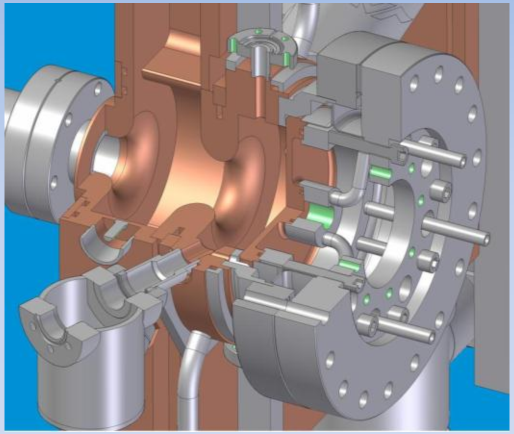
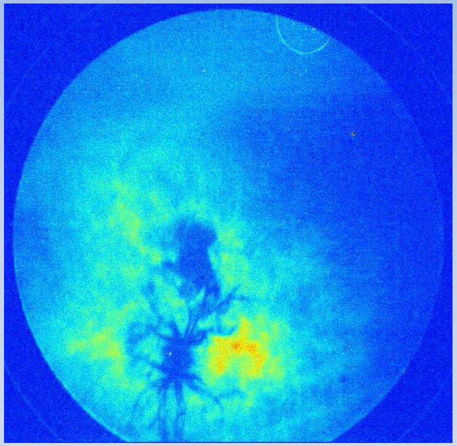
Tutorial on (Generating) High Brightness Beams



David H. Dowell
SLAC & LBNL

PAC11 Tutorial
April 1, 2011

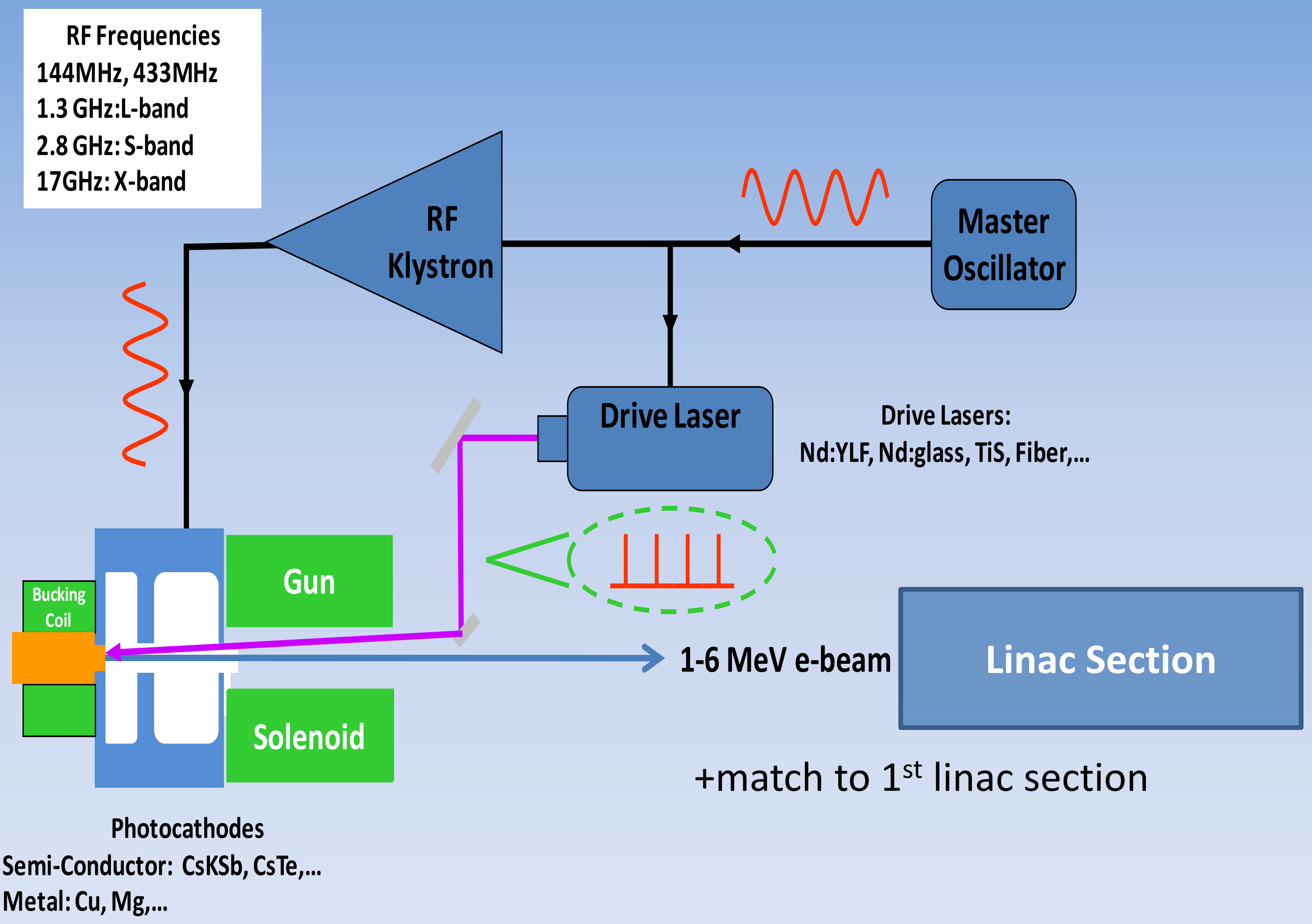


Outline of Tutorial

- *Intrinsic emittance and QE*
- *Space charge limited emission*
- *Simple optical model & RF emittance*
- *Emittance compensation and matching*
- *Solenoid aberrations*
- *"Beam blowout" dynamics & 3rd order space charge*

The Photocathode RF Gun System

Basic components of the photocathode RF gun system



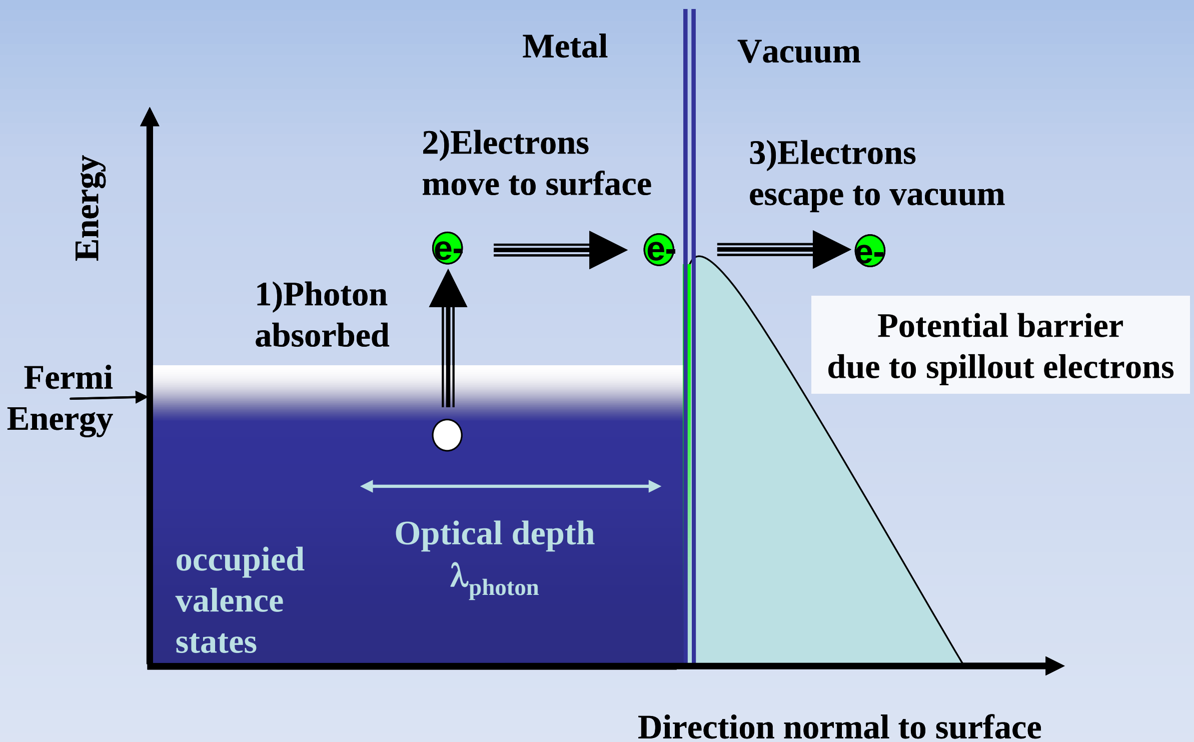
+match to 1st linac section

- *QE and thermal/intrinsic emittance*
- *Space charge limited emission*
- *Simple optical model & RF emittance*
- *Emittance compensation and matching*
- *Solenoid aberrations*
- *“Beam blowout” dynamics & 3rd order space charge*

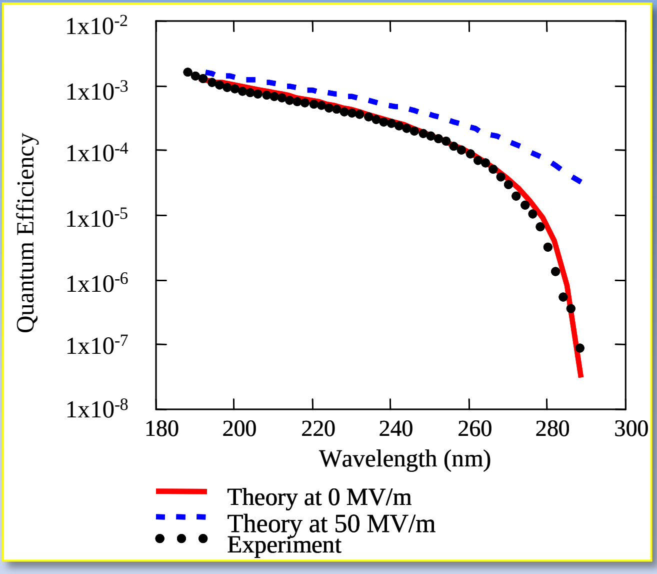
Photo-Electric Emission and the 3-Step Model

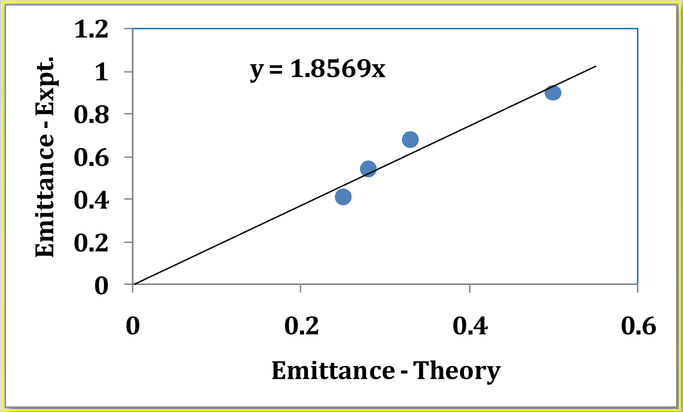
Photoelectric emission from a metal given by Spicer's 3-step model:

1. Photon absorption by the electron
2. Electron transport to the surface
3. Escape through the barrier



Comparison of theory & expt





*D. H. Dowell, K.K. King, R.E Kirby and J.F. Schmerge, PRST-AB 9, 063502 (2006)

