

## ACCURATE TUNING OF 90° CELLS IN A FODO LATTICE\*

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### Abstract

The Superconducting Super Collider was designed with a very exacting emittance budget. In order to avoid emittance dilution in the transfer of beam from the Low Energy Booster to the Medium Energy Booster, it is helpful to ensure that the transfer line connecting the two machines is tuned as designed. We discuss beam-based techniques for ensuring that the transfer line is tuned as designed, and errors associated with this procedure.

### I. BACKGROUND

In order to avoid emittance dilution when injecting a beam into a circular machine, it is necessary to properly match the  $\alpha$  and  $\beta$  functions of the injected beam to those of the machine lattice. These  $\alpha$  and  $\beta$  functions cannot be directly monitored at the injection point to the circular machine, but they may be calculated by measuring beam profiles at a number of points in the transfer line upstream of this injection point. If these measurements are to be used to determine matching to the circular machine, it is necessary to know the transfer matrices between the measurement points and the circular machine injection point with high precision, and it is preferred that these be tuned to design values.

One possible approach to tuning such a transfer line would be to observe beam widths with a large number of beam profile monitors, perhaps as many as one or two per cell. This solution was proposed by some at the Superconducting Super Collider (SSC) Laboratory for the Low Energy Booster (LEB) to Medium Energy Booster (MEB) transfer line, but was rejected because it was too costly and was not necessary. Because of the design of this line as a FODO lattice of 90° cells [1], a simple beam-based procedure relying on beam deflection and measurement of beam displacement may be used to tune the quadrupoles in this line very accurately and precisely. If the quadrupoles have been precisely positioned, this procedure will assure proper tuning of the line.

Since this technique uses the beam as a diagnostic, it does not require that the quadrupoles have well-characterized or repeatable B/I characteristics or that their fields be monitored. It will work equally well with transfer lines containing either laminated or solid-core quadrupoles.

\* Work performed at the SSC Laboratory, operated by Universities Research Association, Inc. for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

<sup>†</sup> Operated by the Universities Research Association, Inc. under contract with the U.S. Department of Energy.

### II. ROUGH TUNING

First consider a single cell of the FODO lattice, as shown in Fig. 1, where the first lens is a horizontally-focusing quadrupole. Nearly every cell contains a horizontal dipole deflector (steering magnet) at the beginning, just after the first quadrupole, and a horizontal beam position monitor (BPM) at the end, just before the first quadrupole of the next cell. In the center, on either side of the defocusing quadrupole, are a vertical BPM and a vertical corrector.

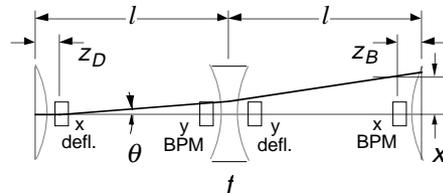


Figure 1: Rough tuning procedure.

In Fig. 1 and the equations to follow,  $l$  is the half-cell length,  $z_D$  is the distance from the lens center to the dipole corrector,  $z_B$  is the distance from the BPM to the lens center, and  $f$  is the focal length of the lens.

The rough tuning procedure is simply to deflect beam at the beginning of the cell, to measure its offset at the end of the cell, and to tune the central defocusing lens to correct the offset.

The transfer matrix from deflector to BPM is the product of three matrices: a drift, a (thin) defocusing lens, and a second drift. The  $(x|\theta)$  element of this matrix may be found, giving the beam offset at the BPM as a function of the horizontal deflection at the dipole corrector:

$$\frac{x}{\theta} = (l-z_D) + (l-z_B) + \frac{(l-z_D)(l-z_B)}{f_1} \quad (1)$$

It can be easily shown that in the thin-lens approximation, a 90° cell is properly tuned when  $f^2 = l^2/2$ . Substituting this into Eq. (1), the offset for a properly tuned cell may be found:

$$\frac{x}{\theta} = (2 + \sqrt{2})l - (1 + \sqrt{2})(z_D + z_B) + \frac{\sqrt{2} z_D z_B}{l} \quad (2)$$

The rough tuning procedure is then simply to adjust each quadrupole until the proper deflection is obtained. By alternating between horizontal and vertical planes, the entire transfer line may be tuned.

The precision of this tuning may be determined by applying first-order perturbation theory to Eq. (1). We assume that distances ( $l$ ,  $z_D$ ,  $z_B$ ) are known to very high

precision, so errors in these are ignored. The pertinent error sources are errors in  $\theta$  (due to calibration or power supply errors, or nonlinearity of the dipole correctors) and errors in  $x$  (due to BPM calibration or resolution).

