STORED BEAM LIFETIME EVALUATION FORMULAE FOR ELECTRON STORAGE RINGS

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Abstract

Two simple analitical formulae to evaluate a lifetime of a stored electron (positron) beam at storage ring have been derived. Comparison between the lifetimes measured under typical operation conditions and calculated values has been carried out.

I. INTRODUCTION

Traditionally, to calculate a beam lifetime along with a bunch volume, revolution frequency, harmonic number, electron energy, bending radius and current, the external parameters such as a vacuum chamber aperture, residual gas pressure and RF voltage have been used. [1], [2], [3]. However, discrepancies more than an order of magnitude may be observed between a calculated and experimental data [2], [4]. In our case we tried to obtain a lifetime estimations based only on beam parameters. Previously, such kind situation has experimentally acknowledged at TANTALUS [5] ring.

II. THE FORMULAE DERIVATION

Consider train of electron bunches moving throughout a magnetic field media supplied by bending magnets. It has been known that in plasma physics the relation of a plasma to magnetic pressure is commonly used to describe plasma behavior. By analogy of plasma pressure we introduce a single bunch electrons pressure P_e as:

$$P_e = \frac{N \cdot E}{V \cdot K_b}$$

Where: N – number of beam electrons, E – electron energy, V – bunch volume, K_b – number of used bunches.
Magnetic pressure P_m can be written as:

$$P_m = \frac{B^2}{2\mu_o},$$

Where: B – magnetic induction of bending magnets, μ_o – permeability of free space.

Then relation of electrons to magnetic pressure β can be written as:

$$\beta = \frac{2 \cdot N \cdot E \cdot \mu_o}{V \cdot B^2 \cdot K_b}$$

Furthermore, let an electron be considered as a sphere of radius r, such as:

$$r = \frac{\lambda_c}{2\gamma},$$

Where: λ_c – Compton wavelength, λ_c=2,42.10^{-12} m,γ – relativistic factor, $\gamma=E/E_o$, E_o – rest energy.

Also, let a scattering crossection of an electron be dependant on only this radius.

Finally, let for β larger than the unit electron bunch behavior be similar to that of ideal gas cloud. When β is lower than the unit a separate spherical electron moving throughout a «medium» whose density is product of a single bunch density and number of used bunches must be considered. The latter condition, in turns, requires a some kind of density memory in a relativistic orbit region. β>1. Lifetime is calculated by means of a commonly used formula [6]:

$$\tau = \frac{1}{n \cdot \sigma \cdot c},$$

Where: τ – beam 1/e lifetime, n – density of particles, σ – scattering crossection of a single spherical electron, c – speed of light.

$$\sigma = \pi d^2 = \frac{\pi \lambda_c^2}{\gamma^2},$$

$$d = 2 \cdot r,$$

$$n = \frac{N}{V \cdot K_b},$$

$$V = (4 \cdot \pi)^{\frac{3}{2}} \cdot \sigma_x \cdot \sigma_z \cdot \sigma_L$$

Where: σ_x, σ_z, σ_L – bunch sizes measured in bending magnets,

$$\sigma_{X,Z,L} = \frac{S_{X,Z,L}}{2,35}$$

$$\sigma_L = c \cdot \tau_L,$$

Where: $S_{x,z,L}$ – bunch FWHM sizes, τ_L – bunch duration.

The followings relations also taken into account:

$$\gamma = 1957 \cdot E(GeV),$$
$$R(m) \cdot B(T) = 3.34 \cdot E(GeV),$$
$$I(A) = 1.6 \cdot 10^{-19} \cdot N \cdot \omega_o(Hz),$$

Where: R – bending radius, I – stored beam current, ω_o – bunch revolution frequency.

On transforming the formula and β for practical use, we have:

$$\tau(hours) = \frac{1{,}41 \cdot K_b \cdot \omega_o(MHz) \cdot \sigma_x(mm) \cdot \sigma_z(mm) \cdot \sigma_L(mm) \cdot E^2(GeV)}{I(mA)}$$

$$\beta = \frac{5 \cdot R^2(m) \cdot I(mA)}{\sigma_x(mm) \cdot \sigma_z(mm) \cdot \sigma_L(mm) \cdot K_b \cdot \omega_o(MHz) \cdot E(GeV)}$$

β<1. Write a total drag force F, which is applied to spherical electron moving in a medium of the density ρ as [7];

$$F = C_D \cdot A \cdot \frac{1}{2} \rho \cdot c^2$$

Where: A – characteristic area, C_D – coefficient of drag.

According to the above suppositions:

$$\rho = \frac{N \cdot E}{V \cdot c^2}$$

$$A = \pi \cdot \left(\frac{\lambda_c}{2\gamma}\right)^2$$

Approximately constant over transcritical region of its dependence on a Reynolds number Re, coefficient of drag C_D [8], [9] equal to 0,15 has been choosen.

Energy E_t, which the electron lost during one turn in a storage ring magnetic field is:

$$E_t = 2\pi \cdot R \cdot F$$

Full energy loss E_τ during lifetime τ is:

$$E_\tau = E_t \cdot \omega_o \cdot \tau$$

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Due to this process a bending radius R become smaller. It is supposed that the electron should be finally lost when it has come full measured bunch width $3\sigma_x$ in the radial direction.

