Application of the Eigen-Emittance Concept to Design Ultra-Bright Electron Beams

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Overview

- Motivation
- Calculating eigen-emittances and correlations
- Numerical results
- Prospects for implementation

Motivation

- Next generation light sources, such as Los Alamos' MaRIE (Matter and Radiation in Extreme) need low transverse emittances, e.g. 0.15 μm or less.
- It has been demonstrated that it is possible to make emittance in one dimension small at the expense of that in another dimension, using a flat-beam transform or emittance exchange (e.g. Kim, 2003; Carlsten & Bishofberger, 2006; Sun et. al., arXiv:1011.1182).
- Eigen-emittance values correspond to the emittances of an uncorrelated beam.
- We want to see if it is possible to tailor the eigen-emittances to small values by introducing correlations at the cathode. We could then remove the correlations and to recover small transverse emittance values.

Eigen-Emittances

- Invariant under linear beam transport.
- Can be obtained from the beam matrix Σ as $|\lambda_j|$ using the characteristic equation (see e.g. Dragt, Neri & Rangarajan, 1992)

$$det(J\Sigma - i\lambda_j I) = 0, \tag{1}$$

where I is the identity matrix and J is the skew-symmetric matrix with non-zero entries on the block diagonal of form,

$$J_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} . \tag{2}$$

Introducing beam correlations

- Canonical coordinates: $\mathbf{s} = (x, p_x, y, p_y, z, p_z)$
- Beam matrix: $\Sigma = \langle s_j s_k \rangle$
- Correlations ("C-matrix") (Yampolsky et. al., arXiv:1010.1558):

$$C = \begin{pmatrix} 0 & 0 & c_{13} & c_{14} & c_{15} & c_{16} \\ 0 & 0 & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & 0 & 0 & c_{35} & c_{36} \\ c_{41} & c_{42} & 0 & 0 & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & 0 & 0 \\ c_{61} & c_{62} & c_{63} & c_{64} & 0 & 0 \end{pmatrix}$$

- Correlated beam: $\Sigma = (I + C)\Sigma_0(I + C)^T$

Two Correlations

- Two is the minimum number of correlations needed to make two eigen-emittances small. This minimal scenario will also require the least optics to remove the correlations and recover small emittances.

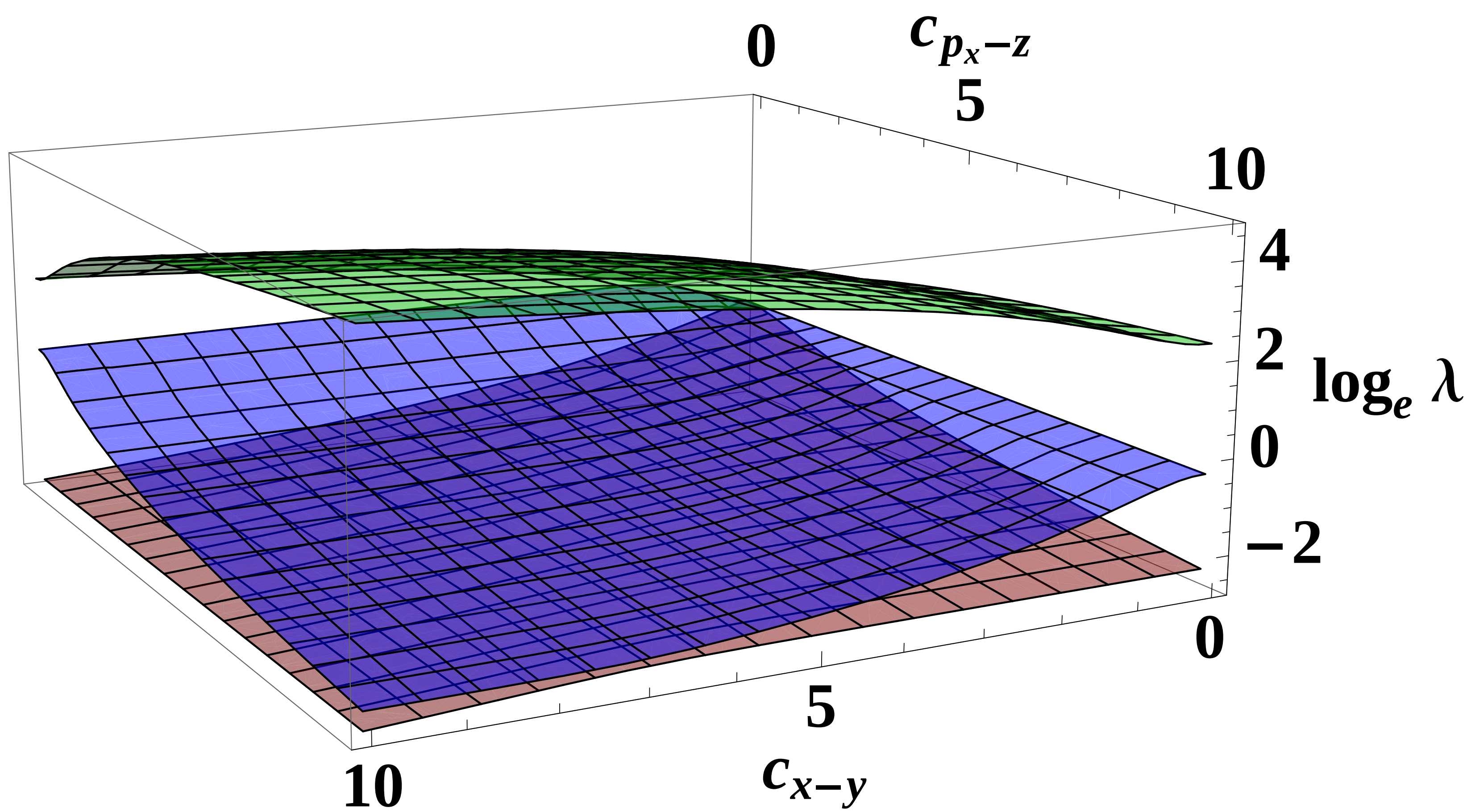
- Two correlations:

