

# Simulation Study of Ion Trapping in PLS Storage Ring\*

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## Abstract

Ion trapping phenomena in the PLS storage ring have been studied by computer simulation in the view point of tune shift effect of the electron beam. Ions created by strong synchrotron radiation in the vacuum chamber are trapped and accumulated in the electron beam potential, and then the beam and the ions move in the potential of each other. Tune shifts in terms of various beam currents show a stable-unstable band structure representing the relationship of the tunes. For the unstable tune values, a partial beam filling and clearing electrodes are used to clear the ions.

## 1 INTRODUCTION

Circulating electron beams in the storage ring generate strong synchrotron radiation. Even though the storage ring maintains an ultra-high vacuum condition, strong radiation and beam itself ionize the residual gas molecules. These ions can be trapped in the electron beam path. This phenomenon, called ion trapping, can lead to degradation of beam performances, and sometimes, can cause beam instability or beam blow-up. Earlier studies on the motion of single ions implied many things about the mechanism and consequences of the ion trapping phenomenon. In this report, the simulation results using many ions and various filling patterns are presented, especially with respect to the tune shift.

## 2 THEORY

### 2.1 Electric Field

A circulating electron beam has the following charge distribution:

$$\rho(x, y) = -\frac{eN_e}{2\pi\sigma_x\sigma_y} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2}\right)}, \quad (1)$$

where  $\sigma_x$  and  $\sigma_y$  are beam sizes in  $x$  and  $y$  direction, respectively, and  $N_e$  for the number of electrons in the beam bunch. This leads to the electric fields such as:

$$E_x - iE_y = -\frac{ieN_e e^{(b-ia)^2}}{2\epsilon_0\sqrt{2\pi}\Delta\sigma^2} [\text{Erf}(b-ia) - \text{Erf}(b/r-iar)], \quad (2)$$

where the error function  $\text{Erf}(z)$  is defined as

$$\text{Erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-\xi^2} d\xi. \quad (3)$$

Here,

$$\Delta\sigma^2 = \sigma_x^2 - \sigma_y^2, a = \frac{x}{\sqrt{2\pi}\Delta\sigma^2}, b = \frac{y}{\sqrt{2\pi}\Delta\sigma^2},$$

and  $r = \sigma_x/\sigma_y$ . Instead of using the complex error function in Ref. [1], we used a real error function. Thus, the result is numerically calculable by the Mathematica™ program.

### 2.2 Equation of Motion

The typical length of the electron beam is about 32 ps for PLS storage ring. This is much shorter than the minimum interaction interval  $t_b$  which is 2 ns. The interaction between ions and electron beams can, thus, be considered to be instantaneous. Ions experience impulse due to the passing electron beam such as

$$\begin{aligned} m_i \Delta \dot{x}_i &= q_i/c E_x(x_i - x_b, y_i - y_b), \\ m_i \Delta \dot{y}_i &= q_i/c E_y(x_i - x_b, y_i - y_b). \end{aligned} \quad (4)$$

At the same time, the motion of the electron bunch is also affected by these ions by

$$\begin{aligned} \gamma m_e N_e \Delta \dot{x}_b &= -q_i/c E_x(x_i - x_b, y_i - y_b), \\ \gamma m_e N_e \Delta \dot{y}_b &= -q_i/c E_y(x_i - x_b, y_i - y_b). \end{aligned} \quad (5)$$

Here,  $m_i$ ,  $m_e$  are masses for the ion and the electron, respectively.  $q_i$  is the electric charge of the ion, and  $\gamma$  is the relativistic factor.  $x_i$  ( $y_i$ ) and  $x_b$  ( $y_b$ ) are  $x$  ( $y$ ) locations for the ion and the center of electron beam, respectively. After the instantaneous interaction, ions drift freely during  $t_b$  or until the next electron bunch arrives in the case of partial filling. The electron beam moves along the beam trajectory with a constant speed  $c$ . During the time interval  $t_b$ , the phase advances of the electron beam are  $\Delta\phi_x = \nu_x ct_b$  in  $x$  direction and  $\Delta\phi_y = \nu_y ct_b$  in  $y$  direction. Here  $\nu_x$  and  $\nu_y$  are tunes in  $x$  and  $y$  direction.

### 2.3 Tune

Tunes of the electron beam can be calculated by Fourier transform of the beam motion. In  $y$  direction, for example, we define the Fourier amplitude as

$$A(\omega) = \int_{-\infty}^t y(t') e^{-i\omega(t'-t)} W(t-t') dt', \quad (6)$$

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where the weight function  $W$  is

$$W(t-t') = \frac{1}{\tau} e^{-(t-t')/\tau}. \quad (7)$$

The weight function is introduced to save computational time. The  $\omega$  resolution is given as  $d\omega \sim 2\pi/\tau$ . If we choose  $\tau \sim 10 \mu\text{sec}$ , about 0.1 MHz frequency resolution can be obtained in the simulation. The tune is obtained by plotting  $A(\omega)$  in terms of the frequency.

