Emittance and Luminosity Evolution during Collisions in the SSC Collider

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Abstract

The nominal beam storage time in the SSC Collider is 24 hours. During this period, there are many co-existing effects that have impacts on the luminosity — intrabeam scattering, synchrotron radiation, *pp* collisions, beam-gas scattering and beam collimation. A computer program has been written which takes these effects into account simultaneously, and calculates the emittances and luminosity as a function of time. These evolution curves are compared for different operation scenarios (i.e., different initial conditions of the beam current, emittances, and beam energies.)

I. INTRODUCTION

The luminosity profile during the 24-hour or so collision period in the SSC Collider is an important issue that needs to be carefully studied. The luminosity is a function of a number of parameters. Among them are the number of particles per bunch (N_b), the beam emittances (ϵ_x, ϵ_y and ϵ_L), and the reduction factor (R) due to the crossing angle. These parameters vary with time. The number of particles will decrease due to beam-beam collisions, beam-residual gas collisions, and beam collimation. The beam emittances will be damped by the synchrotron radiation, but will be blown up by intrabeam scattering, beam-gas scattering, collective instabilities and external perturbations (e.g., ground motion and power supply ripple). The reduction factor (R) is a function of the beam emittances. This picture is further complicated by the fact that the emittance growth rates due to some processes, such as intrabeam scattering and *pp* elastic collisions, are by themselves evolving with time.

In order to model the luminosity evolution accurately, a computer program is written. Based on the best knowledge that we have about the complex processes that will occur during the beam storage time, the program considers *pp* collisions (elastic and inelastic), synchrotron radiation, intrabeam scattering, residual gas scattering and beam collimation simultaneously. It calculates the evolution of the emittances, particle numbers, reduction factors and luminosity. It is interesting to notice that the originally round-shaped beam gradually becomes flat because of the asymmetric feature of intrabeam scattering. The luminosity profile obtained is similar to that of a preliminary study [1].

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II. EVOLUTION CALCULATIONS

A. Basic Formulæ and Parameters

The luminosity per interaction point (IP) is computed from:

$$\mathcal{L} = \frac{N_b^2\, M\, f_0}{4\pi\beta^*\,\sqrt{\epsilon_x\epsilon_y}}\cdot R \qquad \text{(per IP)}$$

in which M is the number of bunches (17424), f_0 the revolution frequency (3.441 kHz) and β^* the beta-function at the IP. The reduction factor is derived from the crossing angle ϕ:

$$R_{x,y} = \left(1+\left(\frac{\phi\sigma_z}{2\sigma_{x,y}}\right)^2\right)^{-1/2}$$

in which $\sigma_{x,y,z}$ are the rms beam size in the horizontal, vertical and longitudinal directions, respectively. In the present design, there are two high luminosity IP's, where β^* is 0.5 meters. The nominal full crossing angle ϕ is 135 μrad. One IP is horizontal crossing, the other vertical.

The emittance is a function of time:

$$\left(\frac{d\epsilon}{dt}\right)_{\text{total}} = \epsilon\left(\frac{1}{\tau_{\text{ibs}}}-\frac{1}{\tau_{\text{rad}}}\right)+\left(\frac{d\epsilon}{dt}\right)_{\text{gas}}+\left(\frac{d\epsilon}{dt}\right)_{pp}$$

in which τ_{ibs} and τ_{rad} are the emittance growth (damping) time of the intrabeam scattering and synchotron radiation, respectively. The last two terms are for the elastic residual gas scattering and elastic *pp* collisions and will be discussed in detail later.

The number of protons per bunch decreases with time:

$$\left(\frac{dN_b}{dt}\right)_{\text{total}} = -\frac{\mathcal{L}_{\text{total}}}{M}\cdot\sigma_{\text{inel}}+\left(\frac{dN_b}{dt}\right)_{\text{gas}}+\left(\frac{dN_b}{dt}\right)_{\text{others}}$$

in which σ_{inel} is the inelastic *pp* cross-section at 20 TeV (100 mb) and the last two terms are the beam loss due to the residual gas nuclear scattering and other effects (e.g., collimation).

