Space-charge Dominated Beam Envelope Transport with Rotatable Axes

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Abstract

In normal paraxial studies of self-interacting accelerator beams, coupling between x- and y-motions that rotate the elliptical beam envelope is considered undesirable. Yet in some applications, such as electron beam lithography and electron beam transmission X-ray computed tomography, external focussing elements cause this coupling by design. To meet this need, the classical 4th order Kapchinskiy-Vladimirsky (KV) differential equations have been generalized to include beam envelope rotation. This set of 10th order generalized envelope equations based on the KV distribution were applied to a 600 mA, 130 keV beam moving through a low-pressure nitrogen gas, where the absence and presence of complete plasma neutralization along its path cause the beam to diverge and converge.

I. INTRODUCTION

The standard equations for the beam envelope are the Kapchinskiy-Vladimirsky equations for the principal axes of an ellipse that does not rotate, because the x- and y- motions are decoupled. However, for certain applications, the x and y motions are coupled deliberately, in order to align the ellipse axes with desired directions. The generalized envelope equations described here allow for rotating elliptical envelopes.

II. THE ELECTRON OPTICS

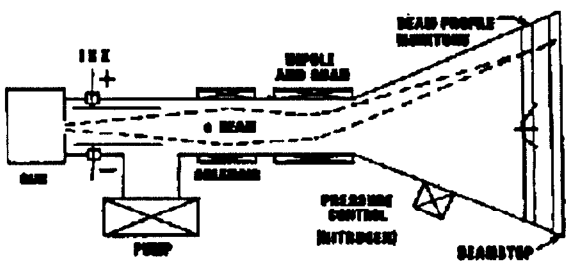

Figure 1. Electron optical system.

A steady space charge limited beam is produced by the gun (Figure 1). By adjusting the nitrogen pressure in the drift tube, the beam is made to ionize the gas in its path on a timescale short compared to its transit time. The ion clearing electrodes[1] sweep the ions out of the beam. The electron beam expands by self-repulsion until it leaves the ICE, when it becomes neutralized and focussed by the solenoid and quadrupole fields, and its own magnetic self-field. The beam is deflected by dipole coils, which add their own inherent focussing fields along its path. The bend plane and beam path consitute a natural coordinate system (Figure 2).

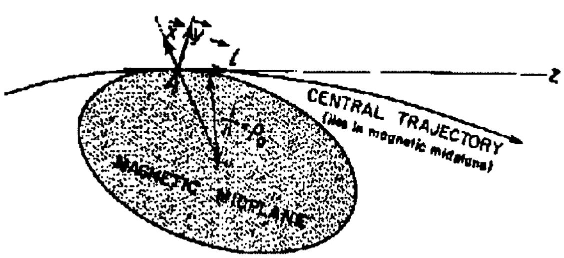

Figure 2. Local beam coordinates.

III. ENVELOPE EQUATIONS

The condition that the beam has a uniform elliptical profile means that the particles be uniformly distributed over the surface of a 4-dimensional ellipsoid in its phase space, according to the Kapchinskiy-Vladimirsky (KV) micro-canonical ensemble ρ:

$$\rho = \delta(1 - I(x, y, p_x, p_y, t)) \tag{1}$$

where

$$I(x,y,p_x,p_y,t) = a_{11}\, x^2 + 2\, a_{12}\, x\, y + 2\, a_{13}\, x\, p_x + 2\, a_{14}\, x\, p_y + a_{22}\, y^2 + 2\, a_{23}\, y\, p_x + 2\, a_{24}\, y\, p_y \tag{2}$$

is a quadratic function of the phase variables and δ is the Dirac delta function. Let the paraxial Hamiltonian be

$$H = (k_1\, x(t)^2 + (p_y(t) + q\, x(t))^2/po + 2\, m\, x(t)\, y(t) + k_2\, y(t)^2 + (p_x(t) - q\, y(t))^2/p_o) / 2 \tag{3}$$

Then the condition that $I(x,y,p_x,p_y)$ be a phase invariant is

$$a_{11}'(t) = 2\,(k_1\, a_{13}(t) + (-\,q\, a_{12}(t) + q^2\, a_{13}(t))/p_o + m\, a_{14}(t))$$

$$a_{12}'(t) = m\,(a_{13}(t) + a24(t) + k_2\, a_{14}(t) + k_1\, a_{23}(t) + (q\,(a_{11}(t) - a22(t)) + q^2\,(a_{14}(t) + a23(t))\,)/p_o$$

$$a_{13}'(t) = k_1\, a_{33}(t) + (-a_{11}(t) - q\,(a_{14}(t) + a_{23}(t)) + q^2\, a_{33}(t))/p_o + m\, a_{34}(t)$$

