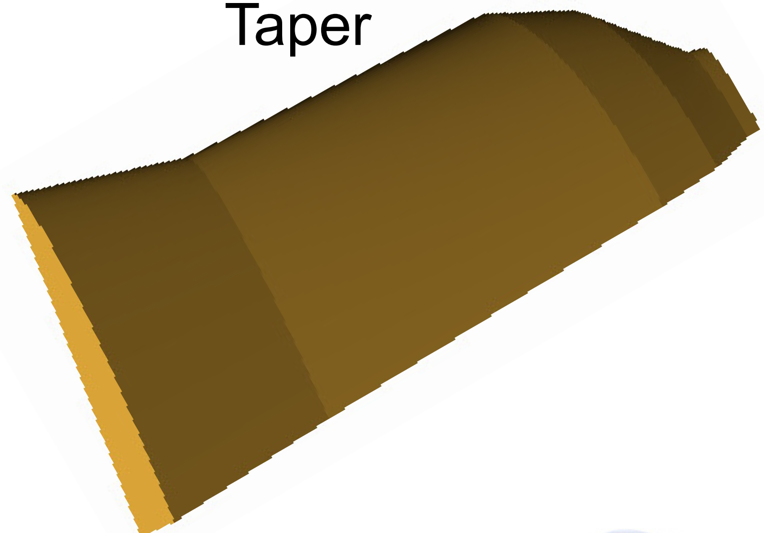


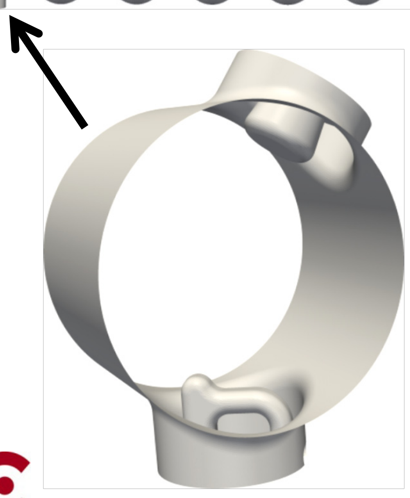
Bellows





Taper





Endgroup in SCRF cavity

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Parallel EM Code Suite ACE3P and User Community

ACE3P (Advanced Computational Electromagnetics 3P) Code Suite

https://slacportal.slac.stanford.edu/sites/ard_public/bpd/acd/Pages/Default.aspx

- conformal, higher-order, C++/MPI parallel finite-element based electromagnetic codes
- supported by SLAC and DOE HPC Grand Challenge (1998-2001), SciDAC1 (2001-06), SciDAC2 (2007-12)

Modules include

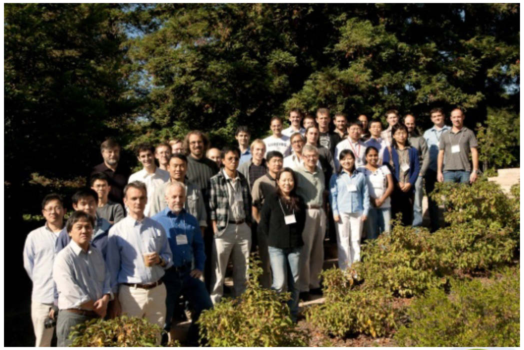
Frequency Domain:	**Omega3P**	**– Eigensolver (damping)**
	S3P	**– S-Parameter**
Time Domain:	**T3P**	**– Wakefields,Transients**
Particle Tracking:	**Track3P**	**– Multipacting, Dark Current**
EM Particle-in-cell:	**Pic3P**	**– RF Gun, Klystrons**
Multi-Physics:	**TEM3P**	**– EM, Thermal & Structural Analysis**

ACE3P Code Workshop

CW09 (15 attendees from 13 institutions) – 1 day

CW10 (36 attendees from 16 institutions) – 2.5 days

CW11 planned for 5 days



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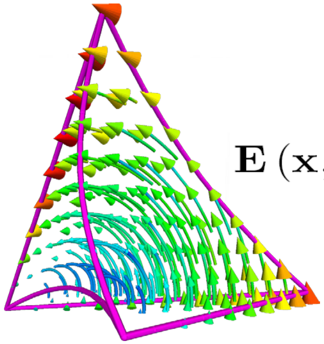
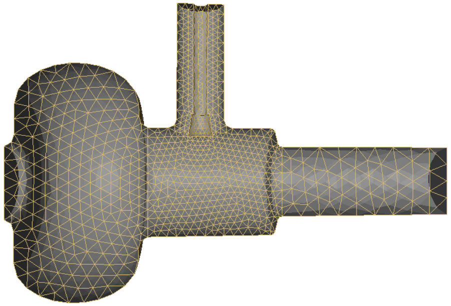
Parallel Higher-order Finite-Element Method