B. Intrabeam scattering

The intrabeam scattering in the SSC has been studied elsewhere [2]. The growth times in the three bunch dimensions are given below [3]:

$$\frac{1}{\tau_s} \equiv \frac{1}{\sigma_s}\frac{d\sigma_s}{dt} = \left\langle A\,\frac{\sigma_h^2}{\sigma_p^2}\,f(a,b,c)\right\rangle$$

$$\frac{1}{\tau_x} \equiv \frac{1}{\sigma_{x\beta}}\frac{d\sigma_{x\beta}}{dt} = \left\langle A\left(f\left(\frac{1}{a},\frac{b}{a},\frac{c}{a}\right)+\frac{D_x^2\sigma_p^2}{\sigma_x^2}\,f(a,b,c)\right)\right\rangle$$

$$\frac{1}{\tau_y} \equiv \frac{1}{\sigma_{y\beta}}\frac{d\sigma_{y\beta}}{dt} = \left\langle A\left(f\left(\frac{1}{b},\frac{a}{b},\frac{c}{b}\right) + \frac{D_y^2\sigma_p^2}{\sigma_y^2}\, f(a,b,c)\right)\right\rangle$$

with

$$A = \frac{r_p^2 c_\ell N_\mathrm{b}}{64\pi^2\sigma_s\sigma_p\epsilon_x\epsilon_y\gamma^4}$$

$$\frac{1}{\sigma_h^2} = \frac{1}{\sigma_p^2} + \frac{D_x^2}{\sigma_{x_\beta}^2} + \frac{D_y^2}{\sigma_{y_\beta}^2}, \qquad \sigma_{x,y}^2 = \sigma_{x_\beta,y_\beta}^2 + D_{x,y}^2\sigma_p^2$$

$$a = \frac{\sigma_h}{\gamma\sigma_{x'_\beta}}, \qquad b = \frac{\sigma_h}{\gamma\sigma_{y'_\beta}}, \qquad c = \sigma_h\sqrt{\frac{2d}{r_p}}$$

The brackets denote the average over the whole circumference. The function f can be found in Ref [2]. The other quantities are: r_p = classical proton radius, c_ℓ = velocity of light, γ = relativistic parameter, $D_{x,y}$ = dispersion functions, σ_s = rms bunch length, σ_p = rms relative energy spread and d = impact parameter $\approx min\{\sigma_x, \sigma_y\}$.

It should be pointed out that what is computed here are the growth times of the beam *amplitudes*. They are a factor of two greater than the growth times of the beam *emittance*.

C. Synchrotron Radiation

The energy lost by a 20 TeV proton in the SSC due to synchrotron radiation is 0.126 MeV per turn. The damping time for energy oscillations works out to be 12.81 hours at the nominal circulation frequency of 3.441 kHz. Over the 24 hour beam storage time of the SSC, this allows for two e-foldings in the longitudinal emittance. The damping partition numbers for the collider are $J_x = J_y = 1$, and $J_\delta = 2$, so that both the horizontal and vertical transverse emittance would also suffer two e-foldings in a 24 hour period [4].

D. Elastic Proton-Proton Scattering

The contribution to transverse emittance growth, for one degree of freedom, resulting from proton-proton elastic scattering is given by

$$\frac{d\epsilon_x}{dt} = \frac{N_\mathrm{b}\, f_0}{4\pi\sqrt{\epsilon_x\epsilon_y}}\cdot\sigma_\mathrm{el} < \theta_x^2 > \qquad \text{(per IP)}$$