$$a_{14}'(t) = k_1\, a_{34}(t) + (-a_{12}(t) + q\,(a_{13}(t) - a_{24}(t)) + q^2\, a_{34}(t))/p_o + m\, a_{44}(t)$$

$$a_{22}'(t) = 2\,(m\, a_{23}(t) + k_2\, a_{24}(t) + (q\, a_{12}(t) + q^2\, a_{24}(t))/p_o)$$

$$a_{23}'(t) = m\, a_{33}(t) + k_2\, a_{34}(t) + (-a_{12}(t) + q\,(a_{13}(t) - a_{24}(t)\,) + q^2\, a_{34}(t))/p_o$$

$$a_{24}'(t) = m\, a_{34}(t) + k_2\, a_{44}(t) + (q\, a_{14}(t) - a_{22}(t) + q\, a_{23}(t) + q^2\, a_{44}(t))/p_o$$

$$a_{33}'(t) = -2\,(a_{13}(t) + q\, a_{34}(t)) / p_o$$

$$a_{34}'(t) = -(a_{14}(t) + a_{23}(t) + q\,(a_{44}(t) - a33(t))) / p_o$$

$$a_{44}'(t) = -2\,(a_{24}(t) - q\, a_{34}(t))/p_o\} \qquad (4)$$

where the solenoidal focussing strength q^2/p_o is external, while the quadrupole strengths k_1, k_2 and m are given by a sum of external(k_{1ext}, k_{2ext}, m_{ext}) and self-forces:

$$k_1 = k_{1ext} + (-2\,\pi^2\, r_d^2/\varepsilon)\,(a_{11} - (a_{14}^2\, a_{33} - 2\, a_{13}\, a_{14}\, a_{34} + a_{13}^2\, a_{44})/(-a_{34}^2 + a_{33}\, a_{44}) + (\pi^2/\varepsilon) / \sqrt{a_{33}\, a_{44}(t) - a_{34}^2}) / \Delta$$

$$k_2 = k_{2ext} + (-2\,\pi^2\, r_d^2/\varepsilon)\,(a_{22} - (a_{24}^2\, a_{33} - 2\, a_{23}\, a_{24}\, a_{34} + a_{23}^2 a_{44})/(-a_{34}^2 + a_{33}\, a_{44}) + (\pi^2/\varepsilon)/\sqrt{a_{33}\, a_{44}(t) - a_{34}^2}) / \Delta$$

$$m = m_{ext} + (-2\,\pi^2\, r_d^2/\varepsilon)(a_{12} + (-(a_{14}\, a_{24}\, a_{33}) + a_{14}\, a_{23}\, a_{34} + a_{13}\, a_{24}\, a_{34} - a_{13}\, a_{23}\, a_{44})/(-a_{34}^2 + a_{33}\, a_{44})) / \Delta$$

$$\Delta = (-(a_{14}^2 + a_{24}^2)\, a_{33} + 2\, a_{13}\, a_{14}\, a_{34} + 2\, a_{23}\, a_{24}\, a_{34} - (a_{13}^2 + a_{23}^2) a_{44})/\sqrt{a_{33}\, a_{44}(t) - a_{34}^2} + (a_{11} + a_2)\sqrt{a_{33}\, a_{44}(t) - a_{34}^2} + 2\,\pi^2/\varepsilon \qquad (5)$$

By Liouville's theorem, the emittance ε is conserved.

$$\varepsilon = \int\int\int\int \delta(1 - I(x, y, px, py, t))\, dx\, dy\, dpx\, dpy$$

where

$$\pi^2/\varepsilon = (a_{14}^2\, a_{23}^2 - 2\, a_{13}\, a_{14}\, a_{23}\, a_{24} + a_{13}^2\, a_{24}^2 - a_{14}^2\, a_{22}\, a_{33} + 2 a_{12} a_{14} a_{24} a_{33} - a_{11}\, a_{24}^2\, a_{33} + 2\, a_{13}\, a_{14}\, a_{22}\, a_{34} - 2\, a_{12}\, a_{14}\, a_{23}\, a_{34} - 2\, a_{12}\, a_{13}\, a_{24}\, a_{34} + 2\, a_{11}\, a_{23}\, a_{24}\, a_{34} + a_{12}^2\, a_{34}^2 - a_{11}\, a_{22}\, a_{34}^2 - a_{13}^2\, a_{22}\, a_{44} + 2\, a_{12}\, a_{13}\, a_{23}\, a_{44} - a_{11}\, a_{23}^2\, a_{44} - a_{12}^2\, a_{33}\, a_{44} + a_{11}\, a_{22}\, a_{33}\, a_{44})^{1/2} \qquad (6)$$

In these equations:

$r_d^2(f) = (1.36J(1-(1. + 0.00196\, E)^2 f))/((2+0.00196\, E)\, E)^{3/2}$, the relativistic perveance term, which can be positive (defocusing) or negative (focussing); E = energy of beam in keV; f = neutralization fraction; f = 1, complete neutralization; f = 0, no neutralization); J = total current in Amps;

$$p_o = \beta\gamma = \beta/\sqrt{1-\beta^2} = 0.0442372\sqrt{(2 + 0.00195693\, E)\, E}$$

IV. NUMERICAL EXAMPLE

The equations above were applied to the system in Sec. II.

