A Compensated Dispersion-free Long Insertion for an FFAG Synchrotron[*]

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Abstract

An FFAG synchrotron differs from a synchrocyclotron by having a large radial field gradient; this large gradient greatly reduces the magnet volume while maintaining stable orbits for all energies, but it does induce substantial nonlinearities. A decade ago, the author proposed a dispersion-free insertion for an FFAG that provides regions of small volume for rf cavities, strippers, and injection and extraction elements, similar to a normal synchrotron.[1] Here we exploit this insertion to provide compensation of the lower order nonlinear driving fields, thereby greatly increasing the dynamic aperture as compared to an FFAG without insertions. The correction fields may be programmed to track the momentum or may be static, providing full compensation at the injection energy.

I. INTRODUCTION

The proposed European spallation source requires an accelerator that can deliver 5 MW protons to the spallation target in a short pulse. In order to deliver such a large beam power with acceptable losses, the accelerator should be dc with a high repetition rate. An FFAG synchrotron would appear to be an ideal solution.[2] The inherent nonlinearities of an FFAG (e.g. $\langle B\rangle \propto \langle R\rangle^k$) overly limit the dynamic aperture for this application, particularly for compact and higher energy machines where the field index, k, is large. All essential resonances are strongly driven. However, if the FFAG were to have dispersion-free insertions, it would be feasible to place a number of correction elements within these insertions to compensate the limiting resonances. This would greatly increase the dynamic aperture.

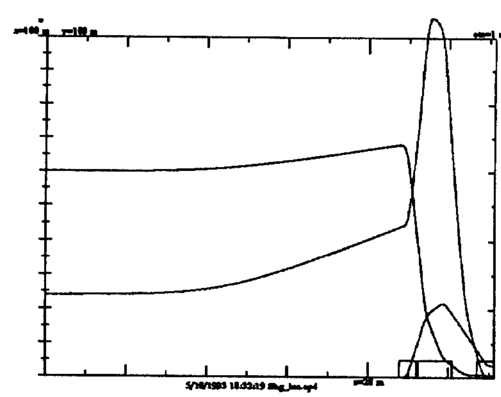
One of the very nice features of an FFAG is the capability of stacking a number of injected pulses to deliver the beam to the target at a much lower rate than the repetition rate of the accelerator. A problem with stacking at such high currents is the development of tails in the particle distribution. These tails, although stable, would be expected to lead to increased activation through scattering. With a dispersion-free insertion, it becomes feasible to add cooling to damp these damaging tails.

One would normally expect the correction magnets to be programmed to follow the acceleration cycle (in the absence of a stacked beam), thereby providing the maximum dynamic aperture throughout that cycle. However, the dynamic aperture needed is greatest at injection, and the time spent at that energy is relatively long, particularly if adiabatic trapping is employed. So one could consider static correction elements optimized for the injection energy. As the energy increases, the total tune spread is reduced, and the amount of required correction is also reduced. This means that a dc correction may prove to be entirely adequate.

Should a dc correction entirely within the dispersion-free drift not be adequate, then one could use the drift spaces immediately adjacent to the central dispersion-free drift space. In these drift spaces, there is dispersion, but the overall width of the beam is still relatively small. Multipole magnets placed in these drift spaces with small, but not zero dispersion, will result in a correction that changes with momentum.

II. THE FFAG DISPERSION-FREE INSERTION

The example shown in the 1983 paper on the insertion for an FFAG is very symmetrical. The central dispersion-free drift space is twice as long as the drift spaces at either end. As is the case with π-2π insertion for normal synchrotrons, it is possible to distribute the total drift space as desired.[3] For example, it is possible to place all of the drift space, except what is needed between magnets, in the central dispersion-free portion. As we approach such a configuration, we are forced to increase the focusing strengths, and the amplitude functions vary more wildly, making it difficult to match the insertion to the remainder of the FFAG. An example of such an asymmetric insertion using rectangular bending magnets is shown in Fig. 1, where the Courant and Snyder beta functions and the dispersion are shown for one half of the insertion.



Cost considerations will surely limit the number of insertions to three or so, and thus the periodicity will be low. This is acceptable because of the compensation. My first

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approach was to start with a symmetrical insertion, adjust it to move all of the magnets toward the ends, add the FFAG section (60° net bends at each end of the insertion for three-fold periodicity) and then try to combine the FFAG magnets and the end magnets to yield a machine with a total of 15 radial bending magnets (no spiral), each of which is assumed to be superconducting, where the coil assemblies would be the major cost item.