Energy change ΔE due to bending radius reduction is:

$$\Delta E = \frac{3\sigma_x E}{R}$$

We obtain τ equating E_τ to ΔE:

$$\tau = \frac{1{,}5\sigma_x \cdot E}{\pi \cdot R^2 \cdot C_D \cdot A \cdot \rho c^2 \omega_o},$$

$$\tau(\text{hours}) = \frac{10{,}46 \cdot \sigma_x^2(\text{mm}) \cdot \sigma_z(\text{mm}) \cdot \sigma_L(\text{mm}) \cdot E^2(\text{GeV})}{I(\text{mA}) \cdot R^2(\text{m})}$$

III. COMPARISON WITH EXPERIMENTAL DATA

In the table storage rings parameters, maximum achieved beam lifetime – τ, calculated lifetime – τ_c , discrepancies between these lifetimes – $\Delta\%$ are listed.

Ring name	E GeV	R m	ω_0 MHz	K_b	σ_x mm	σ_z mm	σ_L mm	I mA	τ hours	β	τ_c hours	$\Delta\%$
ACO	0.54	1.10	13.60	2.00	1.50	0.30	300.00	100.00	16.00	55.00	15.10	5.60
	[10]	[11]	[11]	[10]	[10]	[10]	[10]	[10]	[10]			
ADONE	1.50	5.00	2.85	3.00	0.80	0.40	45.00	100.00	4.00	6.70	3.90	2.50
	[12]	[11]	[11]	[12]	[12]	[12]	[12]	[12]	[12]			
ALLADIN	0.80	2.08	3.36	15.00	0.55	0.07	255.00	100.00	3.90	3.37	4.53	16.00
	[11]	[11]	[11]	[11]	[13]	[14]	[11]	[11]	[11]			
AURORA	0.65	0.50	95.40	2.00	1.20	0.14	30.00	4.00	20.00	0.01	26.60	33.00
	[15]	[15]	[15]	[15]	[15]	[15]	[16]	[15]	[15]			
BESSY	0.77	1.78	4.80	104.00	2.00	1.00	70.00	100.00	6.00[1)]	0.03	5.50	7.50
	[17]	[17]	[17]	[17]	[17]	[17]	[4]	[17]	[17]			
	0.75	1.78	4.80	104.00	0.20	0.50	45.00	600.00	3.00	5.60	3.00	1.00
	[18]	[17]	[17]	[18]	[18]	[18]	[11]	[18]	[18]			
CHESR	5.44	32.00	0.39	6.00	1.08	0.19	16.00	80.00	4.00	9800.0	4.00	1.00
	[19]	[19]	[19]	[19]	[19]	[19]	[19]	[19]	[19]			
DCI	1.85	3.82	3.17	1.00	2.72	1.06	300.00	300.00	40.00	4.30	44.10	10.00
	[11]	[11]	[11]	[11]	[20]	[20]	[11]	[11]	[11]			
MAX	0.55	1.20	9.26	54.00	0.20[2)]	0.20[2)]	45.00	130.00	2.00	1.90	2.95	48.00
	[11]	[11]	[11]	[11]	[21]	[21]	[11]	[11]	[11]			
N-100	0.07	0.50	52.24	1.00	1.00	0.75	600.00	700.00	0.14	0.52	0.13	6.00
	[22]	[22]	[22]	[22]	[23]	[23]	[24]	[22]	[22]			
NIJI-I	0.16	0.70	22.60	7.00	1.10	1.80	170.00	200.00	0.83	0.05	1.05	27.00
	[25]	[25]	[25]	[25]	[26]	[25]	[27]	[25]	[25]			
NSLS-VUV	0.75	1.90	5.88	1.00	0.85	1.00	84.00	155.00	1.83	8.90	2.12	15.00
	[28]	[28]	[11]	[29]	[29]	[29]	[30]	[29]	[29]			
	0.75	1.90	5.88	6.00	0.51	1.30	102.00	286.00	5.33	2.90	6.46	21.00
	[28]	[28]	[11]	[29]	[29]	[29]	[30]	[29]	[29]			
	0.75	1.90	5.88	7.00	0.58	0.20	255.00	800.00	1.66	16.00	1.19	28.00
	[31]	[11]	[11]	[31]	[32]	[32]	[31]	[33]	[33]			
NSLS-XRAY	2.50	6.87	1.75	25.00	0.38	0.12	90.00	57.00	22.50	2631.00	28.40	26.00
	[11]	[11]	[11]	[29]	[28]	[28]	[11]	[29]	[29]			
PEP	7.10	165.00	0.14	68.00	0.29	0.08	5.00	34.00	1.66	$4 \cdot 10^5$	2.30	39.00
	[34]	[11]	[34]	[34]	[35]	[35]	[35]	[34]	[34]			
PF-RING	2.50	8.66	1.61	208.00[3)]	1.70	0.25	15.00	300.00	55.00	14.70	58.40	6.20
	[37]	[37]	[37]	[37]	[37]	[38]	[37]	[37]	[37]			
SIBERIA-I	0.45	1.00	34.50	1.00	1.62	0.13	300.00	100.00	2	0.32	2.10	5.00
	[39]	[40]	[40]	[40]	[39]	[39]	[39]	[39]	[39]			
SOR	0.38	1.10	17.20	1.00	0.98	0.16	81.00	10.00	1.66	0.72	1.55	7.00
	[41]	[41]	[41]	[41]	[41]	[41]	[41]	[41]	[41]			
	0.38	1.10	17.20	7.00	1.95	0.80	170.00	200.00	4.00	0.10	3.25	20.00
	[42]	[42]	[42]	[42]	[42]	[42]	[42]	[42]	[42]			
SORTEC	1.00	2.78	6.55	18.00	1.00	1.00	90.00	200.00	56.00	0.70	74.30[4)]	32.70
	[43]	[43]	[43]	[43]	[44]	[44]	[43]	[44]	[44]			
SRS-2	2.00	5.55	3.12	160.00	1.35	0.12	20.00	150.0	45.00	7.13	61.00	35.00
	[45]	[45]	[45]	[45]	[45]	[45]	[45]	[45]	[45]			
SUPER ALIS	0.60	0.66	17.80	7.00	1.26	0.19	72.00	100.00	1.94	0.17	1.88	3.00
	[46]	[46]	[46]	[43]	[46]	[46]	[43]	[46]	[46]			
SURF-2	0.28	0.84	57.00	2.00	0.64	0.04	300.00	200.00	0.33	2.60	0.52	57.00
	[47]	[48]	[48]	[48]	[48]	[48]	[48]	[47]	[47]			
	0.28	0.84	57.00	2.00	1.30	0.76	300.00	200.00	2.50	0.07	2.34	6.40
	[48]	[48]	[48]	[48]	[49]	[48]	[48]	[48]	[48]			