Strength of Approach – Accuracy and Scalability

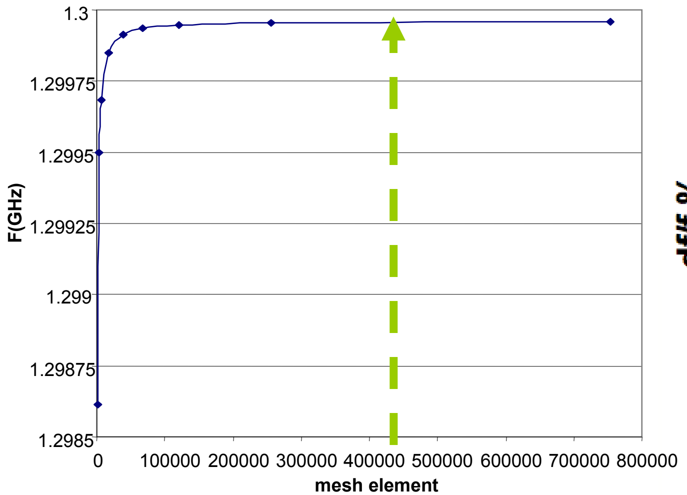
- Conformal (tetrahedral) mesh with quadratic surface
- Higher-order elements (p = 1-6)
- Parallel processing (memory & speedup)

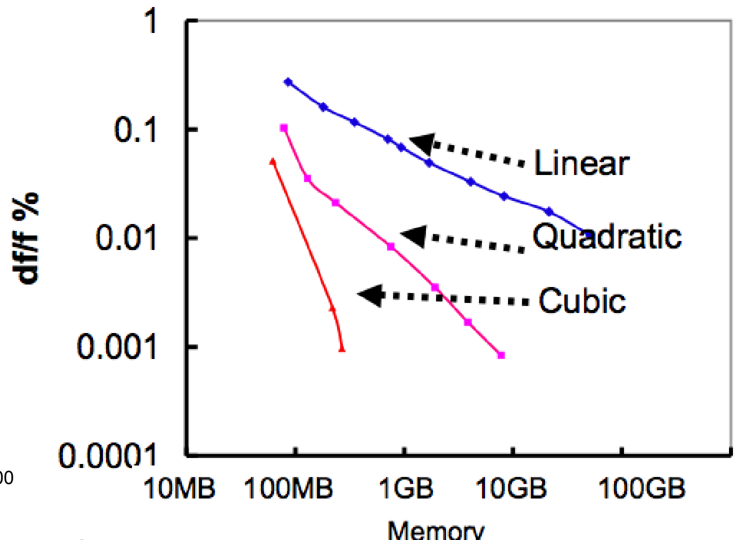


$$\mathbf{E}\left(\mathbf{x},t\right)=\sum_{i}e_{i}\left(t\right)\cdot\mathbf{N_i}\left(\mathbf{x}\right)$$



End cell with input coupler only





67k quad elements (<1 min on 16 CPU,6 GB)
Error ~ 20 kHz (1.3 GHz)

T3P - Finite Element Time Domain Solver

- Maxwell equation in second order

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \vec{\mathbf{E}}\right) + \sigma\frac{\partial \vec{\mathbf{E}}}{\partial t} + \epsilon\frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2} = -\frac{\partial \vec{\mathbf{J}}}{\partial t}$$

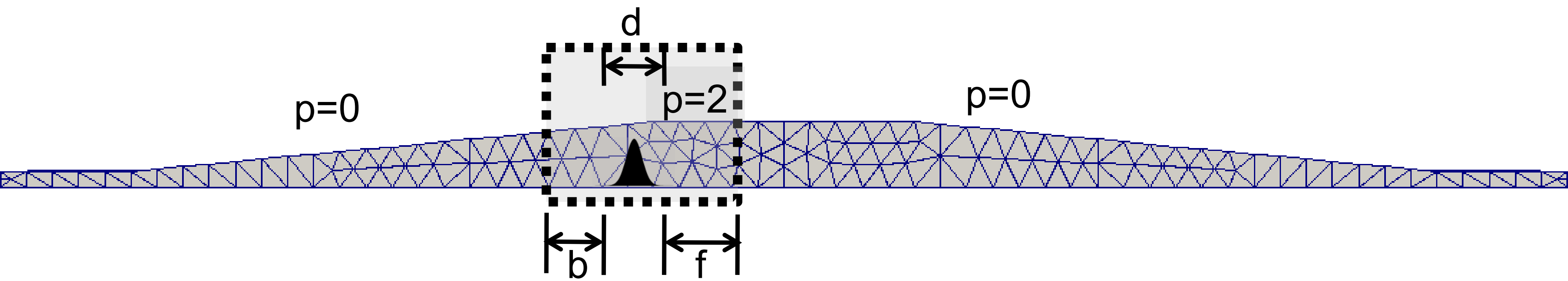
- Discretized form in finite element formulation

