VLASOV AND PIC SIMULATIONS OF A MODULATOR SECTION FOR COHERENT ELECTRON COOLING*

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Abstract

Next generation ion colliders will require effective cooling of high-energy hadron beams. Coherent electron cooling (CEC) can in principle cool relativistic hadron beams on orders-of-magnitude shorter time scales than other techniques [1]. We present Vlasov-Poisson and delta-f particle-in-cell (PIC) simulations of a CEC modulator section. These simulations correctly capture the subtle time and space evolution of the density and velocity wake imprinted on the electron distribution via anisotropic Debye shielding of a drifting ion. We consider 1D and 2D reduced versions of the problem, and compare the exact solutions of Wang and Blaskiewicz [2] with Vlasov-Poisson and delta-f PIC simulations. We also consider interactions under non-ideal conditions where there is a density gradient in the electron distribution, and present simulations of the ion wake.

COHERENT ELECTRON COOLING

Coherent electron cooling (CEC) is an exciting new technique for rapidly cooling high-energy hadron beams [1]. CEC begins in the same way as conventional electron cooling, with hadrons (in this case ions) and electrons co-propagating at matched mean velocities. The proposed Brookhaven CEC consists of three sections: a *modulator*, where the ion imprints a density wake on the electron distribution, an *FEL*, where the density wake is amplified by an FEL, and a *kicker*, where the amplified wake interacts with the ion, resulting in dynamical friction for the ion.

In this paper we consider only the modulator section. We calculate the wakes in the electron distribution due to the presence of the ions. In the modulator section the ions drift in a nearly uniform distribution of electrons. Although these beams are highly relativistic in the laboratory frame, all velocities are non-relativistic in the “beam frame” drifting with the mean speed of the particles.

The response of the electrons is classical Debye shielding, with a few complications. Accelerated electrons have a non-isotropic velocity distribution, and the electron density may also be non-uniform.

In this paper x is the direction of beam propagation. We consider reductions of the 3D beam to 2D and 1D. In a 2D or 1D reduction, the problem has no variation in one or both transverse directions, respectively. A particle in 2D or 1D can be thought of as a line charge or plane of charge. Most equations are valid in dimensions 1 to 3, except as noted.

Simulation results in this paper are calculated using VORPAL [3]. The two models which can be used are delta-f PIC [4], and Vlasov-Poisson. The Vlasov-Poisson solver implemented in VORPAL was adapted from that in Boine-Frankenheim [5].

Wang and Blaskiewicz [2] found exact solutions to the Vlasov-Poisson equations in 3D by assuming a special form of the electron velocity distribution, a kappa-2 distribution. It is nontrivial to derive analogous exact solutions for the 1D and 2D cases, so we present these solutions at the end of this paper. Although the velocity distributions may not be entirely realistic, the exact solutions provide good tests for our numerical algorithms.

VLASOV-POISSON FORMULATION

The electron response to the presence of the ion is classical Debye shielding governed by the Vlasov equation

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f - \frac{e}{m_e}\left[\vec{E} \cdot \nabla_v f\right] = 0, \qquad (1)$$

where $f(\vec{x}, \vec{v}, t)$ is the electron density function, and $\vec{E}$ is the electric field induced by this charge distribution.

We consider that the electron density f can be split into a steady-state solution f_0 plus a perturbation f_1. In the center of the beam we can take f_0 with a uniform spatial distribution and a Gaussian velocity distribution. In this paper we also consider an ion near the edge of the beam, where there is a density gradient in f_0. In this case an external electric field $\vec{E}_0$ is needed to maintain the equilibrium density profile.