$$\begin{aligned}\Sigma &= (I+C_2)(I+C_1)\Sigma_0(I+C_1)^T(I+C_2)^T \\ &\equiv (I+C)\Sigma_0(I+C)^T\end{aligned}$$

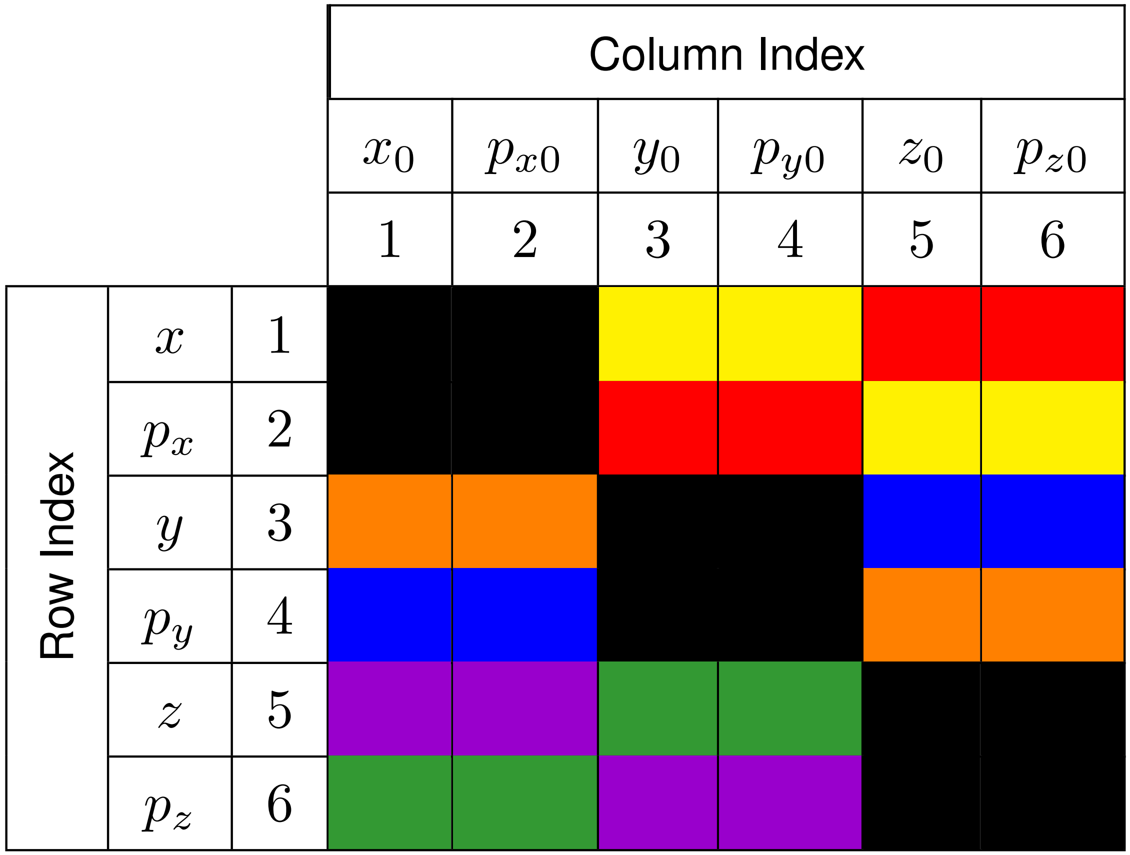
- If $C_1C_2 = C_2C_1$ correlations are *independent*

- If $C_1C_2 \neq C_2C_1$ correlations may be *dependent* or *independent*, depending on the order in which they are applied.

It's Possible!



The “C” Matrix



Matrix entries of the same color (independent correlations) can be combined to produce two small and one large eigenemittance. All combinations of dependent correlations also work.

Possible correlations

- We've identified minimal correlation scenarios that give two small eigen-emittance values.
- Not all realizable in practice.
- Difficult to imagine producing correlations that depend on momentum.
- Angular momentum correlations occur as p_x-y and p_y-x together.
- p_y-z or p_x-z difficult to create at cathode.

Possibilities

- Independent correlations: z-x and p_z-y or z-y with p_z-x.
- Dependent correlations: Possible combinations of coordinate correlations and/or energy with position.

Challenges

- Nonlinear evolution.
- Size of correlation required - practical to implement? Example: aspect ratios of beams.
- Any additional correlations that are inadvertently introduced in practice.

Summary

- Eigen-emittance approach offers an opportunity to tailor a beam's emittance values.
- Possible to achieve two small eigen-emittance values in theory using minimal correlations.
- At least some scenarios would be difficult to implement.
- We have identified possibilities that warrant further investigation.