In this expression σ_el is the proton-proton elastic scattering cross-section, and $\sqrt{<\theta_x^2>}$ is the rms value of the pp elastic scattering angle in the center of mass system, which is projected onto the transverse x-direction. A similar expression occurs for the transverse y-direction. The integrated and center-of-mass-system-differential pp elastic cross-sections are approximately related to the total pp cross-section by the formulae,

$$\sigma_\mathrm{el} \approx (1/4)\sigma_T,$$

and

$$\left(\frac{d\sigma_\mathrm{el}}{d\Omega}\right)_\mathrm{c.m.} \approx \left(\frac{E_\mathrm{c.m.}\sigma_T}{2hc_\ell}\right)^2 \exp\left(-\frac{\theta_x^2}{2\sigma_{\theta_x}^2} - \frac{\theta_y^2}{2\sigma_{\theta_x}^2}\right),$$

where h is Planck's constant and

$$\sigma_{\theta_x} = \sqrt{<\theta_x^2>} = \frac{hc_\ell}{E_\mathrm{c.m.}\sqrt{2\pi\sigma_T}}.$$

For colliding proton beams with $E_\mathrm{c.m.}$ = 20 TeV and $\sigma_T \approx$ 130 mb [5], one finds

$$\sigma_{\theta_x} = \sqrt{<\theta_x^2>} = 6.9\,\mu\mathrm{rad}.$$

E. Elastic Residual Gas Scattering

Following the approach set forth in Ref [6], we have that the scattering of an ultra-relativistic proton beam from the residual gas produces an average emittance growth rate of

$$\frac{d\epsilon}{dt} = \bar\beta\,\frac{d\langle\theta_x^2\rangle}{dt} = \bar\beta\pi c_\ell\left(\frac{2e^2}{E}\right)^2\sum_T n_T Z_T^2\ln(a_T/R_T)$$

where $\bar\beta$ is the average β-function (180 m), E is the energy of the proton beam (20 TeV), e is the elementary unit of charge (esu), and the summation index T labels *atomic* (not molecular) species present in the residual gas, with Z_T the atomic number, a_T the orbital-electron-shielded atomic "radius", R_T the atom's nuclear "radius", and n_T the total number per unit volume of this type of atom which is present in the residual gas.

Partial pressures (as normalized to 0°C) of residual gases in the SSC Collider are projected to be 8×10^{-9} torr of H_2, 10^{-10} torr of CO, and 2×10^{-11} torr of CO_2 [7]. We now assume that for all the atomic types involved (H, C, and O) we may take $\ln(a_T/R_T) \approx \ln(10^5) = 11.5$. With these assumptions we obtain,

$$\frac{d\epsilon}{dt} \approx 4.2\times10^{-17}\ \mathrm{m/s}.$$

F. Particle Loss

The main causes of particle loss are the nuclear interaction between the proton beam and the residual gas, and the inelastic pp collisions at the IP's. For the former, one can define an equivalent luminosity as

$$\mathcal{L}_\mathrm{gas} = N_\mathrm{b}\, M\, c_\ell\cdot n_{\mathrm{H}_2} = 1.2\times10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

in which $n_{\mathrm{H}_2} = 3\times10^8\ \mathrm{cm}^{-3}$ is the density of the residual H_2 in the beam vacuum and $N_\mathrm{b} = 0.75\times10^{10}$. From the measured data of other laboratories (BNL and CERN) it is known that heavy species in the residual gas (CO and CO_2) make a significant contribution to the particle loss. Therefore, the value of $\mathcal{L}_\mathrm{gas}$ above is multiplied by a factor of 3 in the calculations [8]. Other causes of particle loss are beam collimation, collective beam-beam interactions, and noise in the RF system. They account for about 10% of the total particle loss.