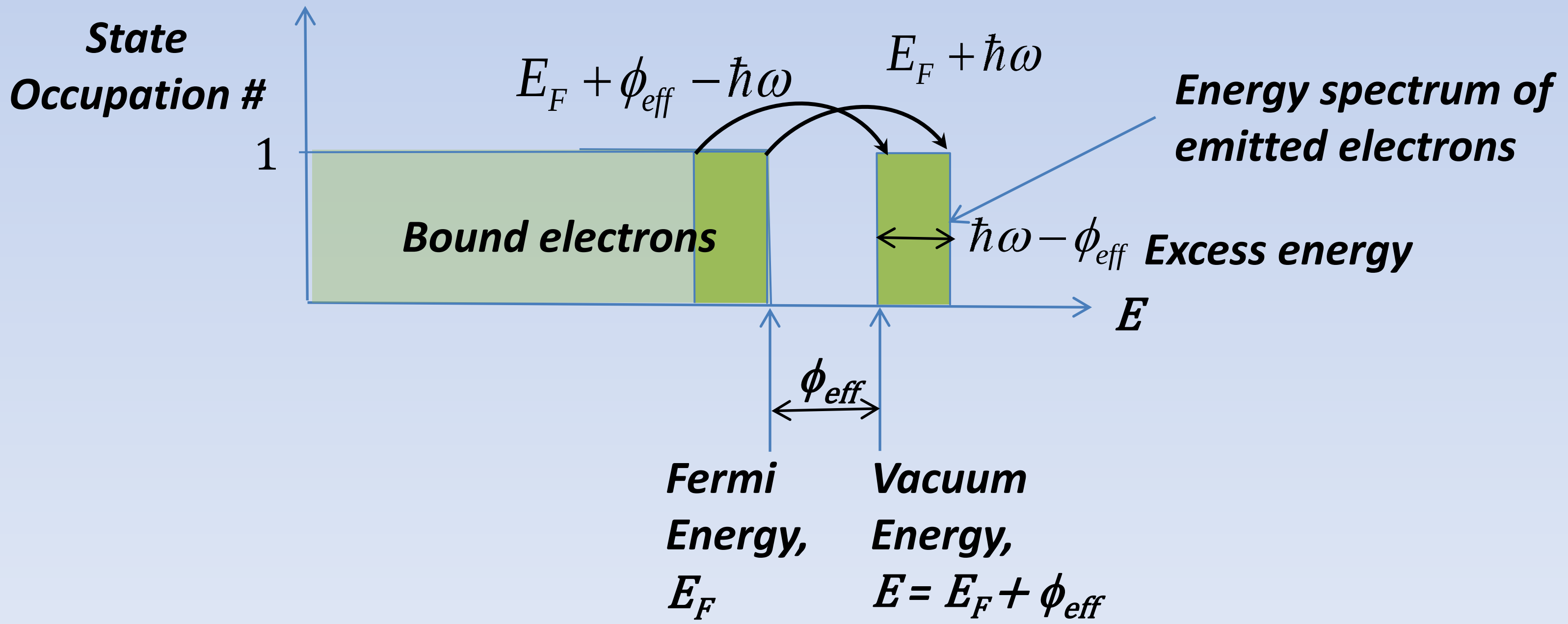
QE and emittance depend upon electronic structure of the cathode

- ***Sum of electron spectrum yield gives QE:*** $QE \propto \int_{E_F+\phi_W-\hbar\omega}^{E_F} EDOS(E)dE$

- ***Width of electron spectrum gives intrinsic emittance*** $\varepsilon_n \propto \sqrt{\hbar\omega-\phi_{eff}}$

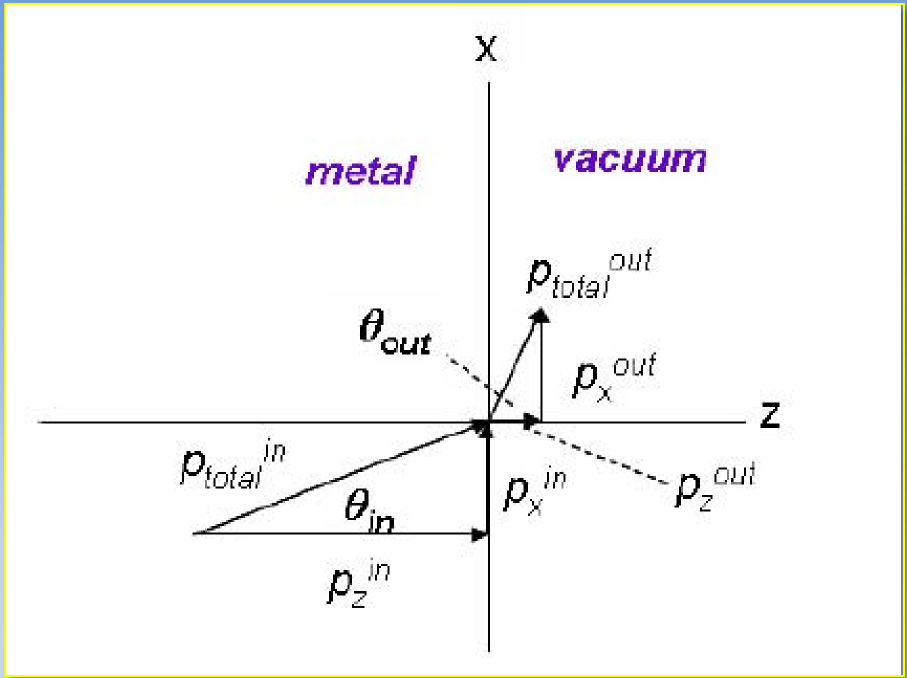
- ***Both are dependent upon the density of occupied states near the Fermi level***

Simple electron gas model of a metal at $kT=0$



Refraction of electrons at the cathode-vacuum boundary

Snell's Law for electrons



Conservation of transverse momentum at the cathode-vacuum boundary

$$p_x^{in} = p_x^{out}$$

$$p_{total}^{in} \sin\theta_{in} = p_{total}^{out} \sin\theta_{out}$$

$$p_{total}^{in} = \sqrt{2m\left(E+\hbar\omega\right)}$$

$$p_{total}^{out} = \sqrt{2m\left(E+\hbar\omega - E_F - \phi_W\right)}$$

Refraction law for electrons:

$$\frac{\sin\theta_{out}}{\sin\theta_{in}} = \sqrt{\frac{E+\hbar\omega}{E+\hbar\omega - E_F - \phi_{eff}}} \equiv \frac{n_{in}}{n_{out}}$$

To escape electron longitudinal momentum needs to be greater than barrier height:

$$p_z^{in} \geq \sqrt{2m\left(E_F + \phi_{eff}\right)}$$

Maximum internal angle for electron with energy E which can escape:

$$\sqrt{2m\left(E+\hbar\omega\right)}\cos\theta_{in} \geq \sqrt{2m\left(E_F + \phi_{eff}\right)} \Rightarrow \cos\theta_{in}^{\max}\left(E\right) = \sqrt{\frac{E_F + \phi_{eff}}{E+\hbar\omega}}$$

Outside angle is 90 deg at θ_{in}^{max} which is typically ~10 deg.

QE for a metal

E is the electron energy
E_F is the Fermi Energy
ϕ_{eff} is the effective work function
$\phi_{eff} = \phi_W - \phi_{Schottky}$

Step 3: Escape over the barrier

$$QE(\omega) = \left(1 - R(\omega)\right) F_{e-e}(\omega) \frac{\int_{E_F+\varphi_{eff}-\hbar\omega}^{E_F} dE}{\int_{E_F-\hbar\omega}^{E_F} dE} \frac{\int_{\sqrt{\frac{E_F+\varphi_{eff}}{E+\hbar\omega}}}^{1} d(\cos\theta)}{\int_{-1}^{1} d(\cos\theta)} \frac{\int_0^{2\pi} d\Phi}{\int_0^{2\pi} d\Phi}$$

Step 1: Optical Reflectivity
~40% for metals
~10% for semi-conductors
Optical Absorption Depth
~120 angstroms
Fraction ~ 0.6 to 0.9

Step 2: Transport to Surface
e-e scattering (esp. for metals)
~30 angstroms for Cu
e-phonon scattering (semi-conductors)
Fraction ~ 0.2

- Sum over the fraction of occupied states which are excited with enough energy to escape, Fraction ~0.04
- Fraction of electrons within max internal angle for escape, Fraction ~0.01
- Azimuthally isotropic emission Fraction =1

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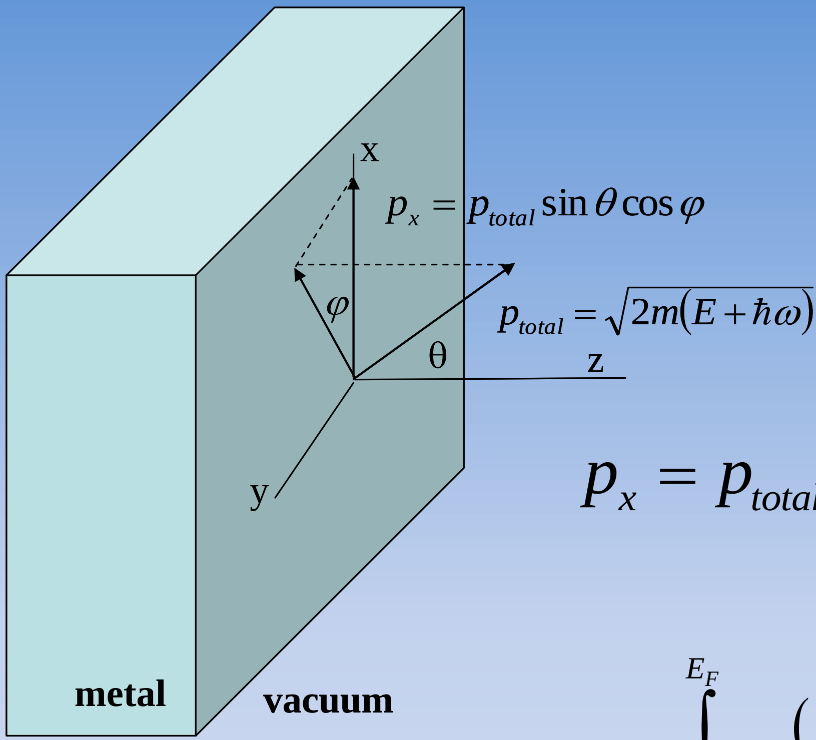
- Sum over the fraction of occupied states which are excited with enough energy to escape,
Fraction ~0.04

QE ~ 0.5*0.2*0.04*0.01*1 = 4x10^{-5}

Derivation of Photo-Electric Intrinsic Emittance



$$\varepsilon_n = \sigma_x \frac{\left\langle p_x^2 \right\rangle^{1/2}}{mc}$$

$$p_x = p_{total} \sin\theta \cos\varphi = \sqrt{2m\left(E + \hbar\omega\right)} \sin\theta \cos\varphi$$

$$\left\langle p_x^2 \right\rangle = 2m \frac{\int_{E_F + \phi_{eff} - \hbar\omega}^{E_F} \left(E + \hbar\omega\right) dE}{\int dE} \frac{\int_{\sqrt{\frac{E_F + \phi_{eff}}{E + \hbar\omega}}}^{1} \sin^2\theta d(\cos\theta)}{\int d(\cos\theta)} \frac{\int_0^{2\pi} \cos^2\varphi d\varphi}{\int d\varphi}$$

Intrinsic emittance for photoemission from a metal

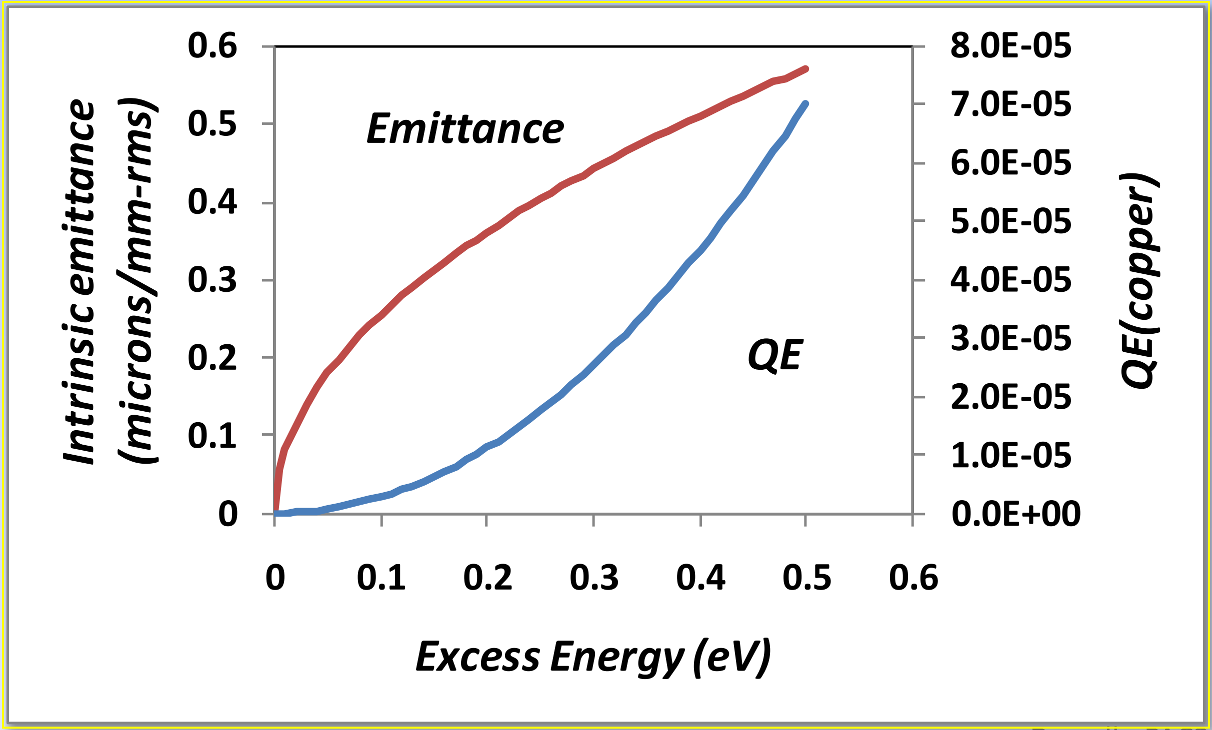
$$\varepsilon_n = \sigma_x \sqrt{\frac{\hbar\omega - \phi_{eff}}{3mc^2}}$$