With the decomposition $f = f_0 + f_1$ and $\vec{E} = \vec{E}_0 + \vec{E}_1$ we have that f_0 satisfies the steady-state Vlasov equation

$$\vec{v} \cdot \nabla_x f_0 - \frac{e}{m_e}\left[\vec{E}_0 \cdot \nabla_v f_0\right] = 0. \qquad (2)$$

If f_0 is uniform in space, then equation (2) is trivially satisfied with $\vec{E}_0 = 0$. For a beam with a Gaussian profile in x and v (in 1D),

$$f_0(x, v) = \frac{n_0}{\sigma\sqrt{2\pi}} \exp\left(-\frac{v^2}{2\sigma^2} - \frac{(x - x_0)^2}{2r_0^2}\right), \qquad (3)$$

where x_0 and r_0 are the center and RMS of the falloff in x, and σ is the RMS velocity of the electron distribution. Equation (2) requires a linear focusing field

$$\vec{E}_0(x) = \frac{m_e}{e}\frac{\sigma^2}{r_0^2}(x - x_0). \qquad (4)$$

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The perturbation density f_1 satisfies

$$\frac{\partial f_1}{\partial t} + \vec{v}\cdot\nabla_x f_1 - \frac{e}{m_e}\Big[\vec{E}_1\cdot\nabla_v(f_0+f_1)+ \vec{E}_0\cdot\nabla_v f_1\Big] = 0. \quad (5)$$

The electric field $\vec{E}_1$ is induced by the density distribution f_1 and the perturbing ion,

$$\nabla\cdot\vec{E}_1 = \frac{\rho(\vec{x},t)}{\epsilon_0}, \quad (6)$$

where

$$\rho(\vec{x},t) = Z_i e\delta(\vec{x}) - e\tilde{n}_1(\vec{x},t), \quad (7)$$

and $\tilde{n}_1(\vec{x},t) = \int f_1(\vec{x},\vec{v},t)d\vec{v}$. In order to match the notation of Wang and Blaskiewicz [2], we have used $\tilde{n}_1(\vec{x},t)$ to denote the shielding response of the electrons to the ion. Here Z_i is the charge number of the perturbing ion ($Z_i = 79$ for a fully stripped gold ion)

Equations (5) and (6) form a coupled system of Vlasov-Poisson equations to be solved. Note that we have made no assumption here that f_1 is small, only that it is the deviation from a steady-state solution.

Simulation Results

We first compare Vlasov and delta-f PIC simulations with the exact solution in equation (13) (details presented in the final section). Figure 1 shows a 1D Vlasov simulation at 0.25 (black), 0.50 (blue), 0.75 (green), and 1.0 (red) plasma periods and have been vertically offset so that they do not overlap (this convention is followed in all the figures). The fit with the exact solutions is excellent and the two curves are nearly indistinguishable.

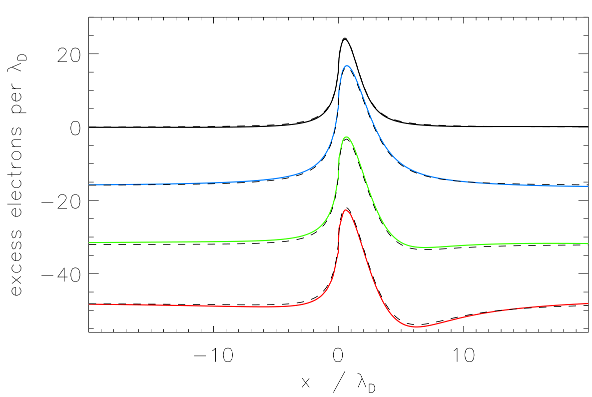

Figure 1: Mountain range plot of the electron response $\tilde{n}_1(x,t)$ from a Vlasov simulation (color) and equation (13) (dashed lines). The curves are snapshots at 0.25 (black), 0.50 (blue), 0.75 (green), and 1.0 (red) plasma periods.

The parameter β in the velocity distribution (8) is analogous to the RMS velocity in the Gaussian case, and should not be confused with a relativistic factor. Parameter values used are $\beta = 1.13\times10^5$ m/s, $n_0 = 1.6\times10^{16}\mathrm{e/m}^3$, and the ion speed $v_0 = -\beta$. The plasma frequency $\omega_p = 7.14\times10^9$ rad/sec and the Debye radius $\lambda_D = \beta/\omega_p = 15.8$ microns. The vertical scale of Figure 1 plot is in units of $\tilde{n}_1(x,t)/\lambda_D$, the area under the curve is the total shielding charge.

