Helical spin rotators and snakes for RHIC

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Abstract

Various possible spin rotator and siberian snake schemes are considered for use at the RHIC collider, based on sequences of four helical dipole magnet modules.

I. Introduction

The RHIC collider, now under construction at BNL, will have the possibility of polarized proton-proton collisions up to a beam energy of 250 Gev. Polarized proton beams of such high energy can be only obtained with the use of siberian snakes, a special kind of spin rotator that rotates the particle spin by 180° around an axis lying in the horizontal plane [1]. Siberian snakes help to preserve the beam polarization while numerous spin depolarizing resonances are crossed, during acceleration. In order to collide longitudinally polarized beams, it is also planned to install spin rotators around two interaction regions.

Schemes based on a sequence of vertical and horizontal bending magnets have been proposed, for use as spin rotators and snakes [2,3]. The main disadvantage of such schemes is a large orbit excursion, especially at the injection energy (about 26 Gev for RHIC). From this point of view a rotator based on helical dipole magnets is more efficient [4]. Several schemes that use multipole helical magnets have been suggested in the last few years [5,6].

This paper discusses snake and spin rotator designs based on sequences of four helical magnets. The schemes that were chosen to be applied at RHIC are presented.

II. Orbital and spin motion in helical magnet

In a helical dipole magnet with a period λ and a field amplitude h, the on-axis field can be written as:

$$H_x = -h \sin kz\ , \quad H_y = h \cos kz\ , \quad H_z = 0 \qquad (1)$$

where $|k| = 2\pi/\lambda$, z is the coordinate along helical magnet axis, and subscripts x and y refer to horizontal and vertical components, respectively. A helical magnet is also characterised by its helicity, $S = k/|k|$.

As a measure of the magnetic field amplitude, it is more convenient to define the field parameter

$$p = \frac{q_0 h}{c|k|} \qquad (2)$$

where $q_0 = e/mc$.

Solving the equations of orbital and spin motion, one obtains the transformation for the proton orbital coordinates and the proton spin after one helical period [7]. For a particle entering the magnet along its axis, the orbit is simply shifted by $\lambda\ p/\gamma$ after one period, in a direction determined by the direction of the magnetic field at the helical magnet entrance and the helicity: $y = y_0 - S\ \lambda\ p/\gamma$. The spin transformation is described by the spin rotation axis:

$$\mathsf{n} = -\frac{1}{\sqrt{1+p^2\ A^2}}\left(A\ p\ \mathsf{e}_y + S\ \mathsf{e}_z\right) \qquad (3)$$

where $A = \left(1 + 1/\nu_0\right) \cdot a$ with $a = (g-2)/2$, and by the spin rotation angle $2\pi\nu$ around this n axis, where ν is:

$$\nu = \sqrt{1 + A^2\ p^2} \qquad (4)$$

These expressions for the one-period transformation of orbit and spin are the basis from which to construct spin rotators that consist of several magnets with integer numbers of periods, but with different helicities and field amplitudes. Note that when one changes the sign of the magnetic field and the helicity, the one period orbit shift does not change while the spin rotation is reversed. This gives additional flexibility during the construction of spin rotators.

III. Siberian snake

The nominal RHIC design includes 2 siberian snakes in each ring. It is assumed that one pair of snakes will be sufficient to overcome the spin depolarizing resonances during beam acceleration. In order to have the spin tune equal to 1/2, the angle between the spin rotation axes of two snakes must be 90°.

On the basis of four helical dipoles, one can construct the analog of a "continuous axis" snake [2,3]. The internal symmetry of such a snake automatically restores the particle orbit at the snake exit, and provides a snake axis in the horizontal plane. The appropriate symmetry conditions, obtained from an analysis of the spin transformation matrix, are:

1. $S_1 = S_4$, $S_2 = S_3$
2. $p_1 = -p_4$, $p_2 = -p_3$
3. $N_1 = N_4$, $N_2 = N_3$, where N_i is the number of periods of the ith magnet
4. The magnetic field at each magnet entrance is vertical

The RHIC lattice imposes additional requirements on the parameters of a siberian snake: the snake must be less than 12 meters long, and the maximum magnetic field must be less than 4 Tesla.

An analysis of all possible snake schemes shows that the best variant, from the point of view of RHIC demands, has the same helicity in all helical magnets, each with one period. Figure 1 shows the relationship between p_1 and p_2, and shows the dependence of longitudinal snake axis projection on p_1, for this variant.