Expanding Eq. (1) to first order in  $x$ ,  $\theta$  and  $f$ , setting the focal length to the proper value, and applying the approximation that  $z_D, z_B \ll l$ , one finds:

$$\frac{df_1}{f_1} \cong (1 + \sqrt{2}) \left( \frac{dx}{x} - \frac{d\theta}{\theta} \right) \quad (3)$$

Assuming a BPM measurement precision of 0.1mm rms and an offset of  $\pm 1$ cm (2cm total swing), the position measurement error is about 0.7%. Assuming an rms error of 1% in setting the angle, the total angular swing is in error by about 0.7%. Thus the lens settings using this procedure will be in error by about 2.4% rms. This should be sufficient for a preliminary tuning of the lattice, but for the LEB to MEB transfer line would not have been precise enough to ensure proper matching of the line to the MEB.

### III. FINE TUNING

Much greater precision can be obtained from a tuning procedure involving two cells, shown in Fig. 2. Upon traversing two  $90^\circ$  cells, the beam will have passed through  $180^\circ$  and will be back on-axis. This will be true at the horizontal corrector two cells from the initial deflection. The BPM is not exactly  $180^\circ$  away from the first corrector, so the beam will have a slight offset as measured at the BPM. The beam position at this BPM is a very sensitive function of the tuning of the central (focusing) quadrupole, and permits very precise tuning of this quadrupole.

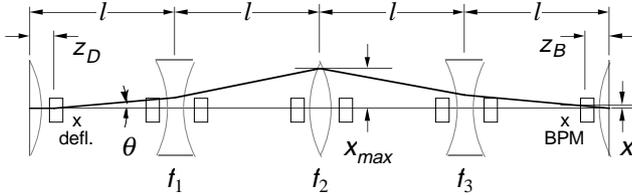


Figure 2: Fine tuning procedure.

The tuning condition may again be calculated. The  $(x|\theta)$  element of the transfer matrix for two cells is:

$$\begin{aligned} \frac{x}{\theta} = & \left[ 1 + \frac{(l-z_D)}{f_1} \right] \left[ l + (l-z_B) + \frac{l(l-z_B)}{f_3} \right] \\ & + \left[ 1 + \frac{(l-z_B)}{f_3} \right] \left[ l + (l-z_D) + \frac{l(l-z_D)}{f_1} \right] \\ & - \frac{1}{f_2} \left[ l + (l-z_B) + \frac{l(l-z_B)}{f_3} \right] \left[ l + (l-z_D) + \frac{l(l-z_D)}{f_1} \right] \end{aligned} \quad (4)$$

For proper tuning, the focal lengths should again be  $f^2 = l^2/2$ . Substituting this into Eq. (4), one finds:

$$\frac{x}{\theta} = (z_D + z_B) - \frac{\sqrt{2} z_D z_B}{l} \quad (5)$$

The tuning procedure is similar to the rough tuning procedure above. One adjusts the central lens to give the proper deflection at the BPM, then moves downstream by a half-cell and does the same with the y-plane to adjust the next quadrupole. Again, each quadrupole may be adjusted in turn along the FODO lattice.

The precision of this two-cell tuning may be determined by applying first-order perturbation theory to Eq. (4). We again assume that distances ( $l, z_D, z_B$ ) are known to very high precision, so errors in these are ignored. Expanding Eq. (4) to first order in  $x$ ,  $\theta$  and  $f$ , setting the focal lengths to the proper value, and applying the approximation that  $z_D, z_B \ll l$ , one finds:

$$\frac{df_2}{f_2} \cong \frac{\frac{df_1}{f_1} + \frac{df_3}{f_3} - \frac{(z_B + z_D)}{\sqrt{2} l} \frac{d\theta}{\theta} + (1 + \sqrt{2}) \frac{dx}{x_{max}}}{6 + 4\sqrt{2}} \quad (6)$$

Note that because of the small size of the deflection,  $x$ , at the BPM, its error has been normalized to  $x_{max}$ , the deflection at the center of the two-cell pair. Note also that the tuning of the central lens is very insensitive to errors in the deflection angle.

Assuming a BPM measurement precision of 0.1mm rms and an offset of  $\pm 1$ cm (2cm total swing) at the central quadrupole, the position measurement error is again about 0.7%. Because of the denominator in Eq. (6) above, this gives an error contribution to  $f_2$  of only 0.06%. Assuming an rms error in setting the angle of 1%, the total angular swing is in error by about 0.7%, which gives a negligible contribution to errors in  $f_2$ . (Assuming that  $z_B$  and  $z_D$  are each about 0.5m, and the half-cell length  $l$  is about 10m, this gives an error contribution to  $f_2$  of about 0.005%.) Assuming that  $f_1$  has already been set very precisely using this two-cell procedure, its errors also give a negligible contribution to errors in  $f_2$  (about 0.02%). Assuming that  $f_3$  has previously been set using the single-cell procedure to an rms precision of 2.4%, its errors give an error in  $f_2$  of about 0.2%. Thus a single application of this two-cell tuning procedure to the FODO lattice allows setting of the quadrupoles to a precision of about 0.2%, dominated by errors in  $f_3$ . A second application of the procedure allows setting of the quadrupoles to a precision of about 0.06%, dominated by errors in measurement of beam position.