$$\begin{aligned}\left[\mathbf{M} + \tfrac{c\Delta t}{2}(\mathbf{R}+\mathbf{Q}) + \beta(c\Delta t)^2\mathbf{K}\right]\mathbf{x}^{n+1} &= \mathbf{b}\\ \left[2\mathbf{M} - (1-2\beta)(c\Delta t)^2\mathbf{K}\right]\mathbf{x}^{n}&\\ -\left[\mathbf{M} - \tfrac{1}{2}c\Delta t(\mathbf{R}+\mathbf{Q}) + \beta(c\Delta t)^2\mathbf{K}\right]\mathbf{x}^{n-1}&\\ -(c\Delta t)^2\left[\beta\mathbf{f}^{n+1} + (1-2\beta)\mathbf{f}^{n} + \beta\mathbf{f}^{n-1})\right]&\end{aligned}$$

- Newmark-β time stepping scheme: unconditionally stable* when $\beta >$ 0.25
- A linear system Ax=b needs to be solved for each time step
- Matrix in the linear system is symmetric positive definite
- Conjugate gradient + block Jacobi / incomplete Cholesky

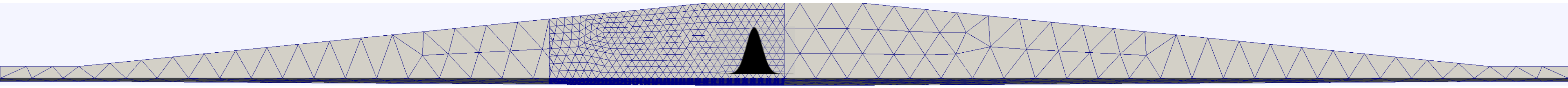
*Gedney & Navsariwala, An unconditionally stable finite element time-domain solution of the vector wave equation, IEEE Microwave and Guided Wave Letters, vol. 5, pp. 332-334, 1995.

Moving Window with p-Refinement

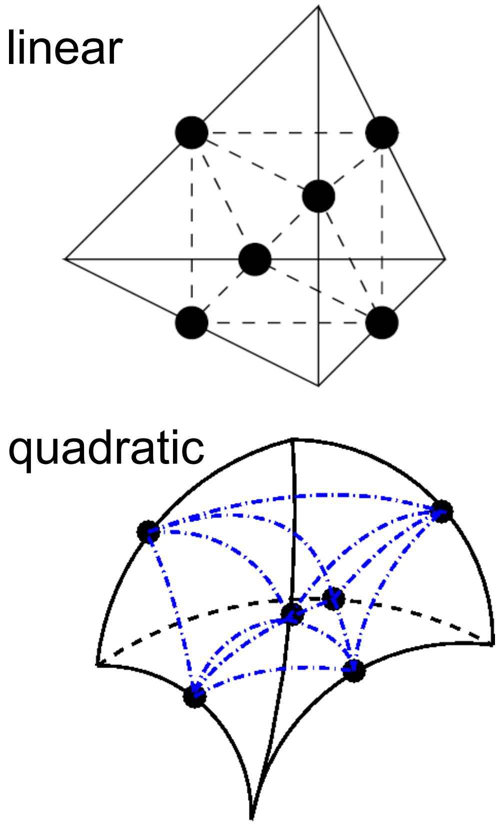


- Use a window to **limit the computational domain**
  - Fields from the left of the window will not catch up the beam
  - Fields on the right of the window is zero
- Use adaptive finite element basis function order p
  - p is nonzero for elements inside the window
  - p = 0 for elements outside the window
- Move the window when the beam reaches its right boundary
  - Prepare the mesh, repartition the mesh, assemble matrices, transfer solution, …

Moving Window with h-Refinement



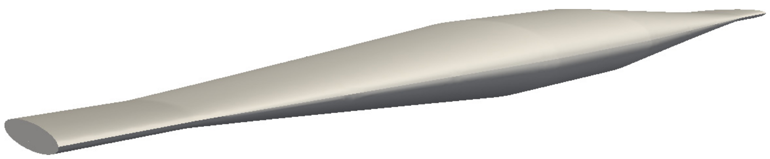
- Online mesh refinement in window
  - Split each tetrahedral element into 8 smaller ones
- Solution transfer from original mesh to new mesh as the window moves through accurate field projection
- Efficient use of computer resources for large problems
  - Mesh generation in serial is time consuming and requires large memory
  - Online mesh refinement eliminates reading of large multiple meshes

