CHARGE SEPARATION FOR MUON COLLIDER COOLING *

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Abstract

Most schemes for six dimensional muon ionization cooling work for only one sign. It is then necessary to have charge separation prior to that cooling. Schemes of charge separation using bent solenoids are described, and their simulated performances reported. It is found that for efficient separation, it should take place at somewhat higher momenta than commonly used for the cooling.

INTRODUCTION

Muon Colliders and Neutrino Factories make muons from pion decay in channels that accept both signs. These muons must have their emittances reduced in all 6 dimensions using ionization cooling[1]. In most schemes to do this, curved orbits with dispersion are required, and these work for only one sign of their charge. Efficient charge separation of the beams is thus required. We have found that separation using bent solenoids works better than methods using dipoles, or rf cavities, for our large emittance beams.

Figure 1 illustrates this method. Capital X, Y, Z are Cartesian coordinates. An initial horizontalbend separates the charges vertically, where a septum can be used to create two independent beams. One beam is immediately bent

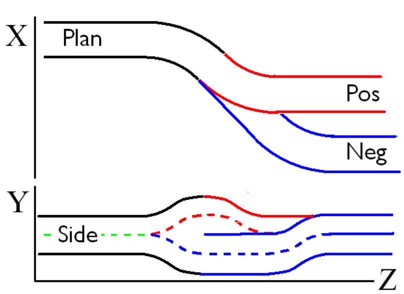

Figure 1: Schematic of charge separation method.

back to remove its dispersion. The other beam is continued straight till it is sufficiently displaced from the first, and then it too is bent back to the forward direction.

BENT SOLENOID DYNAMICS

The field B_ϕ within a bent solenoid, far from its ends, is the same as that around a single vertical current:

$$B_\phi = \frac{\mu_o I}{2\pi\rho}$$

For a trajectory to remain at a fixed radius ρ there has to be an inward bend of approximately that radius. This requirement is satisfied if the trajectory is an upward helix with angle α from the horizontal where

$$\alpha(P_z) = \frac{P_z}{\rho\, B_\phi\, c}$$

where P_z is the momentum along its path in eV, B_ϕ is in Tesla, ρ in meters, and c is the velocity of light in meters/sec. Particles with an initial transverse momentum with respect to this angle will follow helices about it with a wavelength λ:

$$\lambda = \frac{2\pi\, P_z}{B_\phi\, c}$$

where λ is in meters. In addition, there is an upward 'drift' that depends on this initial transverse momentum.

Both effects of the momentum drift and amplitude drift are largely cancelled by having reverse bends. The amplitude drift can also be reduced by using a lower field. But a low B_ϕ requires a longer bend and causes bunch lengthening.

Matching into a Bent Solenoid

In a normal long solenoid, a particle can pass straight down the axis. If the solenoid abruptly starts to bend, then there is a new equilibrium orbit rising with an angle α. This abrupt change in equilibriums represents a mismatch that excites helical motion. On exiting, there is the same effect but with the opposite sign. If the length L of the bend is not a multiple of λ, then a finite helical amplitude remains (see Fig. 2). Here, and in the following figures, lower case z is along the nominal beam direction, x and y are deviations from the nominal beam.

The mismatch can be eliminated by choosing $L = n\lambda$, or as proposed by Norem[3], by initiating, and terminating, the bend in two steps separated by a length of $\lambda/4$, with the bend curvature between them one half of the maximum (see Fig. 3). A suitable adiabatic onset of bending can have the same effect.

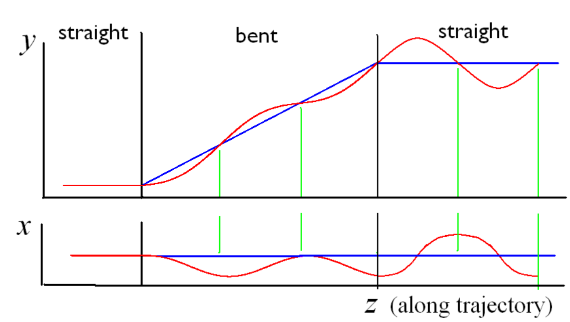

Figure 2: Uncorrected entry and exit from a bent solenoid. x, y, z are transport coordinates.

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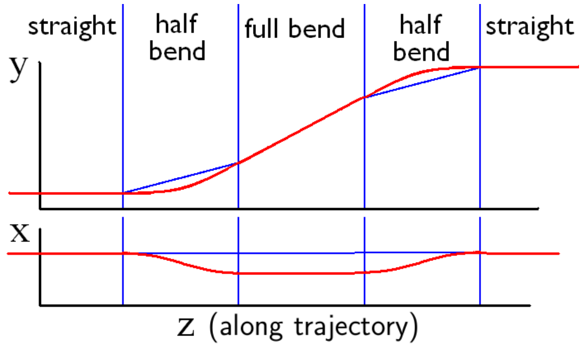

Figure 3: Norem's method to match entry and exit of bent solenoid. x, y, z are transport coordinates.