QE & Emittance are related via the excess energy

Define the excess energy as: $E_{excess} = \hbar\omega - \phi_{eff}$

$$QE = \frac{(1-R(\hbar\omega))}{1+\frac{\lambda_{opt}}{\bar{\lambda}_{e-e}}} \frac{(E_F+\hbar\omega)}{2\hbar\omega}\left(1-\sqrt{\frac{E_F+\phi_{eff}}{E_F+\hbar\omega}}\right)^2 \approx \frac{(1-R(\hbar\omega))}{1+\frac{\lambda_{opt}}{\bar{\lambda}_{e-e}}} \frac{E_{excess}^2}{8\phi_{eff}(E_F+\phi_{eff})}$$

$$\frac{\varepsilon_n}{\sigma_x} = \sqrt{\frac{\hbar\omega-\phi_{eff}}{3mc^2}} = \sqrt{\frac{E_{excess}}{3mc^2}}$$



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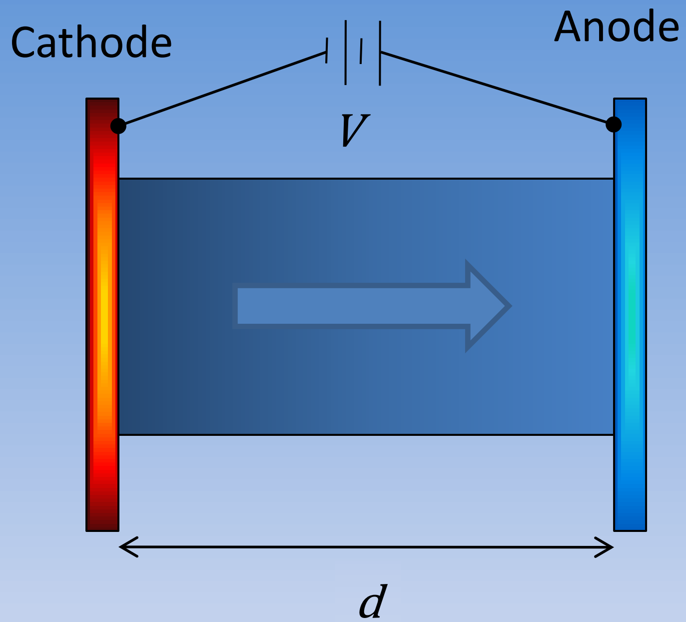
Space Charge Limit (SCL) is different for DC diode and short pulse photo-emission

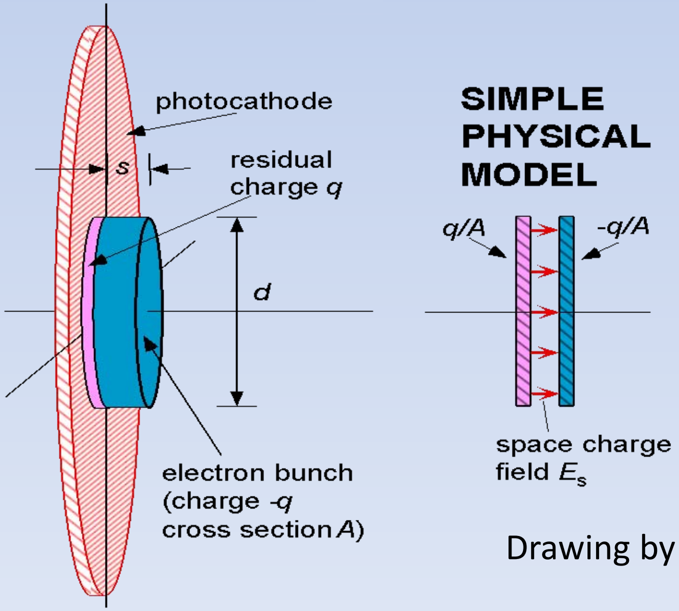
Space Charge Field Across a Diode, Child-Langmuir law:

$$J_{CL} = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{d^2}$$



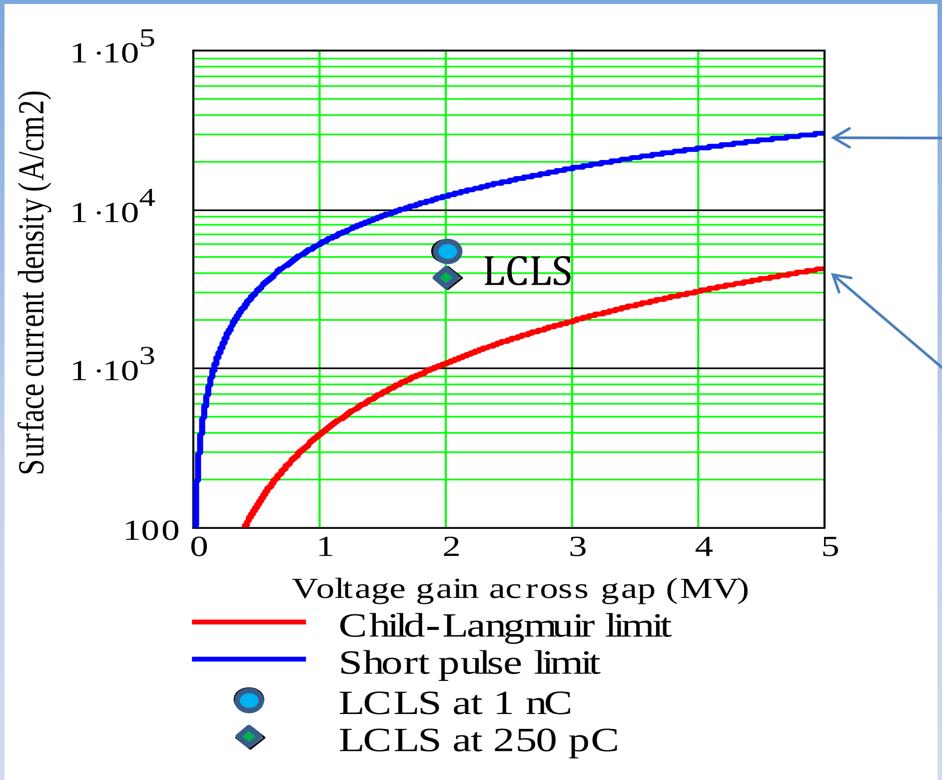
Space Charge Field Across a Short Electron Bunch from a Laser-driven Photocathode, parallel plate (capacitor) model:

$$\sigma_{SCL} = \varepsilon_0 E_{applied}$$



Drawing by A. Vetter

Comparison of space charge limits for Child-Langmuir and Short Pulse Geometries/Conditions

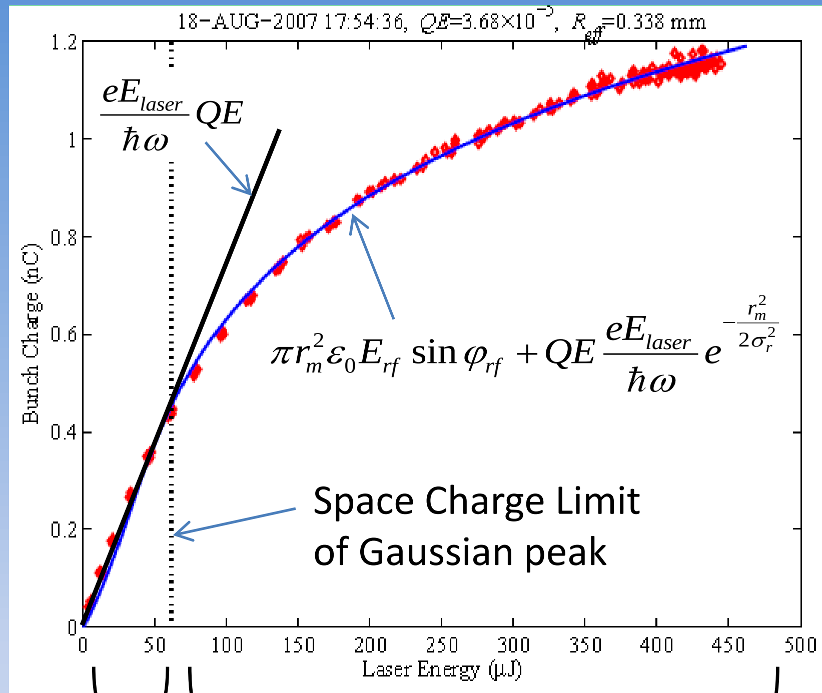


$$J_{SCL} = \frac{\varepsilon_0 E_{applied}}{\tau_{laser}} = \frac{\varepsilon_0 V}{d \cdot \tau_{laser}}$$

$$J_{CL} = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{d^2}$$

LCLS typically operates at approximately half the space charge limit for short pulse emission and a factor of 4 to 5 higher than the space charge limit given by the Child-Langmuir law.

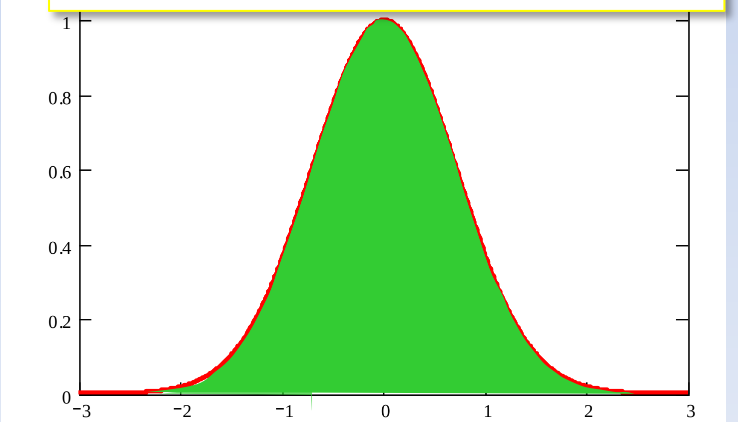
Transverse Electron Beam Shape: The beam core is clipped at the SCL



- ***Produces a flat, uniform transverse distribution in the beam core.***
- ***Flattens hot spots.***

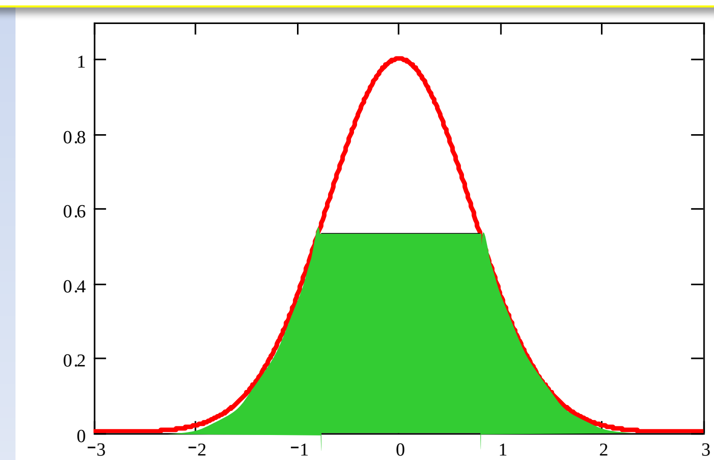
QE Limited Emission

radial distribution follows laser & QE



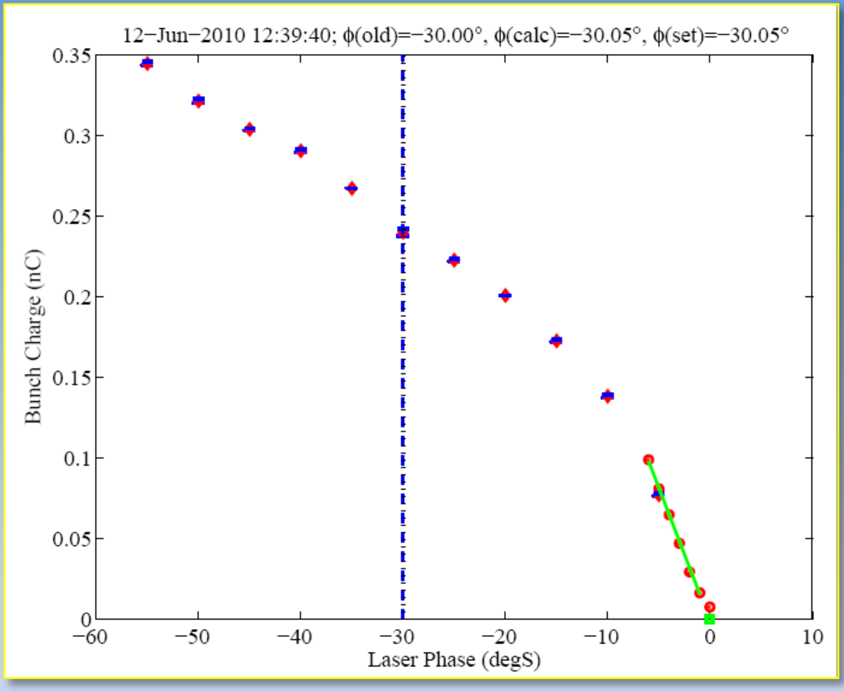
Space Charge Limited Emission

radial distribution saturates at the applied field



Derivation of Schottky Scan Function:

Emitted charge vs. launch phase



Begin with the QE for a metal cathode:

$$QE = \frac{1-R}{1+\frac{\lambda_{opt}}{\bar{\lambda}_{e-e}}} \frac{E_F + \hbar\omega}{2\hbar\omega} \left(1 - \sqrt{\frac{E_F + \phi_{eff}}{E_F + \hbar\omega}} \right)^2$$

where the effective work function is

$$\phi_{eff} = \phi_W - e\sqrt{\frac{e\beta E_{rf} \sin\phi_{rf}}{4\pi\varepsilon_0}}$$

Putting this into the QE formula gives,

$$QE = \frac{1-R}{1+\frac{\lambda_{opt}}{\bar{\lambda}_{e-e}}} \frac{E_F + \hbar\omega}{2\hbar\omega} \left(1 - \sqrt{\frac{E_F + \phi_W - e\sqrt{e\beta E_{rf} \sin\phi_{rf} /(4\pi\varepsilon_0)}}{E_F + \hbar\omega}} \right)^2$$

Everything is known except for material work function, ϕ_W, and the field enhancement factor, β. Fit Schottky scan data to find them.

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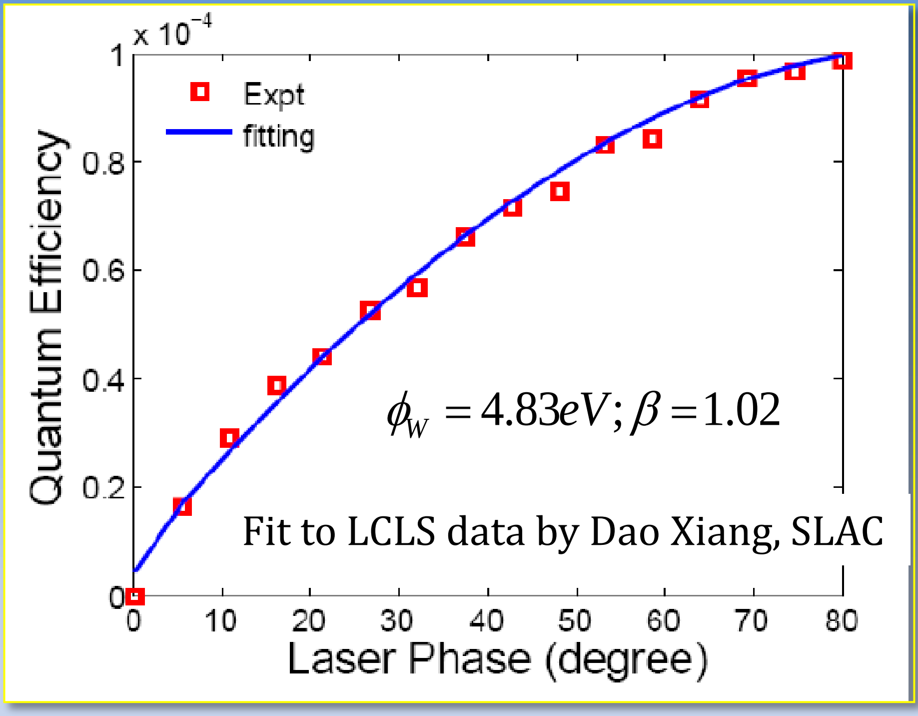

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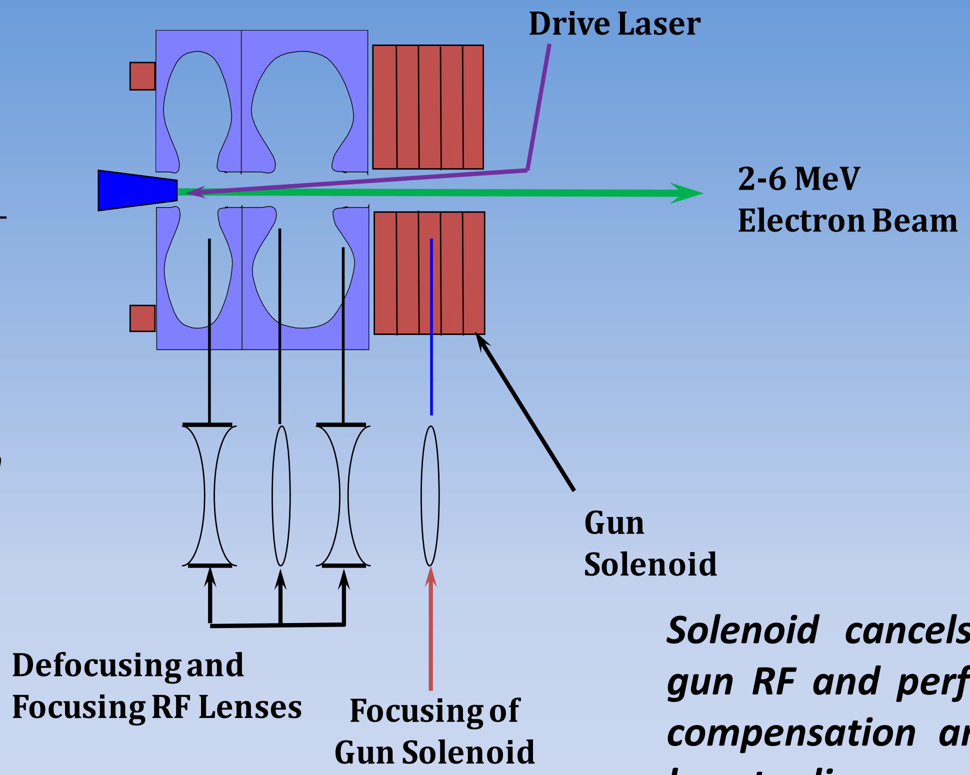
Optical Model of the RF Gun: Solenoid Effects

$$f_{rf} = -\frac{2\gamma mc^2}{eE_0 \sin\phi_e}$$

f_{rf} ~-15 cm for 100 MV/m which needs to be compensated for with solenoid



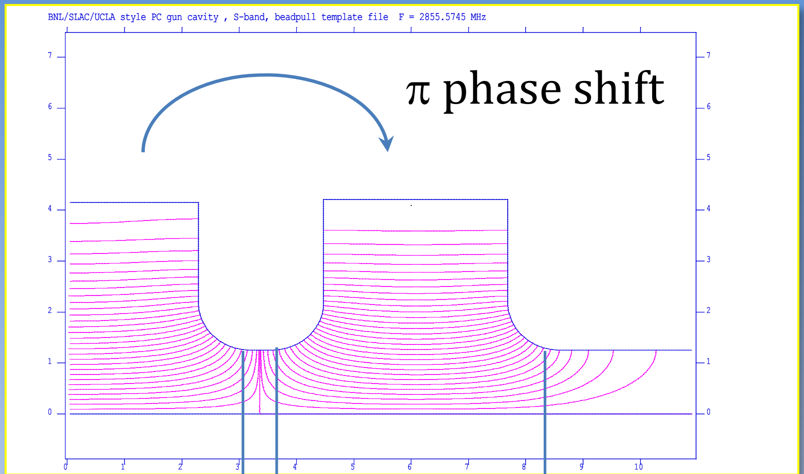
Solenoid cancels defocusing of gun RF and performs emittance compensation and matching to booster linac

Solenoid has focal length of ~15 cm but is ~20 cm long=> thick lens => aberrations

The principal solenoid aberrations can be classified as :

- ***Chromatic***
- ***Geometric***
- ***Anomalous fields***
- ***Misalignment (not discussed here)***

Optical Properties of the Gun's RF Fields



$$\frac{1}{f_1}=\frac{eE_1\sin\phi_1}{2\beta\gamma mc^2}$$

$$\frac{1}{f_e}=\frac{eE_1\sin\phi_e}{2(\beta\gamma)_e mc^2}$$

$$\frac{1}{f_2}=\frac{eE_2\sin\phi_2}{2\beta\gamma mc^2}=\frac{eE_2\sin(\phi_1+\pi)}{2\beta\gamma mc^2}=-\frac{eE_2\sin\phi_1}{2\beta\gamma mc^2}$$

For a π phase shift between cells:

$$\sin(\phi+\pi)=-\sin\phi$$

and with $E_1 \sim E_2$ and $(\beta\gamma)_1 \sim (\beta\gamma)_2$ thus there is no net focusing across the iris between two adjacent π-mode cells:

$$\frac{1}{f_1}+\frac{1}{f_2}=-\frac{eE_1\sin\phi_1}{2(\beta\gamma)_1 mc^2}+\frac{eE_2\sin\phi_1}{2(\beta\gamma)_2 mc^2}\approx 0$$

Hence the optical strength of the gun is dominantly due to the defocus at the exit of the last cell,

$$\frac{1}{f_e}=\frac{eE_1\sin\phi_e}{2(\beta\gamma)_e mc^2}$$

RF Emittance

$$\varepsilon_n = \beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2} \cong \beta\gamma\sigma_x\sigma_{x'}$$

$$f_{rf} = -\frac{2\beta\gamma mc^2}{eE_0\sin\varphi_e} \qquad \Delta x' = -\frac{d}{d\phi_e}\left(\frac{1}{f_{rf}}\right)\Delta x\Delta\phi_e$$

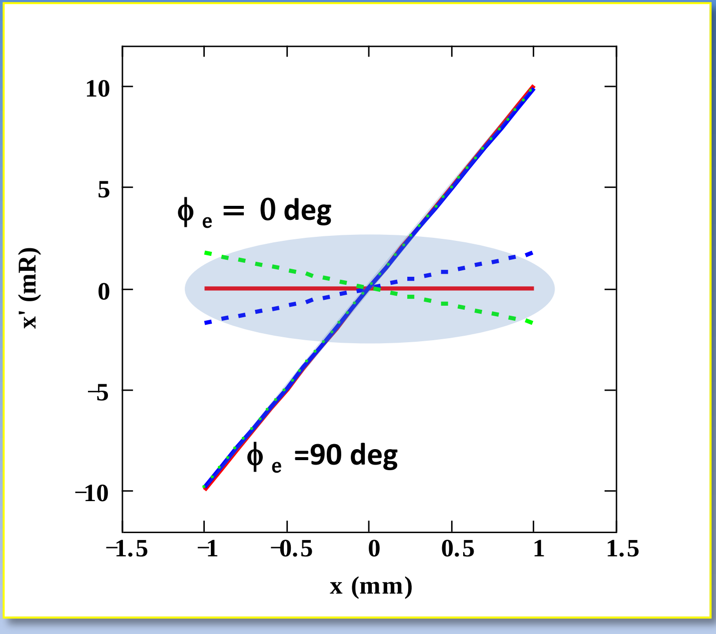
$$\sigma_{x'} = \frac{eE_0\cos\phi_e}{2\gamma mc^2}\sigma_x\sigma_\phi$$