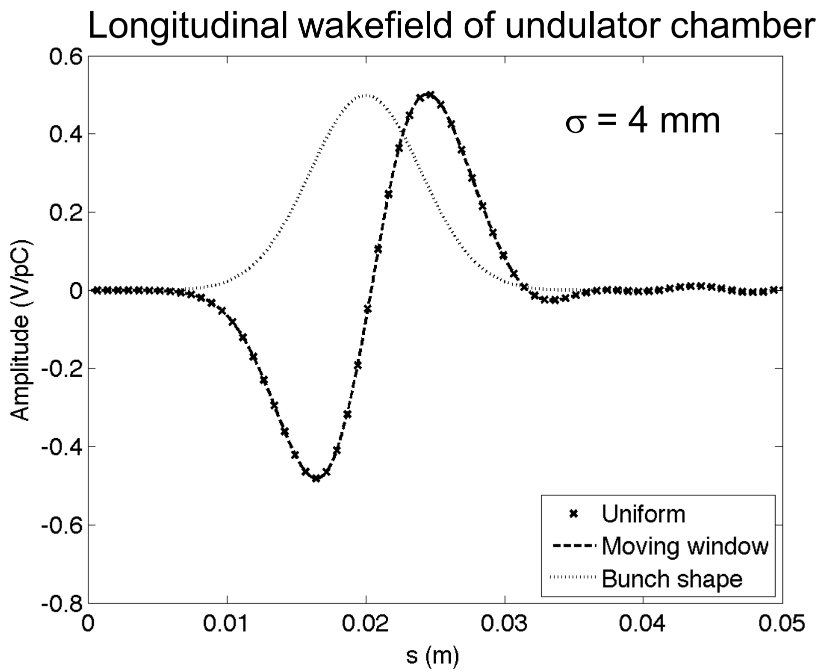


Edge splitting of tetrahedron

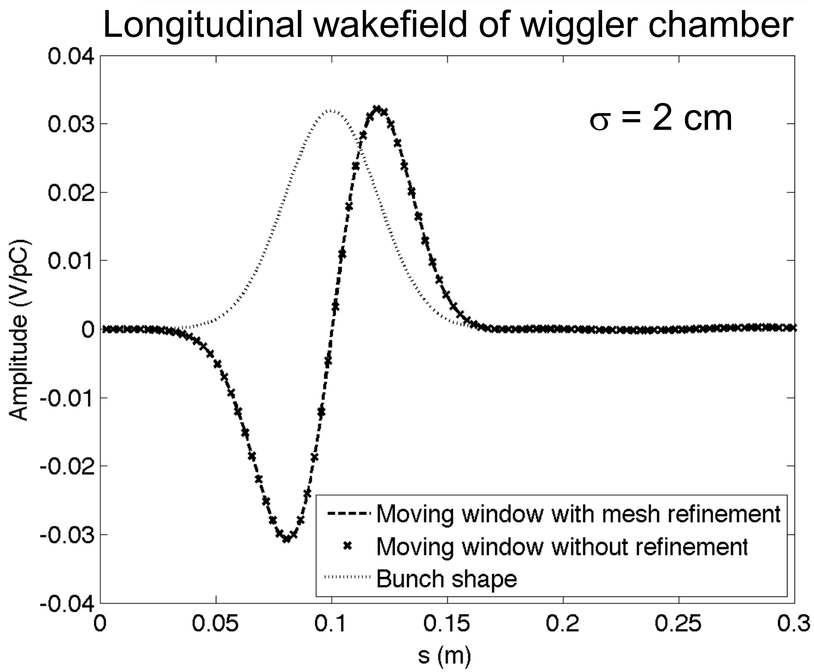
Verification of Moving Windows in T3P

p-refinement





h-refinement



- Excellent agreement in wakefield w/ and w/o moving window
- Computational gain not significant when bunch length is long compared with structure length