Figure 2 shows a similar comparison where delta-f PIC is used. The discrete particle nature of the simulation introduces noise, but the agreement with the exact solution is good.

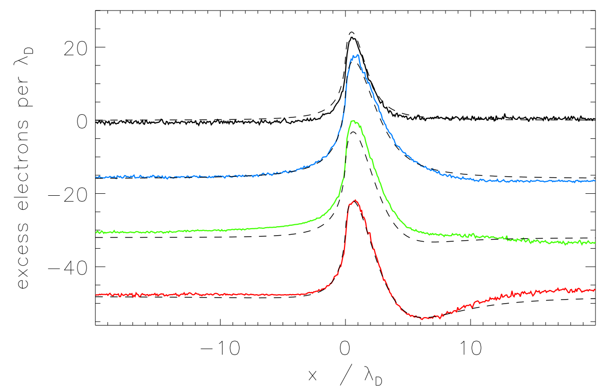

Figure 2: Mountain range plot of $\tilde{n}_1(x,t)$ from a delta-f PIC simulation (color) and equation (13) (dashed lines).

We now consider the case where a density gradient is present. In the run of Figure 3, the density varies according to equation (3) with $n_0 = 1.6\times10^{16}\mathrm{e/m}^3$, $x_0 = 7.5\lambda_D$ and $r_0 = 10\lambda_D$. Consequently, the plasma frequency and the Debye radius vary with x. The values of ω_p and λ_D are fixed by the peak density n_0. The ion in this case is stationary ($v_0 = 0$), and the dashed line shows a comparison with the exact result (13) with no density gradient.

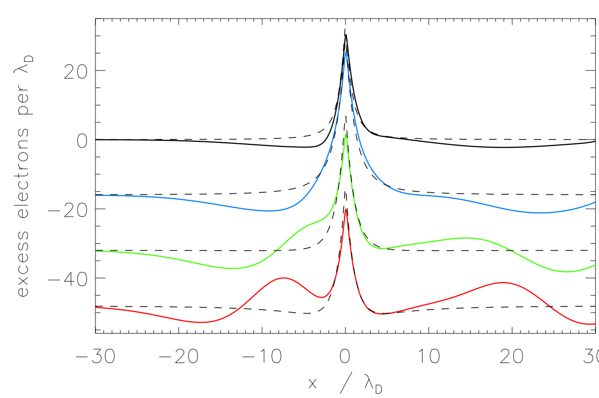

Figure 3: Mountain range plot of $\tilde{n}_1(x,t)$ from a Vlasov simulation in the presence of a density gradient.

In Figure 3 the density is lower to the immediate left of the ion, and higher to the right. The plasma frequency is thus lower to the left of the ion compared to the right, and the electron response can be interpreted as slower to the left of the ion compared to the right. While the shielding

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within a few Debye radii of the ion is similar to the case without the density gradient, farther out there are significant differences. Of concern is that any significant value of the density perturbation that occurs away from the ion will be amplified in the FEL and will contribute to incoherent effects in the kicker.

Exact Vlasov Solutions in 1D and 2D

We use the notation of Wang and Blaskiewicz [2], and derive their results in 1D and 2D. In this section the steady-state density f_0 is uniform in $\vec{x}$, so $\vec{E}_0 = 0$. To obtain exact results, we assume a specific form of the steady-state velocity distribution. In 1D and 2D, this form is

$$f_0(v) = \frac{n_0}{\beta\pi}\left(1+\frac{(v+v_0)^2}{\beta^2}\right)^{-1} \text{ 1D}, \quad (8)$$

$$f_0(\vec{v}) = \frac{n_0}{\beta_x\beta_y 2\pi}\left(1+\frac{(v_x+v_{0x})^2}{\beta_x^2}+\frac{(v_y+v_{0y})^2}{\beta_y^2}\right)^{-3/2} \text{ 2D}, \quad (9)$$

where v_0 (1D) or $\vec{v}_0 = (v_{0x}, v_{0y})$ (2D) is the ion velocity. If we take the 3D kappa-2 distribution (equation (10) in [2]) and integrate it over all z, we obtain equation (9), and integrating this 2D distribution over all y results in equation (8).