### 3 SIMULATION

#### 3.1 Parameters

The simulation is based on PLS storage ring parameters, which are summarized in Table 1. Since ions move very slowly compared to the electron beam, the interaction between ions and electron beams is assumed to be confined to the  $xy$  plane. The electron beam size is also fixed during the simulation. There is no external electric or magnetic field, so the ion trapping takes place in the area such as long insertion straight section where the vacuum pressure is relatively poor. Some important values used in the simulation are listed in Table 2.

Table 1: Parameters for PLS storage ring

Beam Energy	2 GeV
Circumference	280.56 m
Beam Current	400 mA (multibunch)
Natural Emittance	12.1 nm-rad
Natural Energy Spread	$6.8 \times 10^{-4}$
Harmonic Number	468 ( $= 2^2 \times 3^2 \times 13$ )
RF Frequency	500.087 MHz
Betatron Tunes	
Horizontal	14.28
Vertical	8.18
Synchrotron Tune	0.011
Momentum Compaction	$1.81 \times 10^{-3}$
Natural Bunch Length	5.04 mm (rms)
Beam Size†	
Horizontal	348 $\mu\text{m}$
Vertical‡	66 $\mu\text{m}$
Damping Time	
Horizontal	16.6 msec
Vertical	16.6 msec
Longitudinal	8.34 msec

† at the center of insertion straight section.

‡ 10% emittance ratio is assumed.

#### 3.2 Ions

Number of ions created due to the synchrotron radiation can be determined by the neutralization  $\eta$  which is defined as a ratio of ions to the electrons. Here, we assume that  $\eta$  is 0.5%. In the simulation, 117 ions of charge  $Zq_i$  are used. The multiplication factor  $Z$  is determined by the neutralization  $\eta$  and the electron beam current  $I$ . The

mass number of the ion is 28 which represents a nitrogen ion ( $N_2^+$ ) or a carbon monoxide ion ( $CO^+$ ). These are usual background ions found in the storage ring vacuum chamber.

Table 2: Simulation parameters

Beam Size (rms)	
Horizontal ( $\sigma_x$ )	0.35 mm
Vertical ( $\sigma_y$ )	0.16 mm
No. of Ions	117
Ion Mass	28
Neutralization	0.5%
Time Step ( $t_b$ )	1/500.087 $\mu\text{sec}$

#### 3.3 IONTRAP Code

The simulation code IONTRAP is written in the C++ language. Before the code calculates equations of motion, electric fields shown in Eq. (2) are calculated by Mathematica, and the results are used as an input to the IONTRAP. This reduces computational time drastically, so the code runs on an ordinary personal computer. The electric field profile is shown in Figure 1.

## 4 RESULTS

Effects due to the ion-beam interaction can be observed when the beam circulates the storage ring within 100 turns. One hundred turns are equivalent to about 0.1 msec for the PLS storage ring. It is well known that leaving gaps between consecutive bunches can cure ion trapping phenomena [2]. Figure 2 shows tune changes in terms of different gap sizes for different beam currents. The tune change is as bad as 0.1 in this simulation. As expected, the tune change is significant when the gap is not large enough. For a fixed beam current, the tune shift is smaller as the gap is larger, as shown in Figure 4. When the gap size is larger than 5% of the whole bucket, the tune change is reduced to within 0.01. However, Figure 4 shows that there is still a certain range of beam currents which gives relatively larger tune shifts when the gap size is fixed. It suggests that a sequential bucket filling is better than stacking electrons into the partially filled buckets during the injection period when the ion trapping phenomena are considered. A clearing electric field is applied in order to reduce the tune change further as shown in Figure 5. About 1 kV/m DC potential is necessary to decrease the tune change to less than 0.001.

## 5 REFERENCES

- [1] M. Bassetti and G.A. Erskine, "Closed Expression for the Electrical Field of a Two-dimensional Gaussian Charge," CERN-ISR-TH/80-06 (1980).
- [2] M. Q. Barton, "Ion Trapping with Asymmetric Bunch Filling of the NSLS VUV Ring," *Nucl. Inst. and Methods*, **A243**, p. 278 (1986).

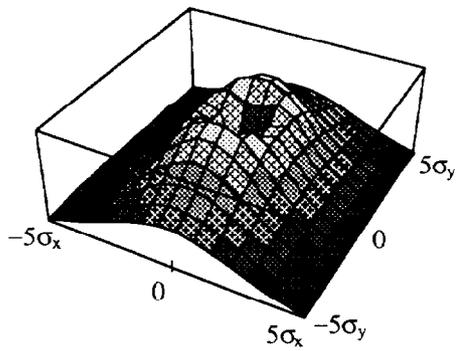


Fig. 1: Electric field profile.