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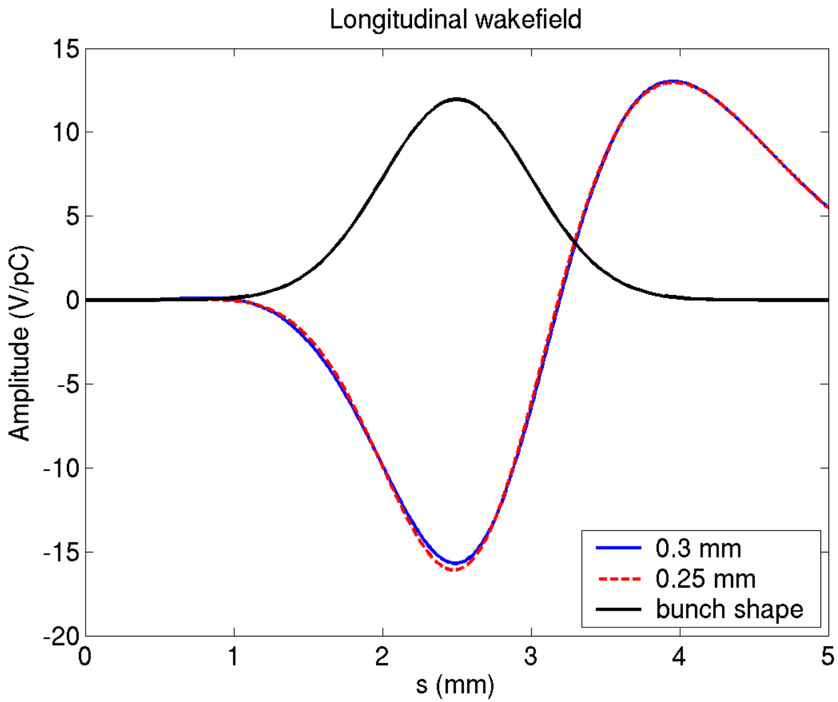
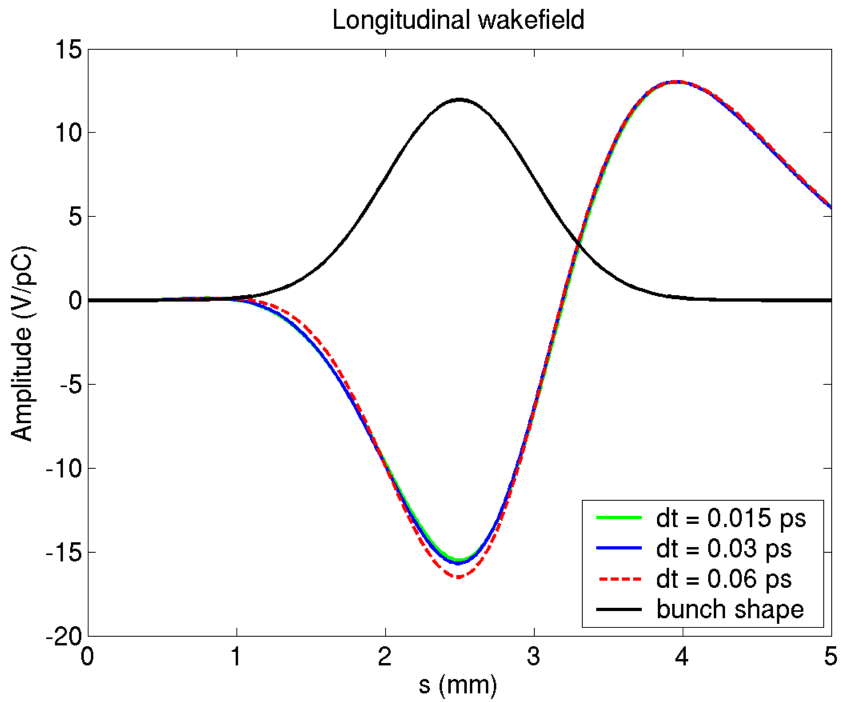
Wakefield of PEP-X Undulator Taper

- σ = 0.5 mm with moving window to cover 20σ
- 0.25 mm element size, 19M quadratic tetrahedral elements, 0.6M elements in window
- p=2 inside the window, p=0 outside
- ~ 3 hours runtime using 512 cores