Ring name	E GeV	R m	ω_0 MHz	K_b	σ_x mm	σ_z mm	σ_L mm	I mA	τ hours	β	τ_c hours	Δ%
TANTALUS	0.24	0.63	32.00	1.00	0.75	0.25	300.00	20.00	3.00	0.09	3.17	5.80
	[50]	[50]	[50]	[50]	[50]	[50]	[50]	[50]	[50]			
TERAS	0.60	2.00	9.55	18.00	0.85	0.21	115.00	70.00	29.00 4)	0.60	25.60 5)	11.00
	[51]	[51]	[51]	[51]	[51]	[51]	[51]	[51]	[52]			
TRISTAN AR	5.70	23.20	0.80	1.00	1.24	0.12	20.00	30.00	2.66	6000.0	3.60	35.00
	[53]	[11]	[11]	[53]	[54]	[54]	[53]	[53]	[53]			
UVSOR	0.75	2.20	5.60	16.00	0.41	0.29	60.00	150.00	4.00	7.50	3.40	15.00
	[55]	[55]	[55]	[55]	[55]	[55]	[55]	[55]	[55]			
VEPP-1	0.04	0.43	110.00	1.00	1.20	0.40	60.00	100.00	0.05	0.67	0.04	28.00
	[56]	[56]	[56]	[56]	[56]	[56]	[56]	[56]	[56]			
	0.13	0.43	110.00	1.00	0.35	0.05	40.00	100.00	0.01	9.00	0.02	42.00
	[56]	[56]	[56]	[56]	[56]	[56]	[56]	[56]	[56]			
VEPP-2M	0.70	1.22	16.60	1.00	0.97	0.60	40.00	200.00	1-2	6.70	1.33	11.00
	[57]	[57]	[57]	[57]	[57]	[57]	[57]	[57]	[57]			
VEPP-3	2.00	6.16	4.00	2.00	0.90	0.06	300.00	250.00	3-4	182.00	2.92	16.00
	[58]	[58]	[59]	[58]	[58]	[58]	[59]	[58]	[58]			
	0.35	6.16	4.00	1.00	0.45	0.20	100.00	20.00	0.20	301.00	0.31	55.00
	[60]	[61]	[61]	[61]	[60]	[60]	[60]	[61]	[61]			

1) halftime, 2) calculated size, 3) partial filling, 4),5) calculated assuming β>1

Dependence of σ_x, σ_z, σ_L, on current I has been taken into account using the beam sizes observed in presence of the current subjected to estimation, provided that such dependence was found. For SORTEC and TERAS rings the formulae give correct results if the gaseous model is applied. We belive that β can be properly changed in these special cases. It should be also mentioned that for TERAS calculated lifetime equal to only the expected one, which five times longer the achieved values if a residual gas wall desorption is suppressed. In many cases the results are in accordance with previously estimated one on the base of the Touschek scattering.

The discrepancies between the experimental and calculated data have been found for SuperACO, SPEAR, PEP (15 GeV), VEPP-4, SPS-1, DORIS, C.E.A. storage rings.These machines were not included in the table. About fifteen new rings were not mentioned because of shortage in the presently available data set.

The uncertainty of measurements of each beam size was evaluated in 5% r.m.s. The mean table uncertainty is 20%.

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