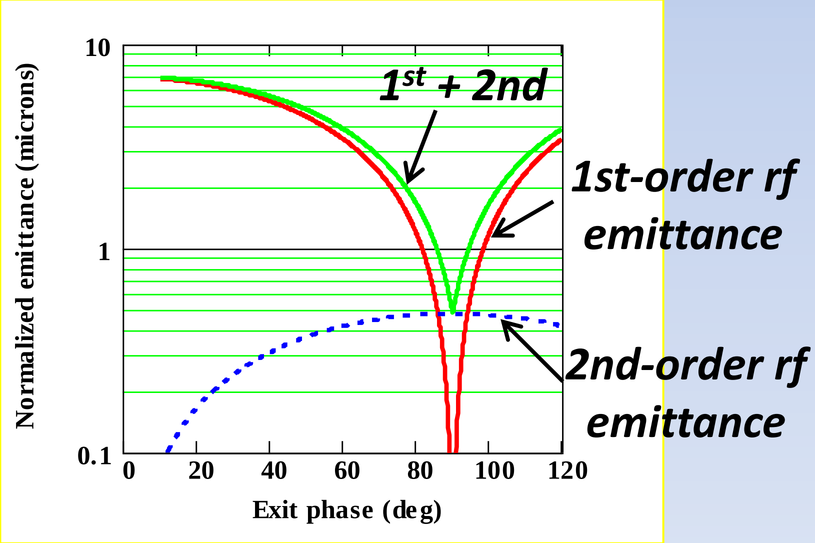
$$\varepsilon_{rf,1} = \frac{eE_0\left|\cos\phi_e\right|}{2mc^2}\sigma_x^2\sigma_\varphi \qquad \text{for } \phi_e \neq 90\,\text{deg}$$

This is the linear part of the emittance, the non-linear part due to the RF curvature is 2nd order in the phase spread of the bunch,

$$\varepsilon_{rf,2} = \frac{eE_0\left|\sin\varphi_e\right|}{2\sqrt{2}mc^2}\sigma_x^2\sigma_\varphi^2 \qquad \text{for } \varphi_e \approx 90\,\text{deg}$$

$$\gamma\varepsilon_{rf,total} = \frac{eE_0\left|\cos\varphi_e\right|}{2mc^2}\sigma_x^2\left[\left|\cos\varphi_e\right|\sigma_\varphi + \left|\sin\varphi_e\right|\frac{\sigma_\varphi^2}{\sqrt{2}}\right]$$





scales as the beam size squared >> a common feature of many emittance sources

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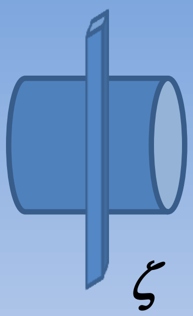
Projected Emittance Compensation*

The radial envelope equation for each slice position, ζ:



External focusing by magnetic and RF fields

emittance acts like a defocusing pressure

$$\sigma_r''(\zeta)+\sigma_r'(\zeta)\left(\frac{\gamma'}{\beta^2\gamma}\right)+k_r\sigma_r(\zeta)-\frac{K(\zeta)}{\sigma_r(\zeta)}-\frac{\varepsilon_n^2(\zeta)}{\beta^3\gamma^3\sigma_r^2(\zeta)}=0$$

acceleration changes magnification of the divergence

Space charge defocusing, K is the generalized perveance

Generalized perveance (Lawson, p. 117)

$$K(\zeta)=\frac{2}{\beta^3\gamma^3}\frac{I(\zeta)}{I_0}$$

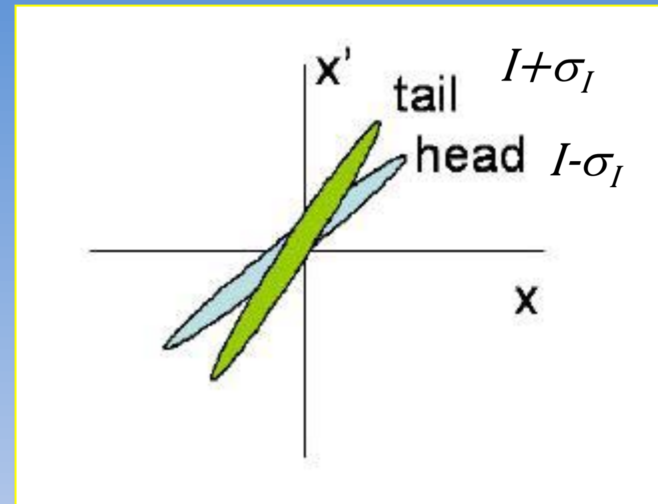
$I(\zeta)$ is the peak current of slice ζ
I_0 is 17000 amps

*Serafini & Rosenzweig, Phys. Rev. E55, 1997, p.7565

Projected Emittance Compensation: How to align the slices?

Beam Envelope Equation:
Assume no acceleration, zero slice emittance

$$\sigma_r'' + k_r\sigma_r - \frac{K}{\sigma_r} = 0$$



Consider solutions for small perturbations from equilibrium radius:

$$\sigma_r = \sigma_e + \delta\sigma$$

Substituting into the envelope eqn. and expanding in a Taylor series we get

$$\delta\sigma'' + k_r\sigma_e - \frac{K}{\sigma_e} + \left(k_r + \frac{K}{\sigma_e^2}\right)\delta\sigma = 0$$

constant terms,
set sum to zero and solve for σ_e:

$$\sigma_e = \sqrt{\frac{K}{k_r}} = \sqrt{\frac{2}{\beta^3\gamma^3}\frac{I}{I_0}\frac{1}{k_r}}$$

Envelope eqn. for small amplitude radial perturbations:

$$\delta\sigma'' + 2\frac{K}{\sigma_e^2}\delta\sigma = 0$$

$$\begin{pmatrix}\delta\sigma_1 \\ \delta\sigma_1'\end{pmatrix} = \begin{pmatrix}\cos k_e z & \dfrac{\sin k_e z}{k_e} \\ -k_e\sin k_e z & \cos k_e z\end{pmatrix}\begin{pmatrix}\delta\sigma_0 \\ \delta\sigma_0'\end{pmatrix}$$

This is known as balanced or Brillouin flow, when the outward space charge force is countered by external focusing, usually a magnetic solenoid. This establishes the invariant envelope of Serafini & Rosenzweig.

Derivation of projected emittance for a current spread in the slices

Envelope eqn. for small amplitude radial perturbations

$$\delta\sigma'' + 2\frac{K}{\sigma_e^2}\delta\sigma = 0$$

This is the wave equation with oscillating solutions:

$$\begin{pmatrix}\delta\sigma_1 \\ \delta\sigma_1'\end{pmatrix} = \begin{pmatrix}\cos k_e z & \dfrac{\sin k_e z}{k_e} \\ -k_e \sin k_e z & \cos k_e z\end{pmatrix}\begin{pmatrix}\delta\sigma_0 \\ \delta\sigma_0'\end{pmatrix}$$

with the equilibrium wave number defined as:

$$k_e^2 = 2k_r = 2\frac{K}{\sigma_e^2}$$

To simplify the emittance calculation let's make the crude approx. that the focusing channel can be replaced by a thin lens such that

$$\begin{pmatrix}\delta\sigma_1 \\ \delta\sigma_1'\end{pmatrix} = \begin{pmatrix}1 & 0 \\ -k_e \sin k_e z & 1\end{pmatrix}\begin{pmatrix}\delta\sigma_0 \\ \delta\sigma_0'\end{pmatrix}$$

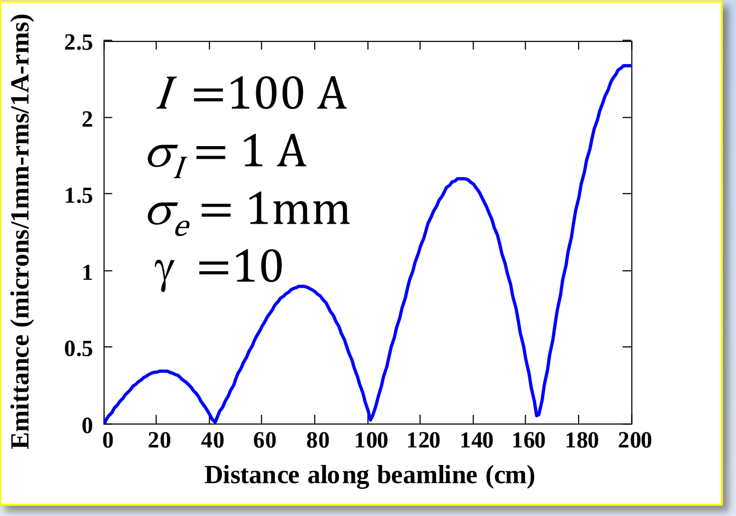
$$\frac{1}{f_e} = k_e \sin k_e z \quad ; \quad k_e^2 = \frac{4}{\beta^3\gamma^3\sigma_e^2}\frac{I}{I_0}$$

The emittance due to lens strength dependence upon beam current:

$$\varepsilon_{n,sc-comp} = \beta\gamma\sigma_e^2\sigma_I\left|\frac{d}{dI}\left(\frac{1}{f_e}\right)\right|$$

Using the expressions for $1/f_e$ and the equilibrium wavenumber in this eqn. gives the projected emittance for a beam with a σ_I rms spread in slice current:

$$\varepsilon_{n,\sigma_I} = \frac{\sigma_e\sigma_I}{\sqrt{\beta\gamma I_0 I}}\left|\sin k_e z + k_e z\cos k_e z\right|$$



Emittance Compensation Summary

- *The beam at the cathode begins with all slices of nearly equal peak current propagating with equilibrium radii in a balanced flow.*

- *The beam envelope equation is linearized and solved for small perturbations about this equilibrium.*

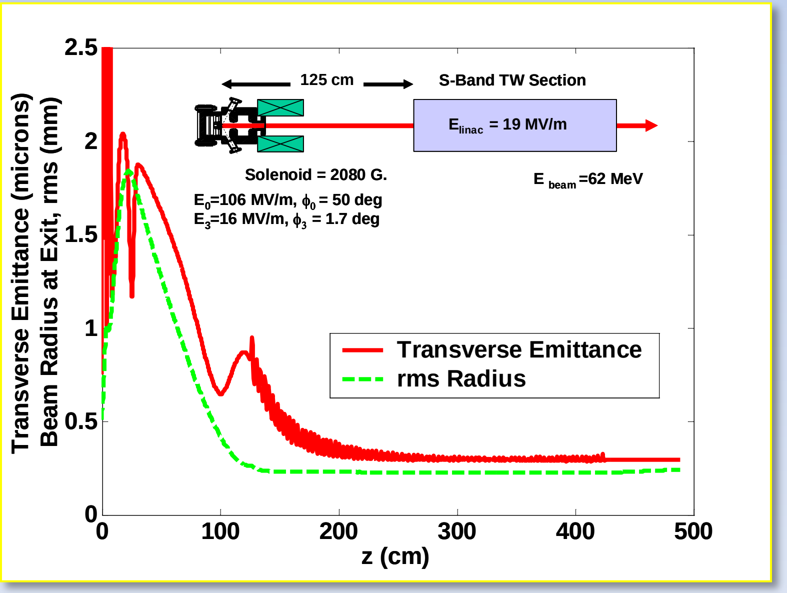
- *The solution obtained shows all slice radii and emittances oscillate with the same frequency (determined by the invariant envelope), independent of amplitude.*

- *Assuming the slices are all born aligned, they will re-align at multiple locations as the beam propagates, with the projected emittance being a local minimum at each alignment. The beam size will oscillate with the same frequency, but shifted in phase by $\pi/2$.*

Emittance compensation includes matching the low energy beam to the booster linac to damp the emittance oscillations

In addition to compensating for the emittance from the gun, it is necessary to carefully match the beam into a high-gradient booster accelerator to damp the emittance oscillations. The required matching condition is referred to as the Ferrario working point and was initially formulated for the LCLS injector.*

The working point matching condition requires the emittance to be a local maximum and the envelope to be a waist at the entrance to the booster. The waist size is related to the strength of the RF fields and the peak current. RF focusing aligns the slices and acceleration damps the emittance oscillations.