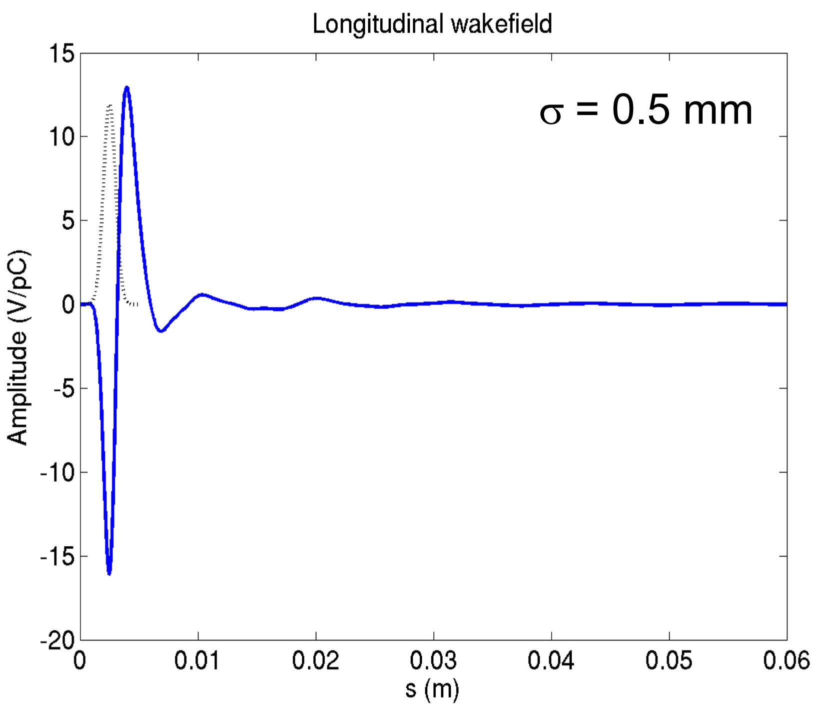
Mesh convergence

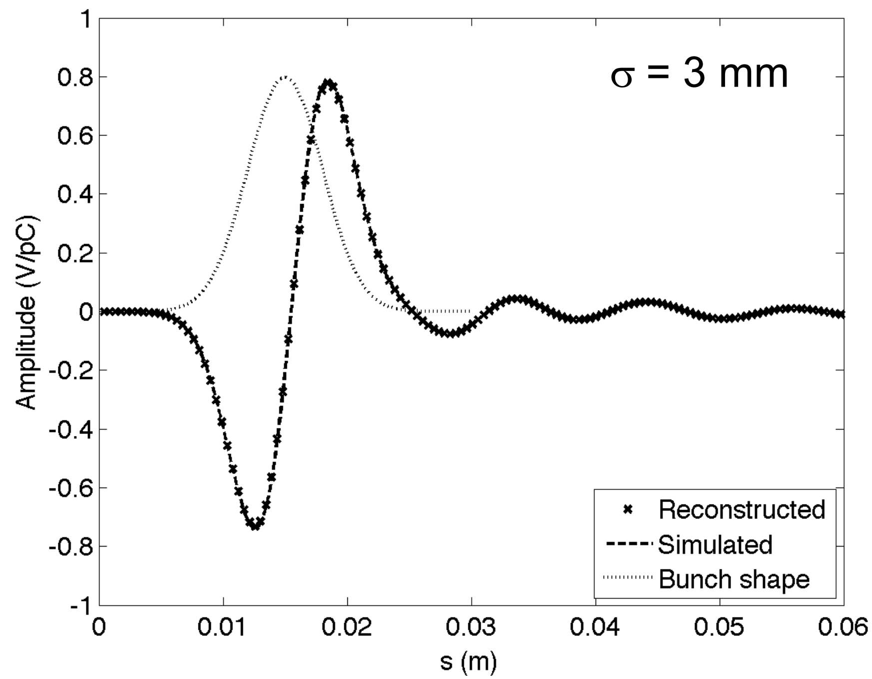


Time step convergence

Pseudo Green's Function Wakefield of Undulator

- σ = 0.5 mm with moving window to cover 120σ
- 0.25 mm element size, 19M quadratic tetrahedral elements, 8.6M elements in window
- p=2 inside the window, p=0 outside
- ~ 15 hours runtime using 8000 cores
- Reconstructed wakefield from Green's function wakefield agrees excellently with direct calculation at σ = 3 mm