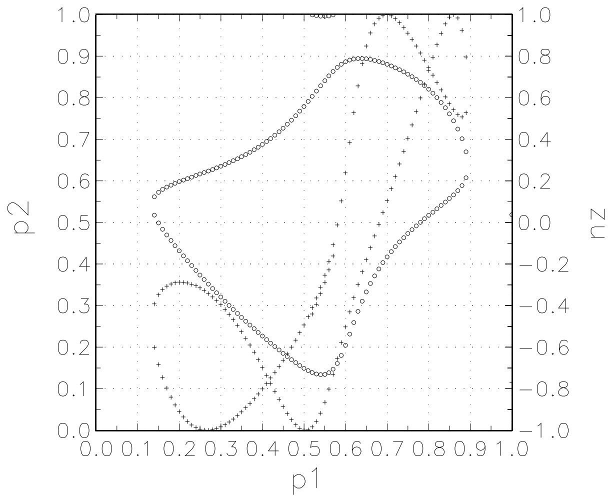

Figure. 1. The dependence of p_2 (circles) and the longitudinal snake axis projection n_z (crosses) on p_1.

Two snakes with 45° and −45° axes have been chosen for RHIC. As Fig. 1 shows, a snake with a 45° axis ($|n_z| = 0.707$) can be obtained in several ways. From the point of view of minimum orbit excursion, the best choice is $p_1 = 0.154$ and $p_2 = -0.493$. Taking the magnets to have a helical period of 2.4 m, $B_1 = 1.26$T and $B_2 = 4.04$T, producing a 3.02 cm orbit excursion inside the snake at the RHIC injection energy. More exact values for the magnetic field and the orbit excursion have been calculated by the direct integration of particle and spin motion in a realistic helical field [8].

IV. Spin rotator

Four pairs of spin rotators are planned at RHIC, in order to have the possibility of longitudinally polarised beam collisions at two collision points.

Due to the presence of dipole magnets inserted between the rotator and the interaction point, where longitudinal beam polarization is desired, the spin orientation angle required in the horizontal plane after the rotator depends on the energy. If the spin angle from the longitudinal axis at the rotator exit is ϕ, then $\phi = 10.2^\circ$ at the lowest RHIC energy ($\gamma = 27$), and $\phi = 101.2^\circ$ at the highest energy ($\gamma = 268$). Thus the rotator must provide a spin orientation angle in the horizontal plane at its exit in the range $10.2^\circ < \phi < 101.2^\circ$.

As described in the previous section on siberian snakes, all rotator designs discussed for use in RHIC included four helical dipole magnets, with an internal symmetry that provides the automatic restoration of the particle orbit at the spin rotator exit.

Consider a spin rotator which consists of four helical magnets, each with just one period. If the direction of the magnetic field at the entrance to each module is characterized by an angle α from the vertical, then automatic restoration of the particle orbit after the rotator implies:

$$\sum_{i=1}^{4} \sin\alpha_i \cdot p_i \cdot S_i \quad = \quad 0$$

γ	ϕ	B_1 (T)	B_2 (T)	max.orbit (cm)
27	10.19	2.13	2.77	2.31
50	18.88	2.38	2.65	1.39
100	37.75	2.87	2.47	0.84
150	56.63	3.22	2.51	0.63
200	75.50	3.41	2.78	0.50
250	94.38	3.50	3.11	0.41

Table I
Required magnetic fields, and maximum orbit excursions, for various RHIC energies.

$$\sum_{i=1}^{4} \cos\alpha_i \cdot p_i \cdot S_i \quad = \quad 0 \qquad (5)$$

It follows from these expressions that one way to introduce symmetry into the scheme is to combine the helical magnets in two pairs, and to require that the orbit shift caused by the first pair is compensated by the second. Asserting that magnets of each pair have the same field direction angle α, it follows from Eqn. 5 that these helical dipoles must have the same field and opposite helicities, or the same helicity and opposite fields. Combining the rotator magnets in pairs does not necessarily mean that two consecutive magnets are connected to each other. For example, one can also relate the first helical module with the third, and the second module with the fourth.

After introducing the symmetry conditions, a rotator scheme depends on two magnetic field values, which must be chosen to satisfy the spin conditions. Specifically, the particle spin after the rotator must be in the horizontal plane, with a particular spin orientation angle. Because the spin transformation matrix for helical dipoles depends on the magnetic fields in a quite complex and nonlinear way, its analysis is performed with the use of a specially written code.

Analysis shows that three variants are the best, from the point of view of minimum orbit excursion and minimum magnetic field. These schemes are shown in Figure 2.

Figure 3 shows how the maximum orbit deviation depends on the final spin angle for all 3 design variants, at a fixed energy of $\gamma = 100$. Since the orbit excursion changes with particle energy as $1/\gamma$, one can obtain the maximum orbit deviation at another energy by appropriately scaling the vertical axis in Figure 3. From these graphs one can see that variant 1 provides least orbit deviation when the spin is close to the longitudinal direction, while variant 3 is best when the spin is close to transverse, and variant 2 is preferable for an intermediate range of spin angles.