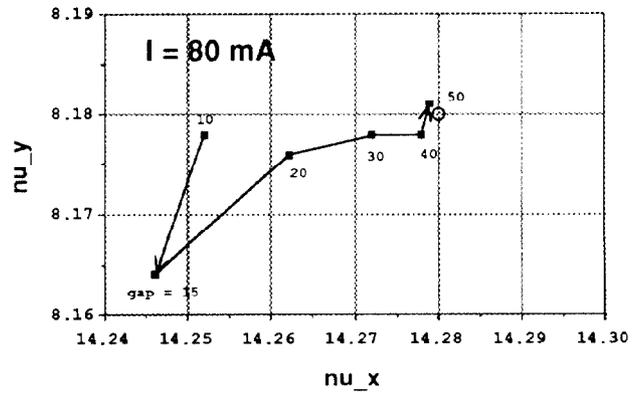


Fig. 3: Tune change in terms of gap size at I=80 mA. The open circle is the operating point.

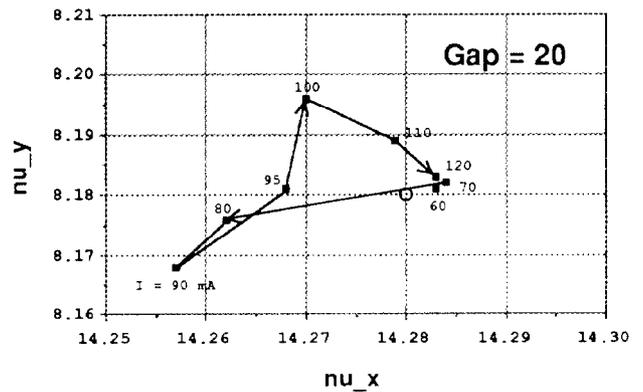
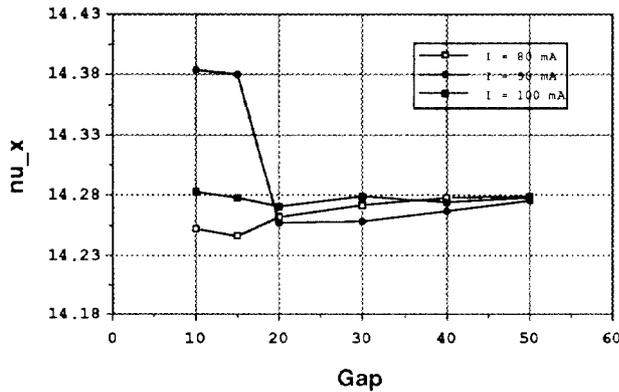


Fig. 4: Tune change in terms of beam current at gap size is 20. The open circle is the operating point.

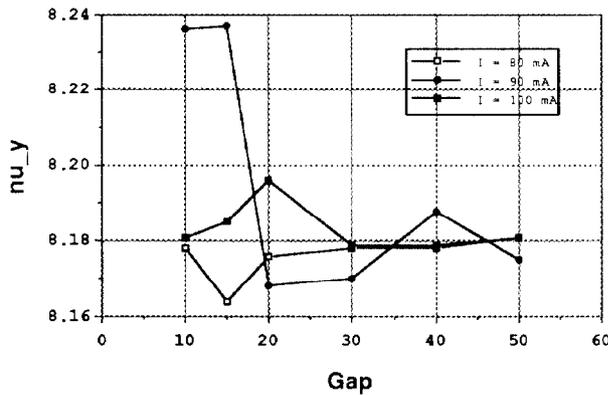


Fig. 2: Tune changes in x(upper) and y(lower) direction.

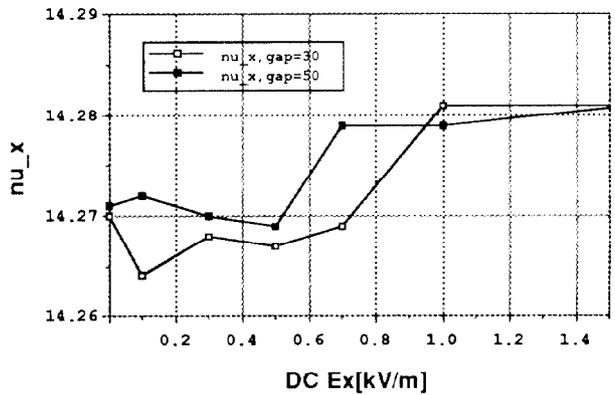


Fig. 6: Tune changes when the clearing voltage is applied.