**M. Ferrario et al., "HOMDYN study for the LCLS RF photo-injector", SLAC-PUB-8400, LCLS-TN-00-04, LNF-00/004(P).*

The Ferrario Working Point

Assume the RF-lens at the entrance to the booster is similar to that at the gun exit with an injection phase at crest for maximum acceleration, $\phi_e=\pi/2$, so the angular kick is,

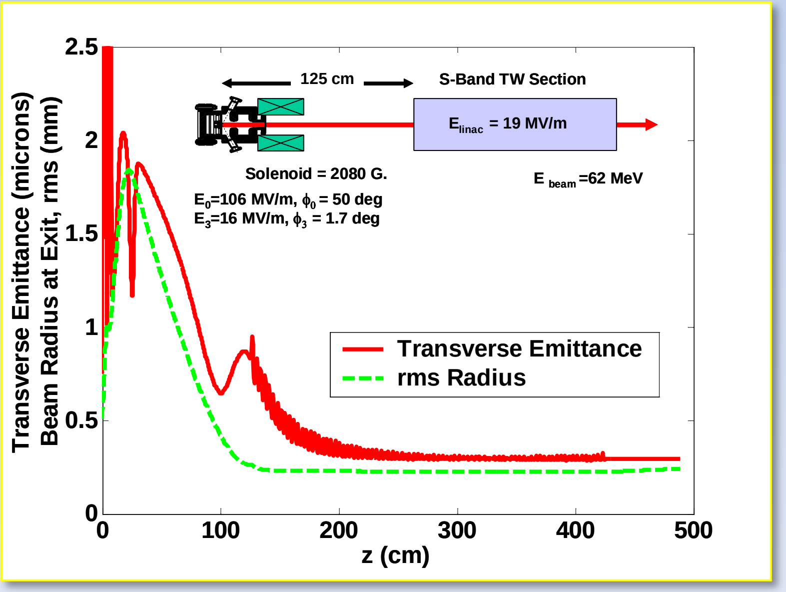
$$\sigma' = \sigma \frac{eE_0}{2\gamma mc^2}$$

Taking the derivative gives the rf term needed for the envelope equation,

$$\sigma'' = -\sigma \frac{eE_0}{2\gamma^2 mc^2}\gamma' = -\sigma \frac{\gamma'^2}{2\gamma^2} \qquad \textbf{since} \qquad \gamma' = \frac{eE_0}{mc^2}$$

with E_0 the accelerating field of the booster. For a matched beam we want the focusing strength of the accelerating field to balance the space charge defocusing force, i.e. no radial acceleration:

$$\sigma'' = -\sigma_{match}\frac{\gamma'^2}{\gamma^2} + \frac{I}{2I_A\gamma^3\sigma_{match}} = 0$$



Solving gives the matched beam size:

$$\sigma_{matched} = \frac{1}{\gamma'}\sqrt{\frac{I}{2I_A\gamma}}$$

$\sigma_{matched}$ is the waist size at injection to the accelerator. The matched beam emittance decreases along the accelerator due the initial focus at the entrance and Landau damping.

- ***Intrinsic emittance and QE***
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- ***Solenoid aberrations***
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Chromatic Aberration in the Emittance Compensation Solenoid

Emittance due to the momentum dependence of the solenoid's focal length:

$$\varepsilon_{n,chromatic} = \beta\gamma\sigma_x^2 \left| \frac{d}{dp}\left(\frac{1}{f}\right) \right| \sigma_p$$

- σ_x is the rms beam size at the solenoid
- f is the focal length of the solenoid
- p is the beam momentum
- σ_p is the rms momentum spread

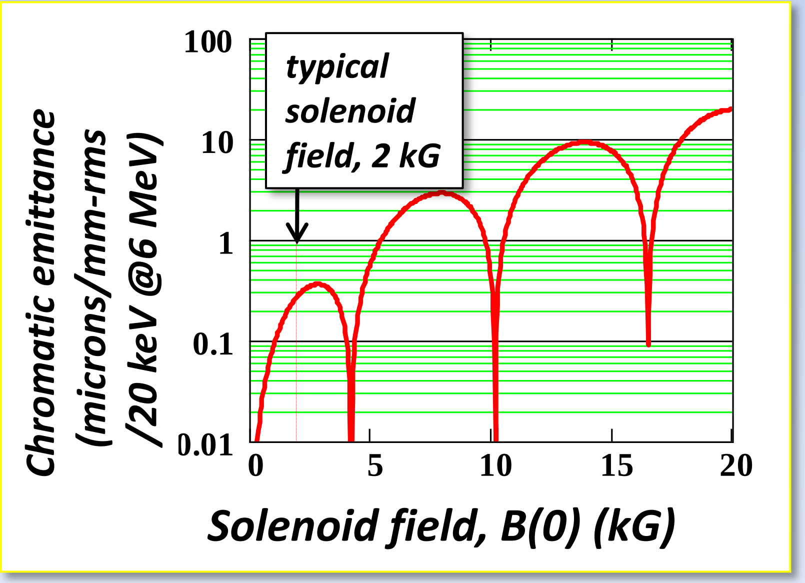
This is a general expression for the emittance produced by a thin lens when the focal length is varied. E.g., it was used earlier to describe emittance compensation.

In the rotating frame of the beam the solenoid lens focal strength is given by

$$\frac{1}{f_{sol}} = K \sin KL, \qquad K \equiv \frac{B(0)}{2B\rho_0} = \frac{eB(0)}{2p}$$

- $B(0)$ is the solenoid field
- L is the solenoid effective length
- $B\rho_0$ is the beam magnetic rigidity

$$B\rho_0 = \frac{p}{e} = 33.356\, p\left(GeV/c\right) kG-m$$



$$\varepsilon_{n,chromatic} = \beta\gamma\sigma_x^2 K \left| \sin KL + KL \cos KL \right| \frac{\sigma_p}{p}$$

The solenoid is achromatic when

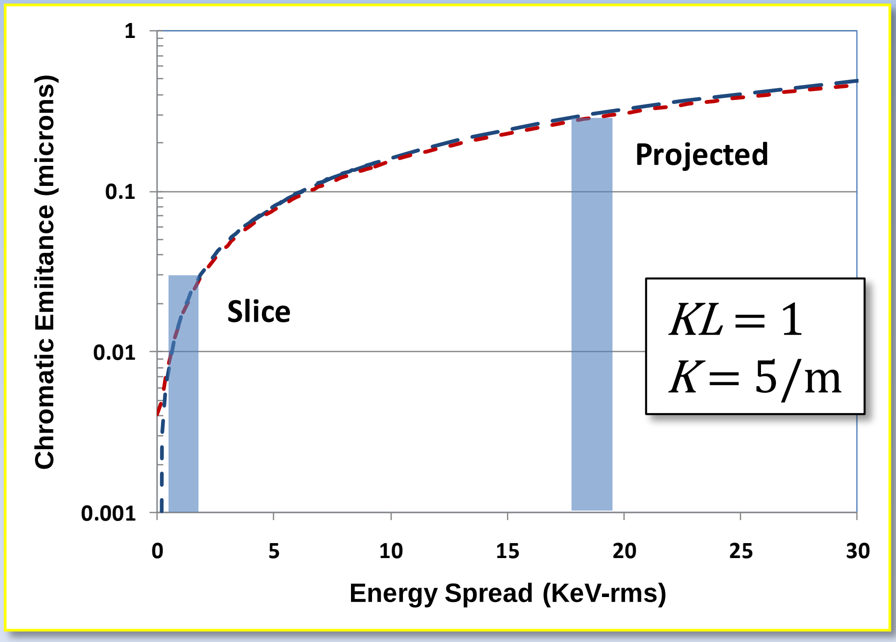
$$\tan KL = -KL$$

Chromatic Aberration: Comparison with simulation & expt.

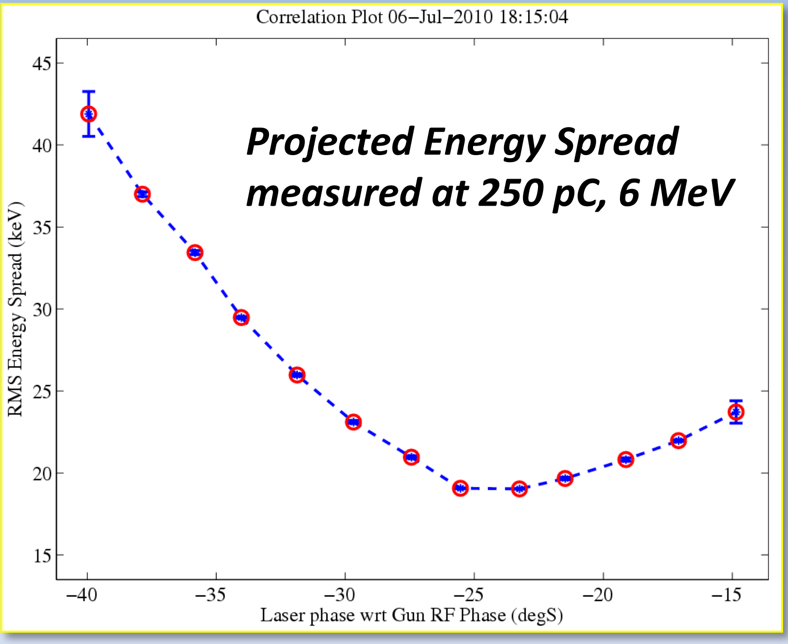
$$\varepsilon_{n,chromatic} = \beta\gamma\sigma_x^2\sigma_p\left|\frac{d}{dp}\left(\frac{1}{f_{sol}}\right)\right| \qquad \frac{1}{f_{sol}} = K\sin KL, \quad \mathrm{K} = \frac{\mathrm{B}(0)}{2\mathrm{B}\rho_0}$$

$$\varepsilon_{n,chromatic} = \beta\gamma\sigma_x^2 \mathrm{K}\left(\sin KL + KL\cos KL\right)\frac{\sigma_p}{p}$$

Comparison of Eqn. with Simulation



Assumes 1 mm rms beam size at solenoid



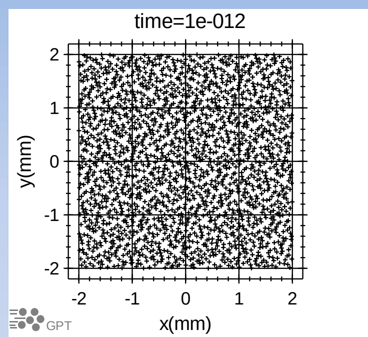
Solenoid chromatic aberration is a significant contributor to the projected emittance. But with a slice energy spread of 1 KeV, the slice chromatic emittance is only ~0.02 microns

Geometric Aberration of the Emittance Compensation Solenoid

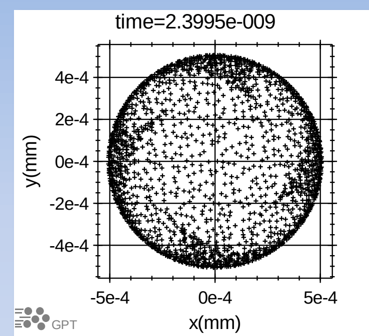
In order to numerically isolate the geometrical aberration from other effects, a simulation was performed with only the solenoid followed by a simple drift. Maxwell's equations were used to extrapolate the measured axial magnetic field, $B_z(z)$, and obtain the radial fields [see GPT: General Particle Tracer, Version 2.82, Pulsar Physics, http://www.pulsar.nl/gpt/].

Transverse beam distributions:

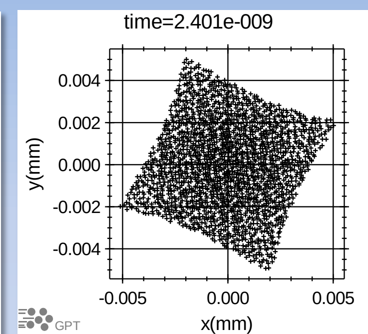
solenoid entrance

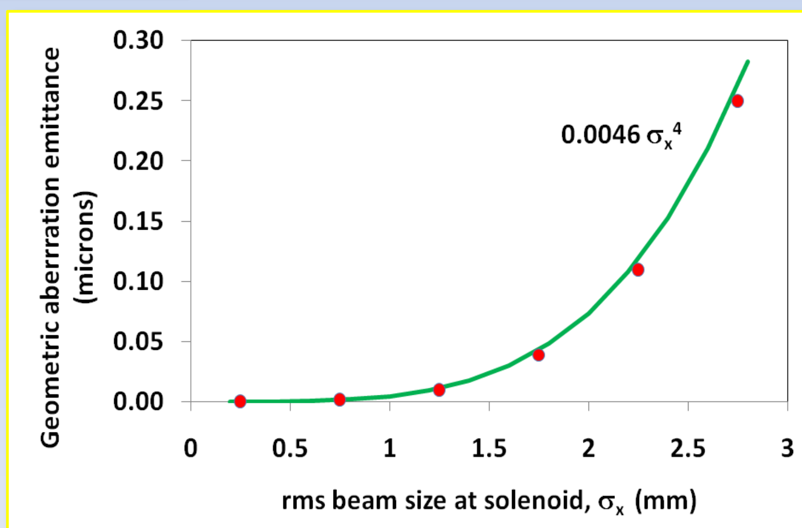


preceding focus

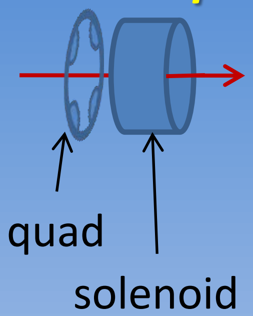


post focus "pincushion" shape





Computing the emittance due to anomalous quadrupole fields of the solenoid



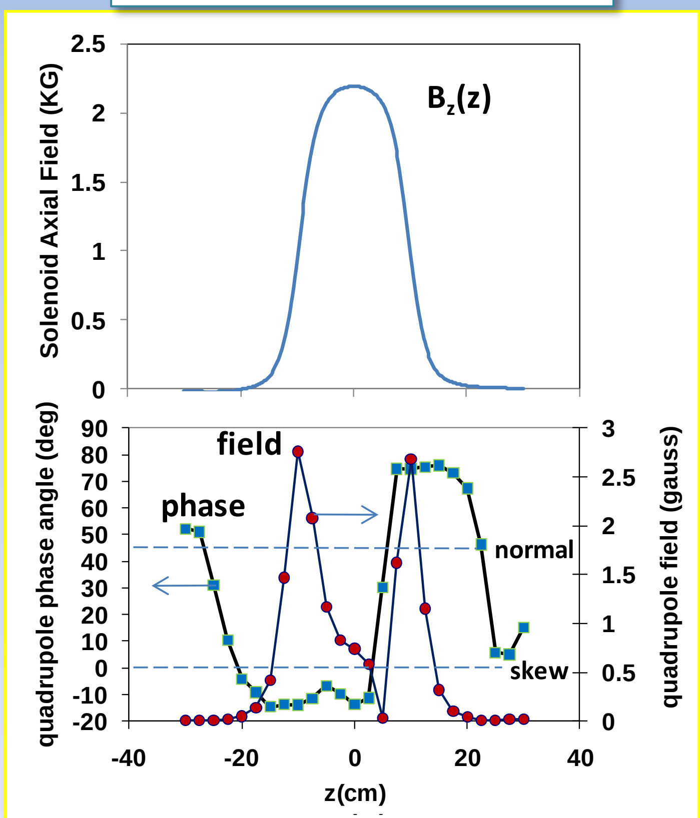
$$R_{sol}R_{quad} = \begin{pmatrix} \cos^2 KL & \frac{\sin KL \cos KL}{K} & \sin KL \cos KL & \frac{\sin^2 KL}{K} \\ -K \sin KL \cos KL & \cos^2 KL & -K\sin^2 KL & \sin KL \cos KL \\ -\sin KL \cos KL & -\frac{\sin^2 KL}{K} & \cos^2 KL & \frac{\sin KL \cos KL}{K} \\ K \sin^2 KL & -\sin KL \cos KL & -K \sin KL \cos KL & \cos^2 KL \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{f_q} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f_q} & 1 \end{pmatrix}$$

$$\sigma(1) = \left(R_{sol}R_{quad}\right)\sigma(0)\left(R_{sol}R_{quad}\right)^T$$

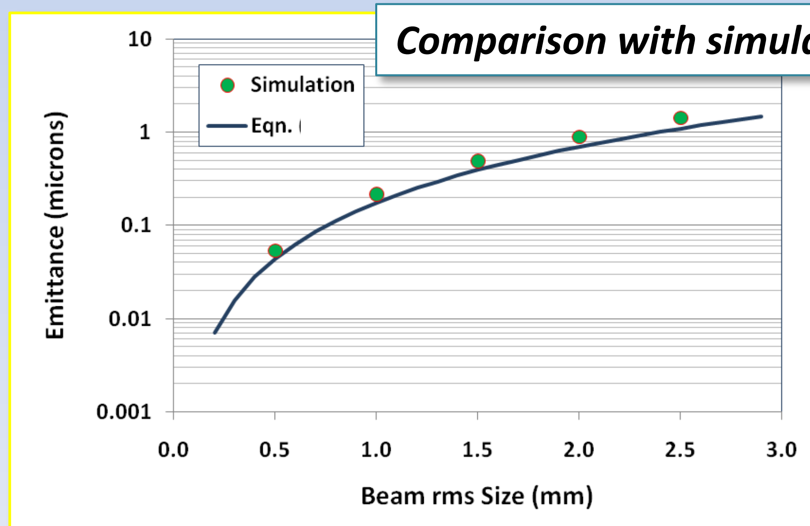
$$\gamma\varepsilon_{x,qs} = \beta\gamma\sqrt{\det \sigma_x(1)} = \beta\gamma\sqrt{\det\begin{pmatrix} \sigma_{11}(1) & \sigma_{12}(1) \\ \sigma_{12}(1) & \sigma_{22}(1) \end{pmatrix}}$$

$$\gamma\varepsilon_{x,qs} = \beta\gamma\sigma_{x,sol}\sigma_{y,sol}\left|\frac{\sin 2KL}{f_q}\right|$$

Measured magnetic fields



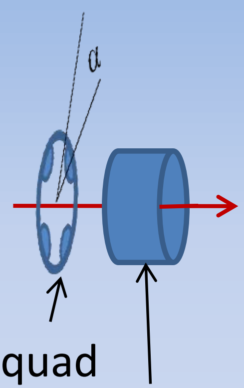
Comparison with simulation

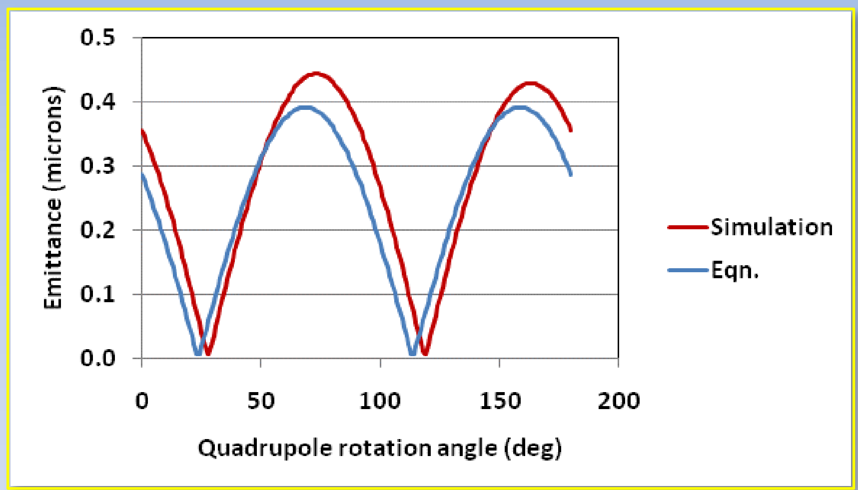


The emittance due to anomalous quadrupole field at the solenoid entrance

The previous expression is for the case of a quadrupole plus solenoid system where the quadrupole itself isn't rotated. When the quadrupole is rotated about the beam axis by angle, α, with respect to a normal quadrupole orientation, then total rotation angle becomes the sum of the quadrupole rotation plus the beam rotation in the solenoid and the emittance becomes

$$\varepsilon_{x,qs} = \beta\gamma\sigma_{x,sol}\sigma_{y,sol}\left|\frac{\sin 2(KL+\alpha)}{f_q}\right|$$





Comparison with simulation for a 50 meter focal length quadrupole followed by a strong solenoid (focal length of ~15 cm). The emittance becomes zero when

$$KL + \alpha = n\pi$$

Adding a normal/skew quadrupole pair allows recovery of the emittance caused by this x-y correlation.

Summary of Emittance Contributions from the Solenoid

Emittance due to chromatic aberration:

$$\varepsilon_{n,chromatic}(\sigma_x) = \beta\gamma\sigma_x^2 K\left(\sin KL + KL\cos KL\right)\frac{\sigma_p}{p}$$

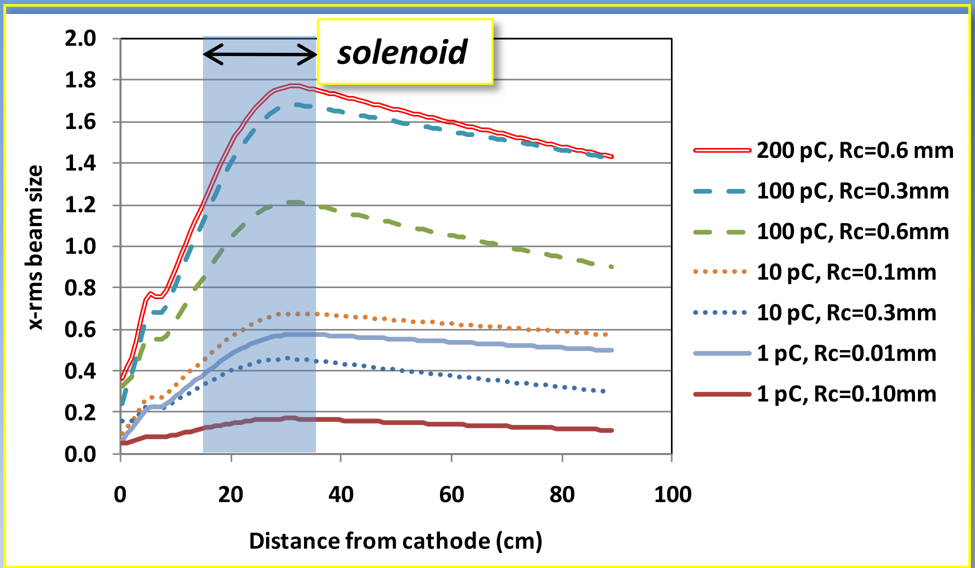
Emittance due to anomalous quad field:

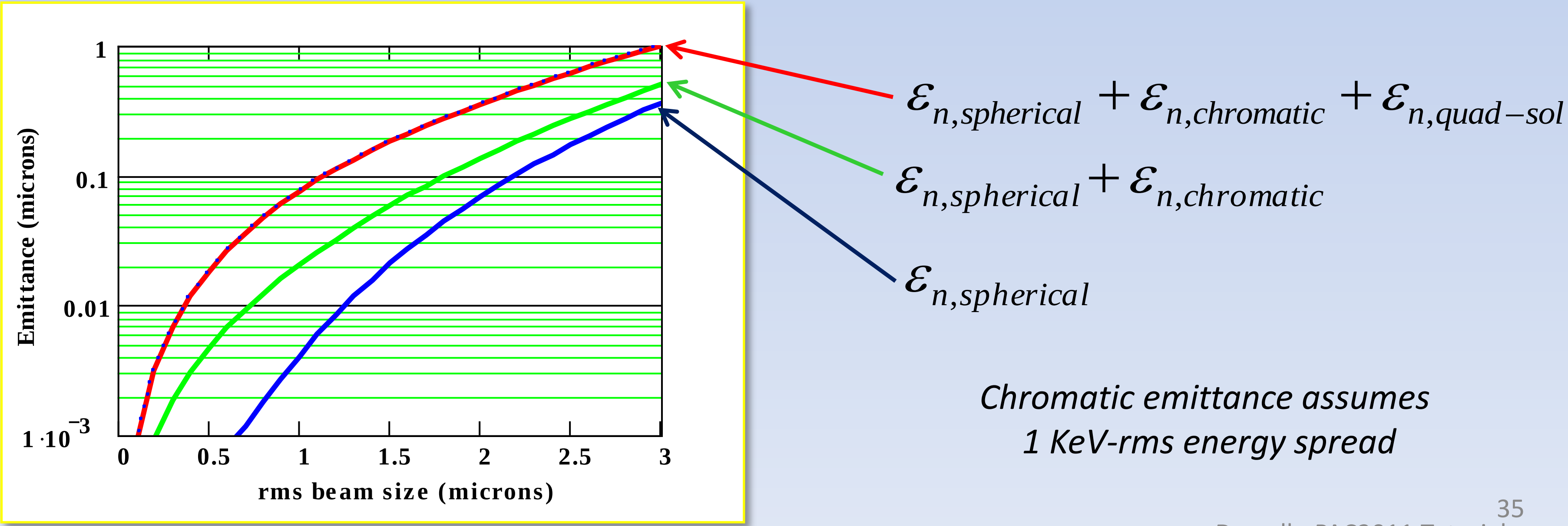
$$\varepsilon_{n,quad-sol}(\sigma_{x,sol}) = \beta\gamma\sigma_{x,sol}^2\left|\frac{\sin 2KL}{f_q}\right|$$

Spherical aberration emittance:

$$\varepsilon_{n,spherical}(\sigma_x) = 0.0046\sigma_x^4$$

rms beam size from gun to linac





Chromatic emittance assumes
1 KeV-rms energy spread

- ***Intrinsic emittance and QE***
- ***Space charge limited emission***
- ***Simple optical model & RF emittance***
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- ***"Beam blowout" dynamics & 3rd order space charge***

Space Charge Shaping*, aka "Beam Blowout!"