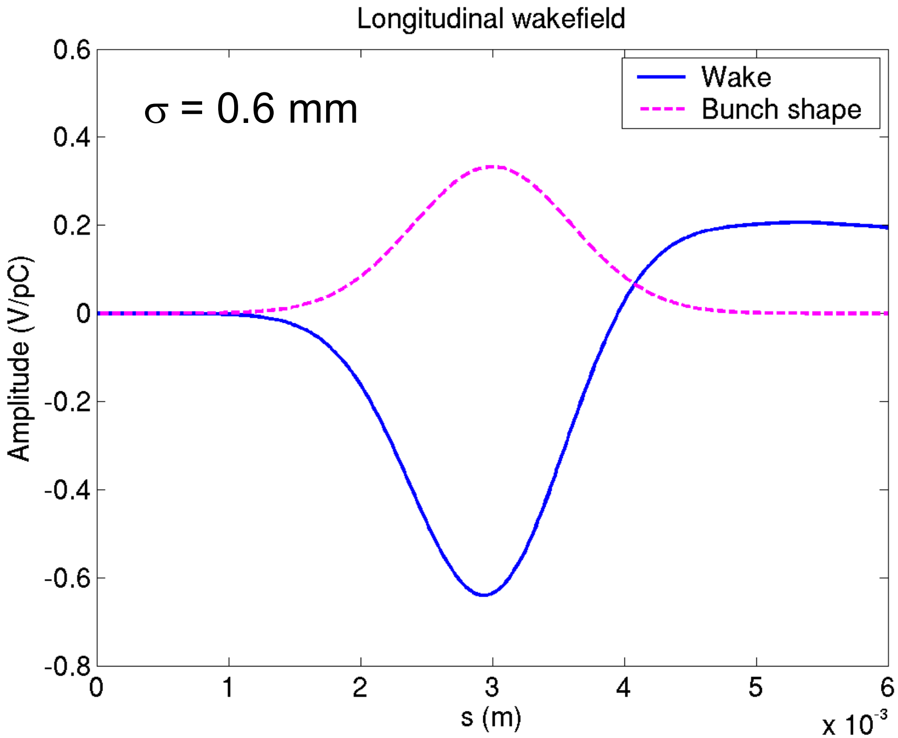




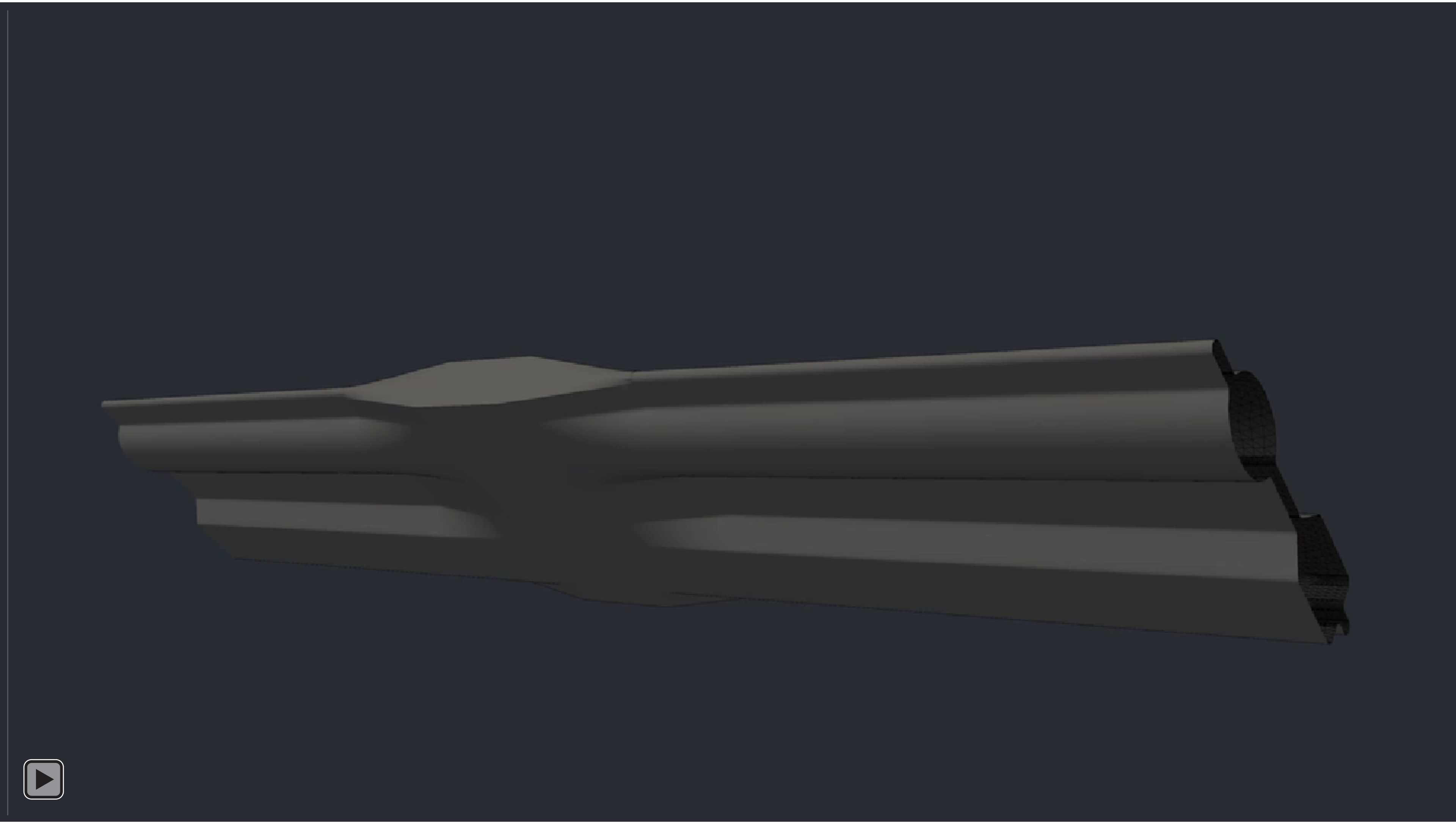
Wakefield in ERL Vacuum Chamber

- σ = 0.6 mm
- Moving window with online mesh refinement
- p=2 inside the window, p=0 outside
- Determine loss factor (~ 0.413 V/pC)

Computer model of Cornell ERL vacuum chamber transition



T3P - Short Bunch Wakefield in ERL

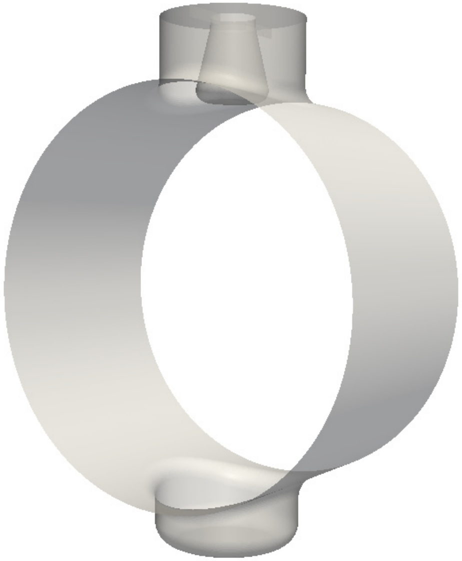


Visualization by Greg Schussman

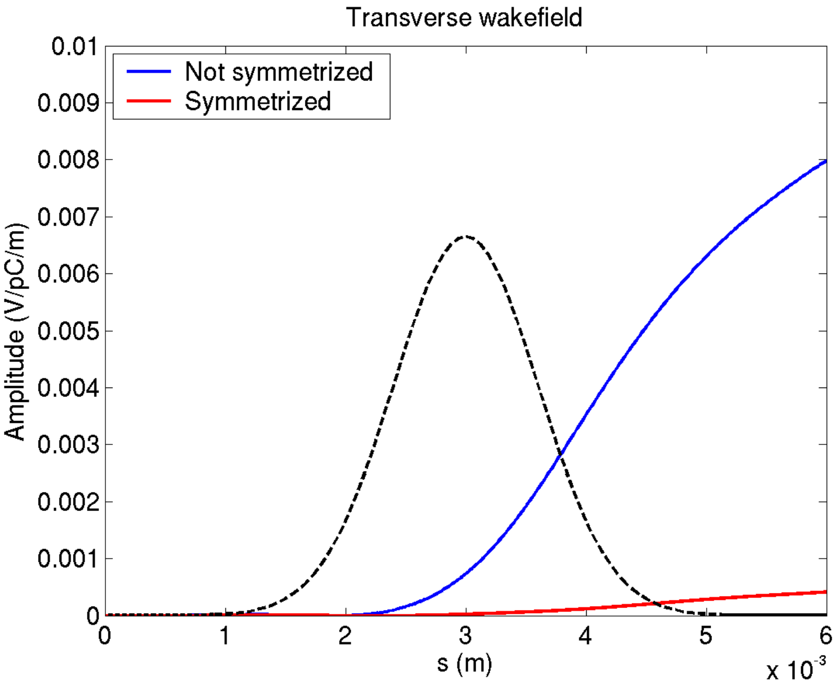
Transverse Wakefield for SCRF Coupler

- σ = 0.6 mm with moving window to cover 20σ
- p=2 inside the window, p=0 outside
- 2 hours runtime using 900 cores
- Transverse wakefield substantially reduced by symmetrized stub at the expense of increase in longitudinal wakefield

Input coupler



Input coupler w/ stub



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Summary

- Parallel computation with T3P allows high-fidelity calculations of wakefields using the high-order finite element method.
- Implementation of moving window techniques along with the beam in T3P substantially reduces computational resources by orders of magnitude for short bunches.
- This new capability of T3P enables the accurate determination of short-range wakefields for accelerator structures such as long, smooth tapers and endgroups in SCRF cavities.
- It will be useful for the design of future light sources and linear colliders operating with short bunches.