CHARGE SEPARATION

Minimizing the increase in longitudinal emittance requires a short length which, for matching, requires a small λ and thus high B_ϕ. But to minimize the amplitude drifts that increase the transverse emittance, a low B_ϕ and thus large λ is preferred. These conflict, but both are helped by raising the energy because dp/p is reduced and the particles are more relativistic.

Examples

Separation is done choosing $L = n\lambda$, or Norem's method. Following the initial bend and charge separation, reverse bends return the beams to their initial direction . There is no rf until all bends are complete, because it would disturb the cancellation of dispersion by the reverse bends.

The parameters of three examples are given in table 1. The parenthesized length is for positives only. $k = 1/\rho$ is the bend curvature. The sections at the end of each example have been added to the simulation for visualization. They are not included in quoted emittance gains. We use improved matching, and greater curvature than in an earlier study[2].

In all examples, the initial beams are Gaussian with rms normalized emittances: $\epsilon_\perp = 15\ \pi$ mm, and $\epsilon_\parallel = 40\ \pi$ mm. Angular momentum is added to set the average canonical angular momentum to zero:

$$P_x = P_{x0} - \frac{y\,c}{2}\,B_\phi \qquad P_y = P_{y0} + \frac{x\,c}{2}\,B_\phi$$

Tracks were simulated using ICOOL[4]. The resulting emittances and bunch lengths are given in table 2. Δx is the horizontal final separation of the two beams.

The first example has a central momentum of 230 MeV/c, similar to that in the earlier study[2]). The bunch length σ_{ct} has risen to 21.2 cm: too long to be captured by the following rf. The longitudinal phase space is severely distorted giving a longitudinal emittance increase of 60%, transverse emittance is increased by 31%, and transmission is only 89%. This is unacceptable.

Raising the energy has multiple advantages: a) for the same longitudinal emittance, dp/p is reduced; b) being more relativistic, the bunch lengthening is smaller; c) with less bunch lengthening, the straight section in the middle can be longer for greater separation; d) it allows weaker bends giving less amplitude drift and non-linearity; and e) for the same transverse emittance $p_\perp/p_\parallel$ is smaller giving less amplitude drift and non-linearity. The performances, as plotted in Fig. 4 are much improved as the momentum

Table 1: Parameters of 3 Examples

p=230 MeV/c, $L=\lambda$, B=3.0 T

Len m	k m^{-1}
1.6	0.4
(1.5)	0
1.6	-0.4
1.0*	0

p=300 MeV/c, Norem, B=1.9 T

Len m	k m^{-1}
1.65	0.15
0.45	0.3
1.65	0.15
(2.0)	0
1.65	-0.15
0.45	-0.3
1.65	-0.15
2.0*	0

p=400 MeV/c, Norem, B=1.9 T

Len m	k m^{-1}
2.2	0.0807
0.8	0.1614
2.2	0.0807
(4.4)	0
2.2	-0.0807
0.8	-0.1614
2.2	-0.0807
2.0*	0

Table 2: Performance of 3 Examples

		Init	Neg	Pos	<incr> %
P=230	$\epsilon_\perp$ (mm)	15.1	17.6	21.1	28%
MeV/c	$\epsilon_\parallel$ (mm)	43.9	59.7	82.1	61%
Δx=	σ_{ct} (mm)	111	151	218	66%
0.9 m	Trans (%)	100	85	89	-13%
P=300	$\epsilon_\perp$ (mm)	15.1	15.4	16.7	6%
MeV/c	$\epsilon_\parallel$ (mm)	38.1	45.8	50.3	26%
Δx=	σ_{ct} (mm)	102	162	187	66%
1.1 m	Trans (%)	100	95	95	-5%
P=400	$\epsilon_\perp$ (mm)	15.1	15.2	15.3	1%
MeV/c	$\epsilon_\parallel$ (mm)	38.1	43.8	47.8	20%
Δx=	σ_{ct} (mm)	102	134	162	45%
2.1 m	Trans (%)	100	98	97	-2.5%

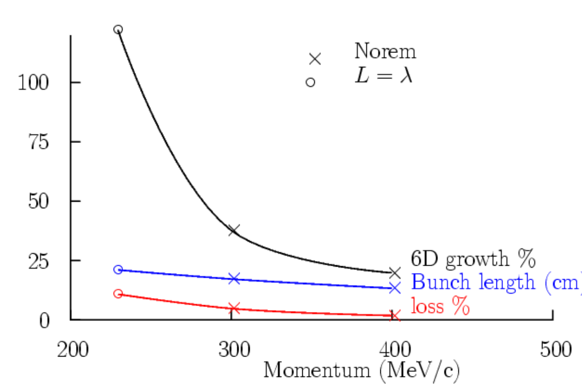

Figure 4: Performance vs. momentum.