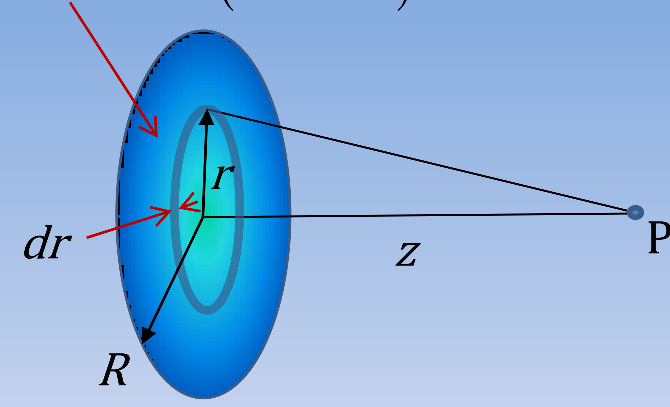
Here we derive the radial force on an electron confined to a thin disk of charge. The surface charge density is assumed to have a radial quadratic surface charge density. The quadratic factor can be adjusted to cancel the 3rd order space charge force of the disk's distribution.

The mathematical technique* is
Step 1: Compute the electrical potential energy on the axis of symmetry.
Step 2: Expand this potential into a power series and multiply each term by the same order of Legendre polynomial to obtain the potential at any point on space.
Step 3: Take the divergence of the potential to get the radial electric field.

*J.D. Jackson, Classical Electrodynamics, 3rd ed., p.101-104

Step 1: Compute the axial potential for a parabolic charge distribution,

$$\sigma(r) = \sigma_0\left(1+\sigma_2 r^2\right)$$



$$4\pi\varepsilon_0 V(z) = \iint \frac{\sigma ds}{\sqrt{z^2+r^2}} = 2\pi\int_0^R \frac{\sigma(r) r dr}{\sqrt{z^2+r^2}}$$

$$V(z) = \frac{\sigma_0}{2\varepsilon_0}\left[\int_0^R \frac{rdr}{\sqrt{z^2+r^2}} + \sigma_2\int_0^R \frac{r^3 dr}{\sqrt{z^2+r^2}}\right]$$

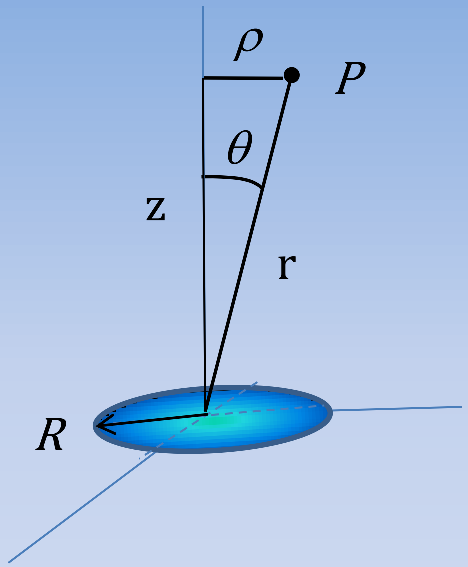
$$V(z) = \frac{\sigma_0}{2\varepsilon_0}\left\{\underbrace{\left(z^2+R^2\right)^{1/2} - z}_{\text{uniform distribution}} + \underbrace{\sigma_2\left[\frac{1}{3}\left(z^2+R^2\right)^{3/2} - z^2\left(z^2+R^2\right)^{1/2} + \frac{2}{3}z^3\right]}_{\text{parabolic distribution}}\right\} = V_0(z) + V_2(z)$$

*Serafini, AIP Conf Proc 279, p645 1992; Luiten et al., PRL, 93, 2004,p.94802

Step 2: Expand V_0 in powers of r and multiply each term by the same order of Legendre polynomial to get the potential for a uniformly charged disk at any point P,

$$V_0(\theta,r)=\frac{\sigma_0}{2\varepsilon_0}\left[RP_0(\cos\theta)-rP_1(\cos\theta)+\frac{1}{2}\frac{r^2}{R}P_2(\cos\theta)-\frac{1}{8}\frac{r^4}{R^3}P_4(\cos\theta)+\cdots\right]$$

$$\text{where } r^2=\rho^2+z^2 \qquad \cos\theta=\frac{z}{r}=\frac{z}{\sqrt{\rho^2+z^2}}$$



In the plane of the disk,

$z=0$ => $\cos\theta=0$ and $r=\rho$,

so the Legendre polynomials are evaluated at zero, i.e. $P_n(0)$.

Then the potential in the plane of a uniformly charged disk is

$$V_0(\rho)=\frac{\sigma_0}{2\varepsilon_0}\left[R-\frac{1}{4}\frac{\rho^2}{R}-\frac{3}{8^2}\frac{\rho^4}{R^3}+\cdots\right]$$

Following the same procedure for the parabolic term gives the potential, V_2

$$V_2(\rho)=\frac{\sigma_0\sigma_2}{2\varepsilon_0}\left[\frac{1}{3}R^3+\frac{1}{4}R\rho^2-\left(\frac{3}{8}\right)^2\frac{\rho^4}{R}+\cdots\right]$$

The total potential in the plane of the disk is given by the sum of these potentials

$$V(\rho)=V_0(\rho)+V_2(\rho)$$

Step 3: Take derivative of potential to get radial field.

Total potential inside the disk: $V(\rho) = V_0(\rho) + V_2(\rho)$

Summing uniform and parabolic potentials and collecting powers of r gives

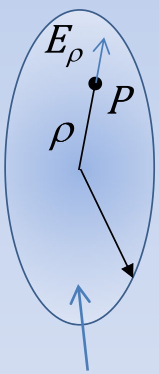
$$V(\rho) = \frac{\sigma_0}{2\varepsilon_0}\left[R\left(1+\frac{\sigma_2 R^2}{3}\right) + \frac{1}{4}\left(\sigma_2 - \frac{1}{R^2}\right)R\rho^2 - \frac{3}{8^2}\left(\frac{1}{R^2}+3\sigma_2\right)\frac{\rho^4}{R} + O\left(\rho^6\right)\right]$$

The radial space charge field is then:

$$E_\rho(\rho) = \frac{\partial V}{\partial \rho} \Rightarrow E_\rho(\rho) = \frac{\sigma_0}{2\varepsilon_0}\left\{ \frac{1}{2}\left(\sigma_2 - \frac{1}{R^2}\right)R\rho - \frac{3}{16}\left(3\sigma_2 + \frac{1}{R^2}\right)\frac{\rho^3}{R} + O\left(\rho^5\right)\right\}$$

Linear force
No emittance

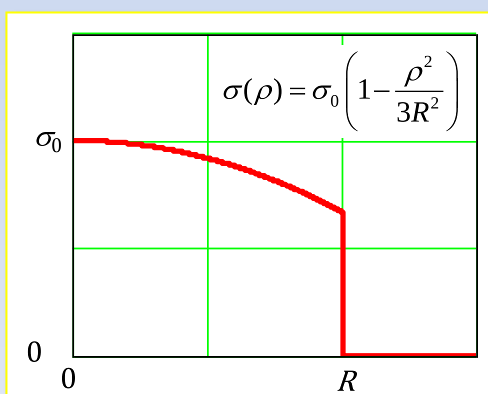
Non-linear force
Emittance growth



E_ρ is the radial space charge field at point P in the plane of the disk for ρ<R

Disk with surface charge density:

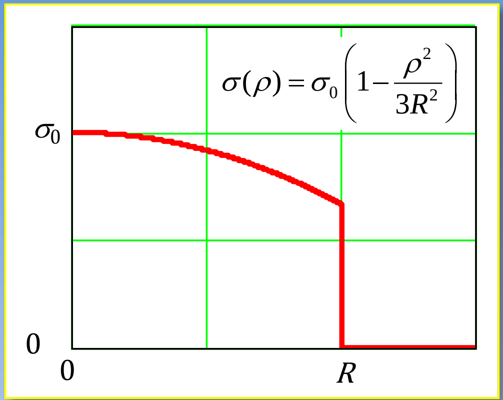
$$\sigma(\rho) = \sigma_0\left(1+\sigma_2\rho^2\right)$$



If we make: $\sigma_2 = -\frac{1}{3R^2}$

Then there's no 3rd order force and no space charge emittance during expansion of beam from cathode!

How large are these radial fields?



$$E_\rho(\sigma_2,\rho)=\frac{\sigma_0}{2\varepsilon_0}\left\{\frac{1}{2}\left(\sigma_2-\frac{1}{R^2}\right)R\rho-\frac{3}{16}\left(3\sigma_2+\frac{1}{R^2}\right)\frac{\rho^3}{R}\right\}$$

For $\sigma_2=-\frac{1}{3R^2}$ ⇨ $E_\rho\left(\sigma_2=-\frac{1}{3R^2},\rho\right)=-\frac{1}{3}\frac{\sigma_0}{\varepsilon_0}\frac{\rho}{R}$

For 250 pC and R=0.6 mm (LCLS-like parameters): $E_\rho=36MV/m$

This is nearly as large as the RF field(!!!) which is $E_{peak}cos\phi_{rf}$ **= 57.5 MV/m**

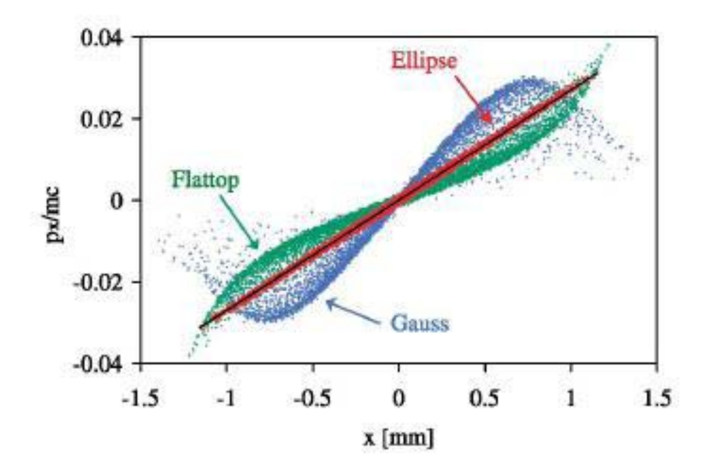


FIG. 2 (color). GPT simulations of the distribution in $x-p_x$ phase space 50 ps after initiation with an ellipsoidal (red dots), a flattop (green dots), and a Gaussian (blue dots) initial radial profile. Black solid line: pancake theory.

Luiten et al., PRL, 93, 2004,p.94802

However, even with these strong fields they are now linear and the space charge emittance is greatly reduced by parabolic shaping of the transverse shape.

Summary

- *Electron emission physics*
  - *Quantum efficiency and intrinsic emittance computed with the 3-step model*
  - *QE agrees but expt. emittance is 2x-larger than theory*

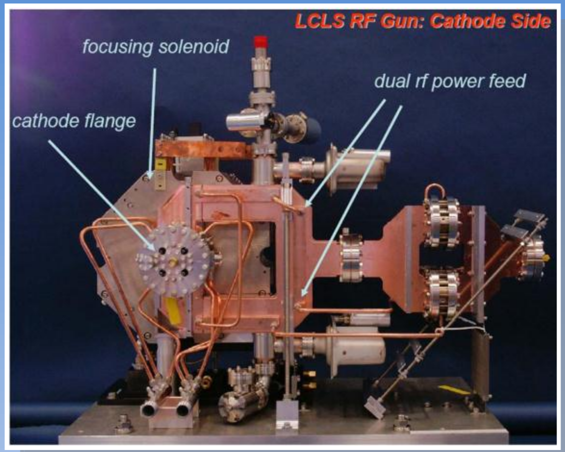
- *Space charge limited (SCL) emission*
  - *Child-Langmuir Law vs. short pulse emission*
    - *short pulse emission has higher space charge limit than C-L law*
  - *Analysis of space charge limited emission data*
    - *SCL flattens the transverse profile*

- *RF emittance*
  - *A simple optical model, RF defocusing at exit of gun canceled by solenoid*
  - *Time-dependent emittance: 1st and 2nd order*

- *Projected emittance compensation*
  - *Balanced flow and plasma oscillations*
  - *Matching to first linac section, Ferrario matching condition*

- *Solenoid aberrations*
  - *Chromatic, projected vs. slice energy spread*
  - *Geometric, scales with beam size to 4th power*
  - *Anomalous quadrupole fields, recoverable emittance with normal/skew quad correctors*

- *Beam blowout dynamics*
  - *Transverse shaping to eliminate space charge emittance due to Serafini and Luiten*
  - *Begin with very short bunch, a single slice which can be modeled as a disk of charge*
  - *Parabolic transverse shaping eliminates 3rd order space charge force*



Thanks for your attention!