Variant 1 has been chosen as the nominal spin rotator design, because it provides the smallest maximum orbit deviation at the RHIC injection energy. The magnetic field values needed to obtain the proper spin direction at the rotator exit can be extracted from diagrams that show how the final spin angle ϕ, and the second field parameter p_2, depend on p_1, the field of the first helical magnet. Such diagrams for the nominal spin rotator design are presented in Figures 4 and 5. Table 1 lists the magnetic field values and the maximum orbit deviation at some RHIC energies (for $\lambda = 2.4m$).

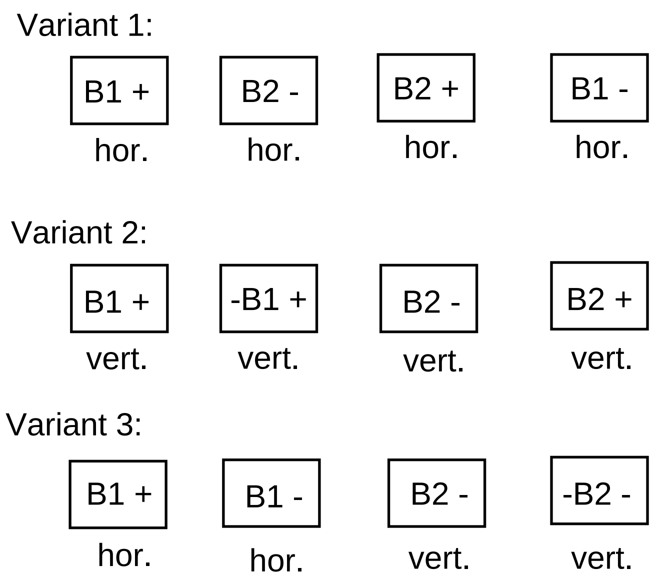

Figure. 2. Three possible symmetric rotator variants. The signs (+, −) denote the magnet helicities, while "vert." and "hor." denote the field direction at the magnet entrance.

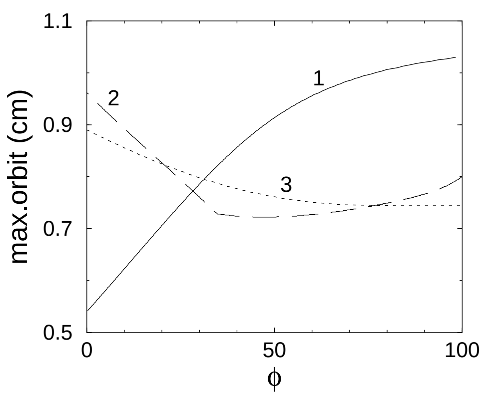

Figure. 3. Maximum orbit deviation versus spin direction angle after each rotator variant, at fixed energy ($\gamma = 100$).

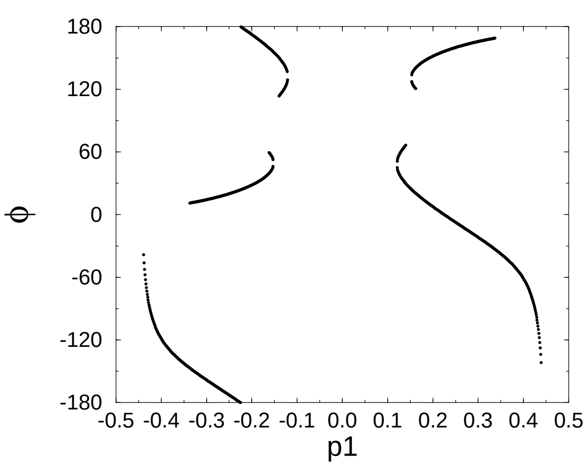

Figure. 4. The relationship between the spin direction angle after the rotator, and the field p_1 of the first magnet.

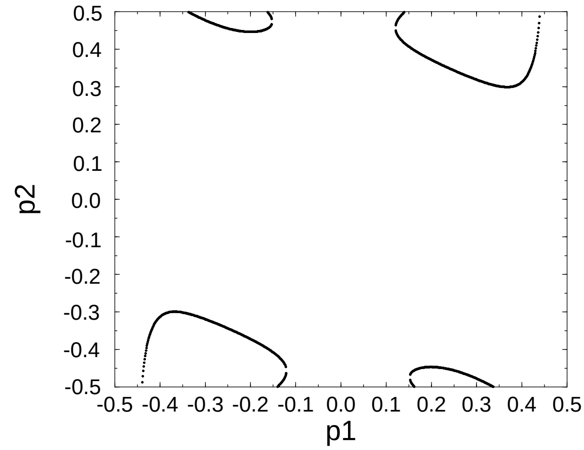

Figure. 5. The relationship between p_1 and p_2, the fields in the first and second magnets.

V. Conclusion

Spin rotator and siberian snake schemes based on helical dipole magnet modules have been adopted at RHIC. The main advantage of these schemes is that the orbit deviation is less than in bending magnet schemes.

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