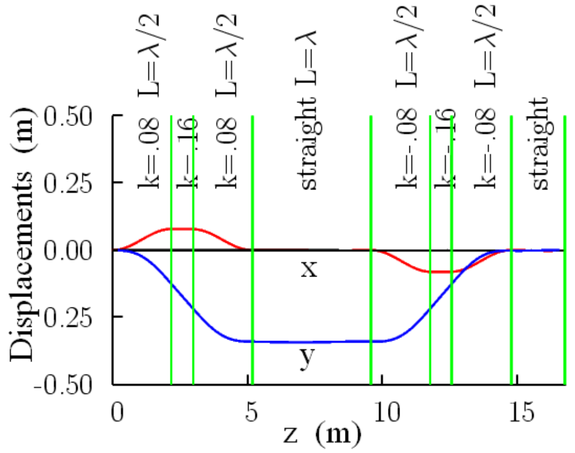

Figure 5: Central track in the 400 MeV example. x, y, z are transport coordinates.

is raised.

At 300 MeV/c, the reduced dp/p allows a lower B_ϕ (1.9 T), and the 6D emittance growth is 38% and the loss 5%, which is possibly acceptable. At 400 MeV/c, the 6D emittance growth is only 24% and the loss only 2.5%. This example also had a better beam separation of 2.1 m.

Figure 5 shows, for the 400 MeV/c example, the x and y deviations for a negative track without initial amplitude. Small deviations in x occur in the bends, but are fully removed both in the central straight section, and at the end. The y is displaced, without oscillations, in the central straight.

Figure 6a and 6b show sample tracks in x and y, respectively. Figure 6c shows scatter plots of momentum vs time at the start and end. The plotted points for each location have been arbitrarily displaced to separate the plots. Some correctable phase rotation at the end is visible, but little other distortion. Figure 6d shows scatter of y vs. x, again with arbitrarily displacements. Good separation in y in the mid section is seen, with no visible distortion.

CONCLUSIONS

Charge separation using bent solenoids can be effective if carefully designed. Bent solenoids can generate dispersion from 'momentum drift', but can spoil emittance from 'amplitude drift'. Abrupt entry into a bent solenoid causes emittance growth, but matching using integral λ lengths, or Norem's method, corrects this problem. Reverse bending removes the dispersion and reduces 'amplitude drift', but only if there is no rf until after all bending. The main problem is bunch lengthening and distortion from the long transports without rf. At 230 MeV/c, even with a higher field of 3 T, non-linearities increase the 6D emittance by 117% and give 13% loss, which is not acceptable.

Raising the momentum from 230 to 300 MeV gives a 6D emittance growth of 38% and the loss 5%, which may be acceptable. Raising the momentum further to 400 MeV/c gives very good results: 6D growth of 24% and 2.5% loss.

Further optimization should include the acceleration to the higher momenta prior to the separation, and the higher momentum cooling immediately after it. The longitudinal phase space prior to the separation should be rotated to minimize the total bunch lengthening.

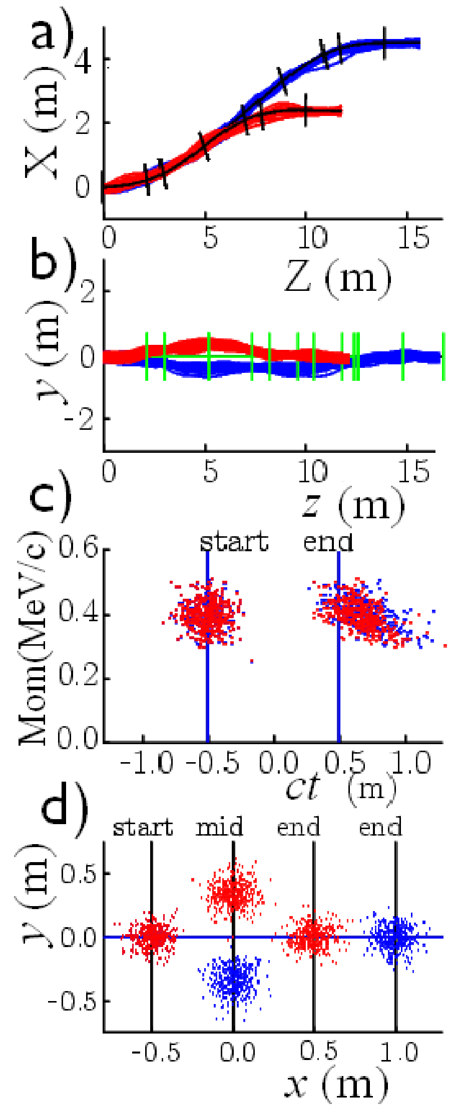

Figure 6: Sample tracks from the 400 MeV/c example. X & Z are Cartesian coordinates; x, y, z are transport.

REFERENCES

[1] G. I. Budker, 1969 Yerevan Conference, AIP Conf. Proc.352 (1996) 4; G. I. Budker, 1970 Kiev Conference, AIP Conf. Proc. 352 (1996) 5.

[2] R. Fernow & J. Gallardo, Proc. 2005 Part. Acc. Conf., p. 2610.

[3] J. Norem, Phys. Rev. Spec. Top. - Acc. Beams 2, 054001 (1999).

[4] R. Fernow, Proc. 2005 Part. Acc. Conf., p. 2651.