A. System parameters

The dipole that bends the beam around runs from 57 to 82.57 cm. The beam enters and leaves the quadrupole at t = 59.45 and 79.93 cm. The focal plane is at 219.67 cm. The beam is deflected 22.67^o away from the gun axis. Deliberate mixing of x- and y- motions is imposed by a non-zero m_{ext}. The location of various optical elements splits the beam path into 8 regions, as shown in Table 1.

Table 1. Separation of beam path into regions.

Region	Start (cm)	External focus power (cm^{-2})
1 ICE	0	0
2 ICE + Solenoid	31.25	q^2/p_o = .00063 to .0018
3 ICE	48.75	0
4 Drift	55.	0
5 Dipole	57	k_{1ext} = .00024
6 Dipole + Quad	59.45	k_{1ext} = .00024 ± 7.5 10^{-4} (Max); - k_{2ext}= ± 7.5 10^{-4} (Max); m_{ext}
7 Dipole	79.93	k_{1ext} = .00024
8 Drift	82.57	0
Image	219.7	NA

We assume no partial neutralization on the path. Within the ICE, we take f = 0 (no neutralization); outside of the ICE, we take f = 1. We assume that the bending magnet field is uniform. All pole face effects are ignored. Other beam parameters are E = 130 keV beam, p_o = 0.76, $\varepsilon_x / (\pi\, p_o) = 8 \cdot 10^{-4}$ cm radian, and $\varepsilon_x = 0.00060585\ \pi$ cm radian. The initial envelope radius r(0) = 0.18 cm and the slope r'(0) = 0.01 radian. It is convenient to work with a natural unit of transverse length = $\varepsilon_x / \sqrt{\pi\, p_o\, |r_d^2(1)|}$ = 0.072 cm, a unit slope = $\sqrt{|[r2d]|/p0}$ = 0.011 radian, and t is normalized w.r.t. a unit longitudinal length = 219.67 cm. A Mathematica® electron beam CAD package SIBER (Self Interacting Beam Envelope Rotater [10]) has been written for interactive integration of the envelope equations.

B. Results

The envelope equations were first integrated with 45^o quadrupole field m_{ext} = 0, from t = 0 to t = 2 for different solenoid $q^2 p_o$ and k_{1ext}. The image plane (t = 1) ellipse amplitudes are shown in Figures 3 and 4.

For a good focus in the y-direction (perpendicular to the dipole bend plane) it is necessary to stay in the black strip in Figure 4. We chose $q^2/p_0 = 0.018$ cm^{-2} and $k_{1ext} = -0.064$ cm^{-2} for the integration with nonzero m_{ext}. The march of the x- and y-amplitude with path parameter is shown in Figure 5.

We next imposed a 45° quadrupole $m_{ext} = 5\times10^{-4}$ cm^{-2} in region 6. The x- and y-amplitudes remain substantially unchanged, but the orientation of the principal axes of the ellipse jumps from 0° to about 2° within the quadrupole and increases only slowly after that (Figure 6).