Recognizing that a scaling FFAG running at its space charge limit is, in reality, not scaling because of the self fields, and knowing that we have the compensation available in the insertions, there does not seem to be any reason to keep the FFAG portion of the machine at all. A fresh start would be to use the minimum number of magnets to achieve the linear solution, including the required net bend, aperture considerations, and betatron phase advances in the dispersion-free drift and then adjust the higher order field components to keep the tunes fixed and maintain the dispersion-free property of the insertion over the entire energy range from injection to extraction.

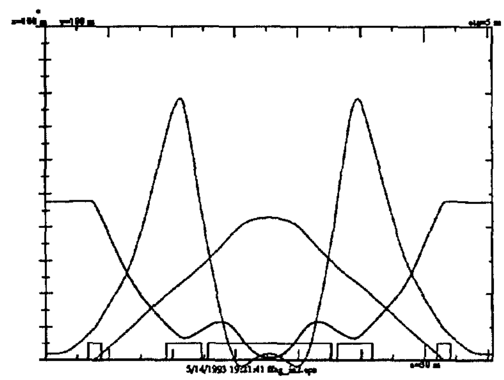
III. DESIGN APPROACHES

The first approach tried with the three-magnet symmetric lattice was to parameterize the three general magnets and then obtain algebraic expressions for the tunes and the slope of the off-energy orbit at the plane of symmetry—all in terms of the particle rigidity. Having these, we would use an algebra-manipulating program to adjust the parameters such that the dependence on rigidity is eliminated.

The second approach considered was to use an orbit integration program to simultaneously calculate the closed orbits and tunes about those closed orbits for several widely separated momenta and to fit the magnet parameters to minimize (eliminate) the differences with momenta.

The third approach, which is relatively simple, is to solve the linear problem with a matrix program with good online graphics and then use Martin Berz's code, COSY INFINITY,[4] to extend the solution systematically to higher and higher orders in $\Delta p/p$. This is the approach adopted.

My version of the LATTICE code was used to adjust three gradient magnets with normal entry and exit to achieve the desired tunes and the desired dispersion-free drift space, subject to minimizing the maxima for the Courant and Snyder beta functions. The bends in the three magnets were arbitrarily chosen to be 10° in, 20° out, and 70° in, respectively, for a net bend of 60°. The three gradients and the magnet locations were adjusted by the code to meet the specified conditions. Although this approach suffices to show the feasibility of the method, one would want to consider different bending angles and the use of edge angles and, perhaps, edge curvature in a real design. Separating the big bend into two magnets, perhaps with reverse "gulley" fields is another consideration. Once a solution has been obtained, LATTICE can be instructed to write an input file for the COSY INFINITY code. An example, which is by no means optimized, is shown in Fig. 2.



The COSY INFINITY code reads the LATTICE deck and produces the same linear results. The next step is to have COSY INFINITY adjust the radial second derivatives of the magnet fields to obtain zero chromaticity and zero second order chromatic dependence of the displacement of the closed orbit in the dispersion-free insertion.

Extending the solution to other momenta requires that we expand each of the linear matrix elements in a power series in the displacements, slopes, and relative momentum shift. However, by starting the sector at the center of the dispersion-free drift space, the displacements and slopes at that point are, by definition, zero. This is an important point because we thus need only consider the pure chromatic derivatives. Moreover, because of the symmetry of the lattice, the Courant and Snyder α functions vanish at the center of the dispersion-free drift space. Thus, in order to achieve a dispersion free drift space with fixed tunes over the entire range of momenta from injection to extraction, we need to achieve the following conditions:

$$\frac{\partial^n M_{11}}{\partial p^n}=0;\quad \frac{\partial^n M_{33}}{\partial p^n}=0;\quad \frac{\partial^n M_{16}}{\partial p^n}=0;$$

$$n=1,2,3,4,\ldots$$

Here, the M_{ij} are the matrix elements in TRANSPORT notation.

Then, systematically, COSY INFINITY is set to calculate to the next higher order and requested to zero the pure chromatic derivatives at that order by optimizing the next higher order radial derivative of the magnetic fields. This process is repeated until we are sure that the order for which we have full correction is adequate for the total momentum spread to be accommodated (injection through extraction).

