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# Experimental Study of Longitudinal Dynamics of Space-Charge Dominated Parabolic Bunches \*

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#### Abstract

Longitudinal dynamics of space-charge dominated parabolic beams has been studied at the University of Maryland Electron Beam Transport Facility. In this paper we report results which emphasis experiment, while numerical simulation studies are presented in another paper in this proceedings [1]. Excellent agreement is found between experiment, simulation and theoretical prediction.

## I. INTRODUCTION

The longitudinal dynamics of space-charge dominated beams has been theoretically, experimentally and numerically studied at the University of Maryland. One of the objectives is to study the parabolic bunch which is important to both theory and application. The longitudinal bunch model with parabolic line charge density has been developed by L. Smith and D. Neuffer [2,3]. Since the self-forces are linear this model plays a similar role for the longitudinal dynamics as the uniform density K-V distribution for the transverse dynamics. We note that a parabolic beam was observed as the final equilibrium state at the Indiana University Cyclotron Facility [4]. A systematic experimental and numerical study will check the validity of the theoretical model and can contribute to better understanding of the beam physics.

## II. THEORETICAL MODEL

The longitudinal space-charge field can be approximately calculated as follows [5,6]

$$E_{z}(z, s) = -\frac{g}{4\pi\varepsilon_{0}\gamma^{2}}\frac{\partial\Lambda(z, s)}{\partial z},$$
 (1)

where z is the distance from the center of the beam bunch,  $\Lambda(z,s)$  is the line charge density at a longitudinal distance s traveled by the beam bunch center, and the geometry factor g is a function of the ratio of the pipe radius b over the beam radius a, where 2Z is the bunch length. g can also depend on the ratio Z/a when Z/a become comparable to unity [6]. Neuffer found a self-consistent distribution function of the longitudinal phase space that gives a linear force and velocity distribution and a preserved parabolic bunch shape. It is a solution of the Vlasov equation. The evolution of the bunch length is described by the longitudinal envelope equation:

$$\frac{d^{2}Z}{ds^{2}} - \frac{2gZ_{i}I_{p}(0)}{\beta I_{0}}\frac{1}{Z^{2}} - \frac{\varepsilon_{L}^{2}}{Z^{3}} = 0, \qquad (2)$$

where 2Zi and  $I_P(0)$  are the initial bunch length and peak current, respectively,  $I_0=1.7x10^4$  amperes is the characteristic current for electrons,  $B_0=v_0/c$ ,  $v_0$  is the beam center velocity, c is the speed of light, and  $\varepsilon L$  is the longitudinal emittance of the beam. If the space-charge term is dominant over the emittance term, (20 times larger in this experiment), then an analytical expression for Z is given in Ref. [7]. If the initial velocity is a linear function of z, then it remains linear and its slope is determined by Z'.

In this one-dimensional model the longitudinal and transverse dynamics can be coupled through the geometry factor g, which reflects the boundary condition and bunch dimensions.

## III. EXPERIMENTAL SETUP AND METHOD

The experimental configuration is illustrated in Ref. [7]. The electron bunch is produced by a newly developed electron beam injector which consists of a variable-perveance electron gun, an induction acceleration module, and three matching lenses. The parabolic beam profiles can be generated by controlling the pulse shape applied to the grid of the gun. An initial velocity tilt can be imparted to the beam by the timevarying acceleration voltage of the induction gap, as shown in Fig. 1. Then, the beam is matched into a 5-m long periodic focusing channel consisting of 36 short solenoidal lenses.

The typical initial beam parameters in this experiment are 2.5 keV energy, 20 mA peak current, 26 ns pulse duration. The beam current profiles are measured by fast beam current monitors and a fast Digitizing Signal Analyzer. The beam center velocity  $cB_0$  and energy can be obtained by the time of flight method. The acceleration gap voltage at the beam center can be calculated by subtracting the 2.5 keV beam energy before the gap. Matching it in Fig. 1, one can figure out the initial velocity tilt.