This precision is more than sufficient for the tuning of the LEB to MEB transfer line of the SSC. Simulations show that 0.1% rms errors on the quadrupoles give less than 1% emittance growth [1].

### IV. COMPLICATIONS

#### A. Thick Lenses

In the above analyses, a thin-lens approximation was used. This approximation may be improved by modeling the quadrupoles as thick lenses using the principal plane construction of classical optics. For a reasonably thin quadrupole, the principal planes are very close to the center

of the quadrupole. To first order, their displacement from the center of the quadrupole is given by:

$$z_P \cong \frac{l_Q^2}{24f} \quad (7)$$

where  $z_P$  is the distance between the principal plane and the quadrupole center,  $l_Q$  is the effective length of the quadrupole, and  $f$  is the focal length of the quadrupole.

Distances ( $z_D, z_B, l$ ) should be taken from the principal planes rather than from the centers of the quadrupoles. The principal planes are on opposite sides of the lens center for focusing and defocusing quadrupoles. Because of this, the effective distance  $l$  between a focusing and defocusing lens does not change, so neither should the lens focal lengths  $f$ . The distances  $z_D$  and  $z_B$  in the focusing condition (Eqs. (2) and (5)) are modified slightly, however.

This effect can be estimated for the parameters of the LEB-MEB transfer line of the SSC. For a 0.5m quadrupole, tuned to a focal length of about 7m, the principal planes are offset about 1.5mm from the quadrupole centers, so that the sum ( $z_D + z_B$ ) is changed by about 3mm. A deflection angle of about 0.3mrad gives a deflection of about 1cm maximum at a focusing quadrupole 90° away. Using this deflection, the shift in transverse beam position,  $x$ , in the focus equations (Eqs. (2) and (5)) is on the order of 1μm, which is well below the resolution assumed for the BPMs. Thus, the thin lens approximation is perfectly adequate for this case, and introduces no detectable errors.

### B. Bends

Transfer lines are not always straight. Generally, there are bends in the lines to assist in injection to a circular machine. These bends are accomplished by large dipole magnets inserted in drift spaces of the basic FODO lattice. If these magnets are horizontally bending and have flat ends, there is no effect on horizontal focus. There is, however, a slight vertical focusing effect, which can be modeled as a weak, thin lens at each end of the dipole. For small angular deflections, the focal length of each of these lenses is given by:

$$f_D \cong \frac{2l_D}{\alpha^2} \quad (8)$$

where  $f_D$  is the focal length of the equivalent lens,  $l_D$  is the effective length of the bend dipole, and  $\alpha$  is the bend angle.

If a dipole is about 2m long, and the bend angle is about 50mrad, the focal length is about 800m, and the composite (both ends) is about 400m. If this bend is near a quadrupole, which has a focal length of about 7m, it represents about a 2% effect. This additional focusing must be taken into account in the tuning equations (Eqs. (4)-(6)), but since the cells still have nearly 90° phase advance, this extra focusing does not destroy the precision of the tuning procedure.

### C. Missing Correctors or BPMs

It has been assumed that each half-cell contains a correction dipole and a BPM. While this is desirable, it is not absolutely necessary. If a dipole corrector is missing from a half-cell, and the upstream quadrupoles have been adjusted correctly, a corrector located two cells (180°) upstream may be used. Similarly, if a BPM is missing and the downstream quadrupoles are adjusted correctly, a BPM located two cells (180°) downstream may be used.

### D. Matching Sections

Special consideration must be given to the transition from the transfer line into the circular machine. The tuning procedure for the last two or three quadrupoles in the line would have been modified slightly, with a number of BPMs in the MEB used to detect displacements, but this should not have affected the precision of the procedure.

In general, transfer lines may have matching sections at their downstream end to match  $\alpha$  and  $\beta$  functions from the line to the circular machine, which would further complicate tuning considerations at the end of the line. The LEB-MEB transfer line design had no such downstream matching section (implying that beam was "mismatched" in the transfer line, exhibiting large "beta waves"), although pairs of quadrupoles at the downstream end may have been used for dispersion matching [1].

## V. CONCLUSIONS

It has been shown that a transfer line composed of 90° FODO cells can be tuned very accurately and precisely by a procedure based on deflecting beam and observing its offset approximately two cells (180°) downstream. For the LEB-MEB transfer line at the SSC, this would have allowed the quadrupoles to be tuned to a precision of about 0.06% rms, resulting in less than 1% emittance growth due to  $\alpha$  and  $\beta$  function mismatch. This answers the concerns of some at the SSC that this line would be difficult to tune due to its solid core magnets, large beta waves, and small number of beam profile monitors.

## VI. ACKNOWLEDGMENTS

We wish to thank Karl Brown, John McGill, and Rodney Gerig for helpful discussions and for their encouragement in formulating these techniques.

## VII. REFERENCES

- [1] N. Mao et al, "Beam Optics of LEB-MEB Transfer Line for Superconducting Super Collider", *Proceedings of the 1993 Particle Accelerator Conference, Washington, D.C.*, vol. 1, pp. 333-335.