III. RESULTS AND DISCUSSION

A. Initial Conditions and Evolution Curves

The initial conditions of the baseline design are: $\mathcal{L} = 1\times10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ per IP, $N_\mathrm{b} = 0.75\times10^{10}$, $\gamma = 21315.8$ (20

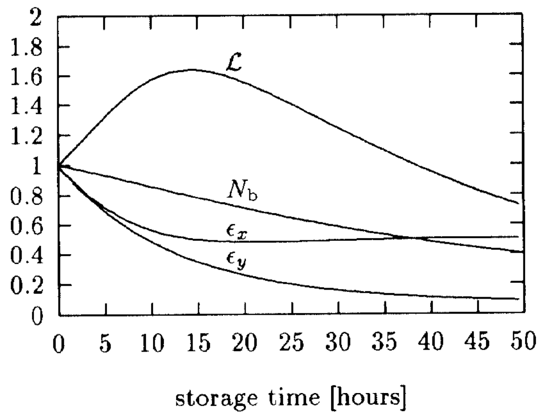

Figure 1. Evolution curves for luminosity, number of protons per bunch, and horizontal and vertical emittances. Parameters used are for baseline design.

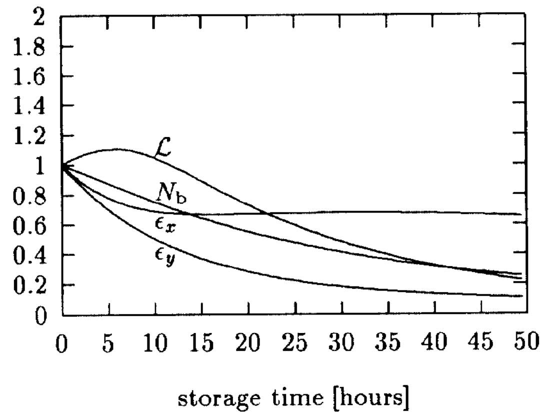

Figure 2. Evolution curves for luminosity upgrade by a factor of 10.

TeV), the normalized transverse emittances $\epsilon_{Nx} = \epsilon_{Ny} = 10^{-6}$ m-rad, $\sigma_s = 6$ cm, $\sigma_p = 0.58 \times 10^{-4}$. The evolution curves are shown in Fig. 1.

- The luminosity curve first rises because the emittances decrease due to synchrotron radiation. It reaches a peak, and then drops because of particle loss and intrabeam scattering. This feature is qualitatively similar to that in Ref [1].

- The transverse emittance in the x-direction gradually becomes larger than that in the y-direction. This is because the average dispersion function D_x (1.5 m) is much bigger than D_y (which is virtually zero). This results in different intrabeam scattering growth rates in x and y. Therefore, the round beam will become flat. After 24 hours, the aspect ratio is about 2.4 to 1; after 50 hours, 5.7 to 1.

- The transverse emittances converge toward equilibrium values where damping effects are balanced by antidamping effects.

B. *Different Operational Scenarios*

The Collider may be operated under different initial conditions for various reasons. Fig. 2 is for the case when the luminosity is upgraded by a factor of 10 (by increasing N_b by a factor of $\sqrt{10}$), whereas Fig. 3 shows the curves for an energy upgrade from 20 TeV to 23 TeV, while keeping the same beam current.

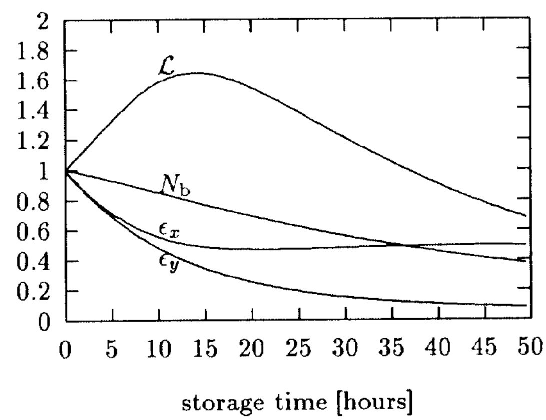

Figure 3. Evolution curves for energy upgrade from 20 TeV to 23 TeV.

IV. ACKNOWLEDGEMENTS

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