The procedure can be written as follows:

$$\frac{\partial^2 B_k}{\partial x^2} \Rightarrow M_{116}=0;\ M_{336}=0;\ M_{166}=0;$$

$$\frac{\partial^3 B_k}{\partial x^3} \Rightarrow M_{1166}=0;\ M_{3366}=0;\ M_{1666}=0;$$

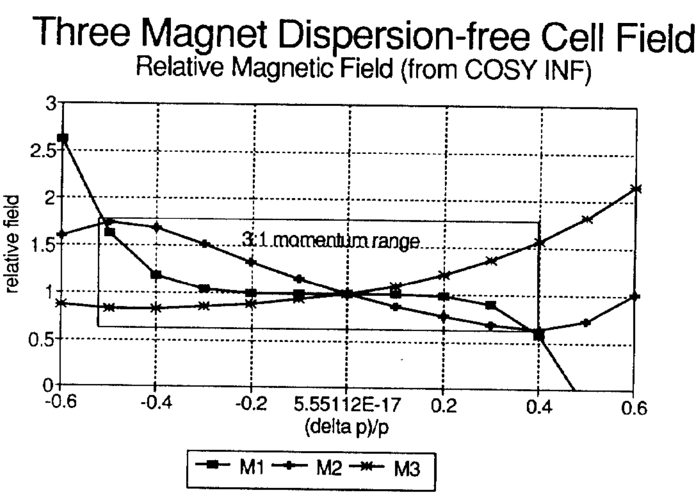
$$\frac{\partial^4 B_k}{\partial x^4} \Rightarrow M_{11666}=0;\ M_{33666}=0;\ M_{16666}=0...$$

Here, the index k represents the three (or more) different bending magnets whose field we have at our disposal. We have this flexibility because we have dropped the requirement that the field scale with $\langle R\rangle^k$. Having the field and its radial derivatives, we can, of course, obtain its radial profile. The solution, which yields fixed tunes and dispersion-free drifts through the fifth order in the relative momentum shift is:

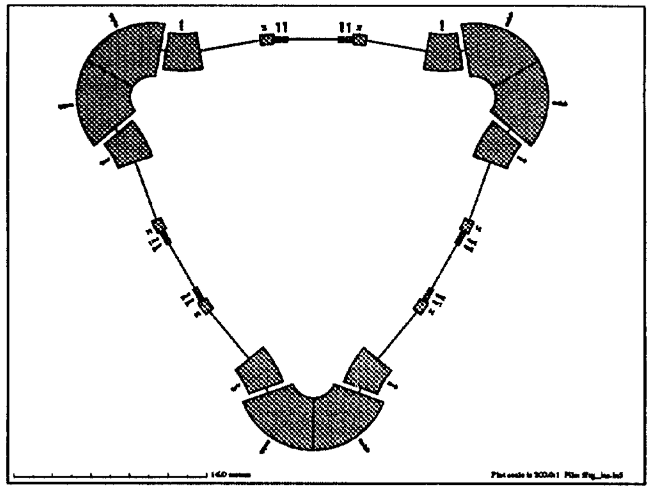
$$\frac{B_1}{B_0}=1+2.4\frac{x}{\rho}-78.7\left(\frac{x}{\rho}\right)^2-1.83E5\left(\frac{x}{\rho}\right)^3 -8.49E7\left(\frac{x}{\rho}\right)^4-3.43E10\left(\frac{x}{\rho}\right)^5+...$$

$$\frac{B_2}{B_0}=1-7.77\frac{x}{\rho}+35\left(\frac{x}{\rho}\right)^2-144\left(\frac{x}{\rho}\right)^3 -785\left(\frac{x}{\rho}\right)^4+5.1E4\left(\frac{x}{\rho}\right)^5+...$$

$$\frac{B_3}{B_0}=1+1.4\frac{x}{\rho}+3.7\left(\frac{x}{\rho}\right)^2+7.5\left(\frac{x}{\rho}\right)^3 +12\left(\frac{x}{\rho}\right)^4-17.3\left(\frac{x}{\rho}\right)^5+...$$



When graphed over the magnet region used by a momentum range of 3:1, the field profiles are as shown in Fig. 3. This example, which is by no means optimized, has a circumference of 106.1 meters and radial and vertical tunes of 2.25 and 2.30, respectively. The floor plan for this ring is shown in Fig. 4.



The magnets close to the dispersion-free straight section are small, whereas those well away from the insertion are large because of the dispersion there.

The solution sequence can be modified so that COSY INFINITY determines the multipole components for the corrections magnets after the corresponding components for the bending magnets have been determined. This is yet to be done.

We have created a simple ring with fixed fields that contains a wide momentum range of 3:1 from injection to extraction with fixed tunes and dispersion-free insertions. The multipole correction magnets in the insertions can correct several resonances thereby providing a large dynamic aperture.

IV. REFERENCES

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[2] D. W. Kerst, K. R. Symon, L. J. Laslett, L. W. Jones, and K. M. Terwilliger, *"Fixed Field Alternating Gradient Particle Accelerators"*, CERN Symposium, 32 (1956)

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