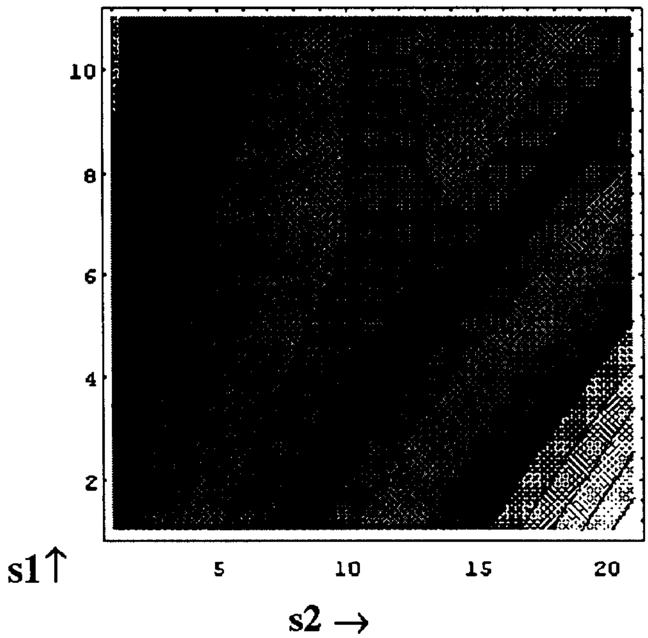

Figure 3. Contour plot of ellipse x-amplitude as a function of focus and quadrupole parameters s1 and s2 respectively. Solenoid focussing strength $q^2/p_0 = 1.2\times10^{-4}$ s1 + 5.1×10^{-4} cm^{-2}. Quadrupole focussing strength $k_1 = 7.5\times10^{-5}$ s2 - 8.25×10^{-4} cm^{-2}. 10 equally spaced contours between 0.46 (black) and 86.91 (white) x 0.72 mm.

V. CONCLUSIONS

A set of 10th order envelope equations was applied to a space-charge dominated electron beam system. Mixing was introduced by imposing a 45° quadrupole strength, and the change in orientation of the elliptical envelope followed through to the image plane and beyond. A electron beam optical CAD program SIBER now exists for investigating basic electron beam design and beam tuning sensitivities.

VI. REFERENCE

[1] R. E. Rand and E. Y. Tsiang, U. S. Patent 5,193,105, March 9, 1993.

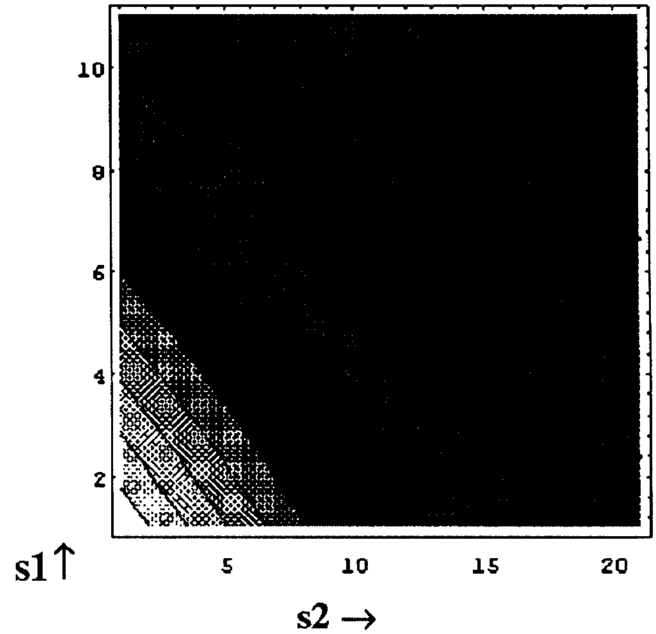

Figure 4. Same as Figure 3. Contour plot of ellipse y-amplitude in units of 0.72 mm. 10 equally spaced contours between 0.46 (black) and 117.64 (white) x 0.72 mm.

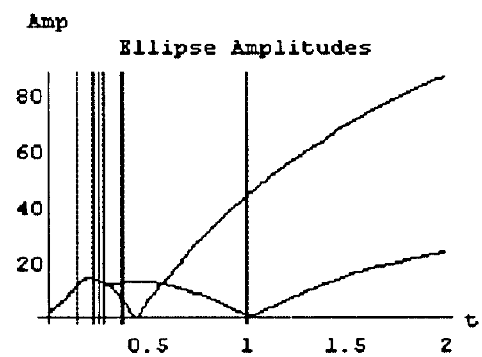

Figure 5. X and y amplitudes(upper and lower), in units of 0.072 cm, as t. 0° quad strength = 6.375×10^{-4} cm^{-2}, solenoid strength = 1.8×10^{-3}cm^{-2}. Vertical lines divide regions.

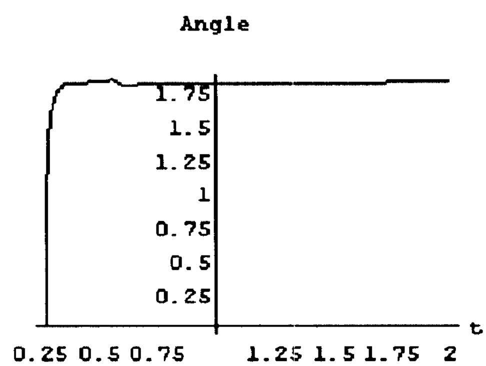

Figure 6. Ellipse orientation in degrees, from regions 6 to 8 of Table 1. 0° quad strength = 6.375×10^{-4} cm^{-2}, solenoid = 1.8×10^{-3}cm^{-2}, 45° quad strength = 5×10^{-4} cm^{-2}.