In equation (17) of Wang and Blaskiewicz [2], they derive a compact formula for the evolution of the density perturbation in wavenumber space $\vec{k}$,

$$\frac{d\tilde{n}_1}{dt}(\vec{k},t) = Z_i\omega_p \sin(\omega_p t)\exp\left(\lambda(\vec{k})t\right), \quad (10)$$

where $\lambda(\vec{k})$ is given by their equation (13), in 1D and 2D $\lambda(\vec{k})$ is given by

$$\lambda(k) = ikv_0 - |k|\beta \text{ 1D}, \quad (11)$$

$$\lambda(\vec{k}) = i\vec{k}\cdot\vec{v}_0 - \sqrt{(k_x\beta_x)^2+(k_y\beta_y)^2} \text{ 2D}. \quad (12)$$

We now invert equation (10) using the inverse Fourier transform. In 1D the formulas are straightforward and Fourier inversion gives

$$\tilde{n}_1(x,t) = \frac{Z_i\omega_p}{\pi\beta}\int_0^t \frac{\psi\sin(\omega_p\psi)d\psi}{\psi^2+\frac{(x+v_0\psi)^2}{\beta^2}}. \quad (13)$$

The limiting case where $t\to\infty$ can be evaluated exactly if we assume $v_0 = 0$ (stationary ion). This gives the steady-state result

$$\tilde{n}_1(x) = \frac{Z_i\omega_p}{2\beta}\exp\left(-\frac{\omega_p|x|}{\beta}\right). \quad (14)$$

Unlike in 3D, the steady-state density perturbation does not diverge near the ion, and in fact $\tilde{n}_1(0) = Z_i\omega_p/2\beta$. By integrating the steady-state perturbation $\tilde{n}_1(x)$ over all x we find that the total shielding charge is Z_i, as expected.

In 2D we note that the real part of $\lambda(\vec{k})$ in equation (12) is a function only of the magnitude of the scaled vector wavenumber $(\beta_x k_x, \beta_y k_y)$. The Fourier inversion can therefore be reduced to a single integral over this magnitude, which can be evaluated exactly (see [6] for the general theory on this). In 2D, Fourier inversion and time integration of equation (10) yields

$$\tilde{n}_1(\vec{x},t) = \frac{Z_i\omega_p}{2\pi\beta_x\beta_y}\int_0^t \frac{\psi\sin(\omega_p\psi)d\psi}{\left[\psi^2+\frac{(x+v_{0x}\psi)^2}{\beta_x^2}+\frac{(y+v_{0y}\psi)^2}{\beta_y^2}\right]^{3/2}} \quad (15)$$

We can obtain an exact steady-state result for a stationary ion $\vec{v}_0 = 0$,

$$\tilde{n}_1(x,y) = \frac{Z_i\omega_p^2}{2\pi\beta_x\beta_y}\int_0^\infty \frac{\alpha\sin\alpha}{(\alpha^2+r^2)^{3/2}}d\alpha, \quad (16)$$

where $r^2/\omega_p^2 = x^2/\beta_x^2 + y^2/\beta_y^2$ and $\alpha = \omega_p\psi$. Evaluating the integral gives

$$\tilde{n}_1(x,y) = \frac{Z_i\omega_p^2}{2\pi\beta_x\beta_y}K_0(r), \quad (17)$$

where K_0 is a Modified Bessel function of the second kind. We note that $K_0(r) \approx -\ln r$ for small r, and $K_0(r) \approx \sqrt{\pi/(2r)}\exp(-r)$ for large r. Therefore the perturbed density in 2D diverges logarithmically near the ion. Finally, integrating this over all x and y gives the total shielding charge Z_i.

Future Work

We will develop Vlasov-Poisson simulations in 2D, including a possible density gradient as well as external fields. We are also developing delta-f PIC models for these cases, in 1D, 2D and 3D.

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