The parabolic beam is produced and matched into the channel. Without the induction acceleration the bunch expands and the current reduces, while with the acceleration the bunch is compressed to a waist then expands after that.

## IV. EXPERIMENTAL RESULTS

A. Longitudinal envelope measurement

To verify the envelope equation, the beam current profiles are measured at 5 different locations, shown in Fig. 2 for the expansion case and in Fig. 3 for the compression case. They are fitted by ideal parabolic curves to obtain the bunch width, i.e. the longitudinal envelope. The envelope evolution is plotted in Fig. 4 for different initial velocity tilts in comparison with the calculation of the envelope equation. They are in very good agreement.

<sup>\*</sup> Research Supported by the U.S. Department of Energy.

### B. Velocity distribution measurement

To check the linearity of the space-charge force and velocity distribution the beam energy is measured at 3 different locations. A typical beam velocity vs. time is plotted in Fig. 5 and fitted by a straight line. The slopes are plotted and compared with the envelope calculation in Fig. 6 for the expansion case and in Fig. 7 for the compression case. Again, good agreement is found.

#### C. The transverse coupling

To explore the transverse coupling effect we have done the measurements with the same longitudinal conditions but different transverse focusing, i.e. different beam radius. The experimental results plotted in Fig. 8 show that due to different transverse focusing the envelope evolution changes but still obeys the envelope equation, however g values are different for each case.

In calculating the theoretical curves we assumed that the g-factor remains constant along the channel. This assumption gave surprisingly good agreement with the experimental data. However, since the beam radius varies along the bunch and also along the channel one does not expect g to remain constant. In practice it is not an easy task to determine the g factor without experimental measurement or 2-D computer simulation. Nevertheless, work is underway to calculate the g factor by numerical methods.

We also have looked into how critical the transverse matching condition is to the longitudinal beam behavior. In the experiment we deliberately mismatched the beam by changing the matching lens focusing. We found that the current profiles showed very little change over quite a wide range of mismatch conditions. When the mismatch became too large we started losing beam. These results are confirmed by a 2-D simulation. In the simulation the longitudinal dynamics remains almost the same though there is a very significant transverse mismatch.

### VI. CONCLUSION

An experimental study of a space-charge dominated parabolic beam has been carried out. Excellent agreement has been achieved between the experiment and the longitudinal envelope model. In the experiment we have demonstrated that the parabolic current profile is preserved and the velocity distribution remains linear as a result of the linear space-charge force. A constant g along the transport channel used in the 1-d model was found to be a good approximation. However, g changes significantly when the transverse focusing changes and can be measured or calculated by a 2-D code. The experiment shows that the transverse mismatching has little effect on the longitudinal dynamics, and this was confirmed by the 2-D simulation.

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Fig. 1 Voltage waveform of induction gap (top) and current pulse shape of initial parabolic beam (bottom).



Fig. 2 Current profiles of parabolic bunch without initial velocity tilt at five different locations of the channel.



Fig. 3 Current profiles of parabolic bunch with initial velocity tilt at five different locations of the channel.

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Fig. 4 Relative parabolic bunch lengths Z/Zi vs. drift distance with different initial velocity tilts, where the circles represent experiment and the solid curves theory.



Fig. 5 Velocity distribution of parabolic bunch without initial velocity tilt measured at s=3.746 m, where the circles represent experiment and the solid line is linear fitting.



Fig. 6 Slopes of linear velocity distribution without initial velocity tilt, measured at s=0.473, 3.746, and 5.42 m. The circles represent experiment and the solid curve theory.



Fig. 7 Slopes of linear velocity distribution with initial velocity tilt Z'i=-0.06, where the circles represent experiment and the solid curve theory.



Fig. 8 Relative bunch length vs. drift distance with Z'i=-0.092 for different transverse focusing strength, where the circles represent experiment and the solid curves theory.