Numerical Modelling of Time-Space Behavior of High-Current Relativistic Electron Beam in Plasma Waveguide

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Abstract
The physical processes in the magnetized plasma waveguide, in which the high-current relativistic electron beam (Sudan parameter $S > 1$) is injected, are investigated by the developed 2.5-D relativistic electromagnetic code. The computer study has shown that the charge and current compensations of the high-current beam are considerably different from those of the low-current beam. In addition the self-consistent electromagnetic fields generated by a beam at the initial state radically alter both the linear and the nonlinear instability stages.

I INTRODUCTION

The advantages of a use of the high-current relativistic electron beam (HCREB) for development of a powerful electromagnetic radiation sources, and the new type accelerators based on the collective methods of the particles acceleration, and etc. are presented in [1]–[3]. It was noted that the simultaneous growth both of the beam energy and the non-equilibrium degree of a system defining by the Sudan parameter $S = (n_b/n_e)^{1/3}\gamma$ (n_b, n_e are the beam and plasma densities, γ is the relativistic factor) is a very important fact. In the unbounded systems this can results in the decrease of the energy (at $S > 1$) transferring from a beam to the plasma for the oscillations excitation. In the system bounded in the radial direction this fact decreases the efficiency of the electromagnetic waves radiation due to the oscillations excitation with the small ratio between the transverse and longitudinal components [3, 4]. Čerenkov mechanism was proved to be changed to the anomalous Doppler mechanism for HCREB [5]. This allows to remain the high efficiency of the electromagnetic radiation at the certain parameters of a system such as the magnetized plasma waveguide (MPW) – HCREB.

The equilibrium and stability conditions of HCREB were investigated in many works (see Refs. in [1]–[5]). The injection of the low-current ($S \ll 1$) stringent REB is also studied in detail both with the magnetic field and without field. It was shown if the beam radius a to be greater than the skin-depth $\lambda_E = c/\omega_e$ (c is the light velocity, ω_e is the Langmuir plasma frequency) the beam current is compensated by a return plasma current damping in a plasma with the finite conductivity. The external magnetic field modifies the current compensation condition to the form $a \gg \lambda_E(1 + \Omega_e^2/\omega_e^2)$ (Ω_e is the Larmor electron frequency).

In this work both the charge–current compensation and the stability of HCREB ($S > 1$) in MPW are investigated. The hollow narrow beam of the a radius and of the Δ_b thickness (so that $\Delta_b < c/\omega_e$, but $a > c/\omega_e$) is considered, and the hollow wide beam with $\Delta_b \approx r_L$ (r_L is the waveguide radius) is also. In the both cases the Čerenkov resonance condition of the beam with the plasma wave is not valid i.e. $\gamma > \omega_e/(ck_\perp)$ ($k_\perp$ is a transverse wave number). For comparison, the study of the low-current beam is also presented.

II MODEL AND EQUATIONS

In order to study the dynamics of a collisionless plasma with the relativistic electron beam in both the self-consistent and the external electromagnetic fields in axisymmetric ($\partial/\partial\theta = 0$) geometry, we use the set of relativistic Vlasov's equations for the distribution functions of the given type of particles $f_s(\vec{p}, \vec{R}, t)$. Here $\vec{p} = m_s\vec{v}\gamma$, $\vec{v} = \{\dot{r}, r\dot{\theta}, \dot{z}\}$, $\gamma = [1 - (|\vec{v}|/c)^2]^{-1/2}$ is the relativistic factor, $\vec{R} = \{r, z\}$. The self-consistent electromagnetic fields in Vlasov's equation are determined by Maxwell's equations in the form of wave equations for the dimensionless scalar ϕ and vector $\vec{A}$ potentials in which the right side is defined by the total charge and current densities [6].

We will consider the infinite value of the uniform external magnetic field $H \to \infty$ then the motion of the particles can be treated as the one-dimensional. Thus we have the only equation of the motion and the equations for the potentials $\phi(r, z)$ and $A_z(r, z)$.

In these equations the quantities involved are used in the dimensionless form: $[v] = c$; $[r, z] = c/\omega_e$; $[t] = \omega_e^{-1}$; $[n] = n_{0e}$; $[q] = e$; $[m] = m_0$; $[\phi, A] = \mathcal{E}_{ch}/e$; $[E, B] = (4\pi n_{0e}\mathcal{E}_{ch})^{1/2}$; $[J] = en_{0e}c$, where $\omega_e = (4\pi n_{0e}e^2/m_0)^{1/2}$ is the electron plasma frequency, $\mathcal{E}_{ch} = m_0c^2$ is the rest energy of the beam electron, n_{0e}, m_0, e are the initial density, rest mass and charge of the electrons respectively.

The dimensionless equation of motion, obtained as characteristic equation of Vlasov's equation, was written as

$$\frac{du_z}{dt} = -\frac{q}{m}\left(\frac{\partial A_z}{\partial t} + \frac{\partial \phi}{\partial z}\right)$$

where $u_z = \gamma v_z$, $\gamma = [1 + u_z{}^2]^{1/2}$.

The boundary conditions for the potentials are

$$r = 0: \quad \partial\phi/\partial r = \partial A_z/\partial r = 0;$$
$$r = r_L: \quad \phi = A_z = 0;$$
$$\left.\begin{array}{l} z = 0 \\ z = z_L \end{array}\right\}, \phi = \frac{\partial A_z}{\partial z} = 0$$

The initial conditions for the self-consistent fields are $\Delta\phi = -\rho, A_z = 0$. Here Δ is Laplassian.

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The boundary conditions for the distribution functions set the Maxwellian distribution for plasma particles with the temperature T at $(z=0, z=z_L)$ and the hollow electron beam injection at $z=0$: $f_b(m_0 u_z, \vec{R}, t) = \delta(u_z - u_{0b})$ at $r_{min} \leq r \leq r_{max}$ and $u_z > 0$, they are equal to zero at $z = z_L$. Here, r_{min} and r_{max} are the minimum and maximum beams radii respectively, $u_{0b} = V_b/(1-V_b^2)^{1/2}$, V_b is a beam velocity. At the initial time, the distribution functions are equal to Maxwellian for the plasma particles and equal to zero for the beam electrons.

The present model is a particular case of the general 2.5-dimensional cylindrical model [6]. We have used the version of the general computer code for the investigation of a relativistic beam dynamics in the plasma waveguide.

III COMPUTER SIMULATION

Let a hollow magnetized electron beam with velocity V_b be injected along the z-axis into the plasma waveguide under the external magnetic field. The beam current density is equal to $q_b n_b V_b$. In the calculations we assumed the mass ratio to be $m_i/m_e = 100$, $m_e = 20 m_0$, $m_b = m_0$, the number of particles in the cell was $N_e = N_i = 4$, $N_b = 100$ for narrow beam and $N_b = 20$ for wide beam. The electron beam velocity was supposed $V_b \approx 0.995$, i.e $\gamma = 10$. The length and radius of the waveguide were $z_L = 127.5$ and $r_L = 7.5$. The plasma temperature is equal to $T = 2 \cdot 10^{-3}$. The number of points and the time step for solving Maxwell's equations were $(J_r \times J_z) = (16 \times 256)$ and $\Delta\tau = 0.025$. The time step for solving of the equation of the motion is equal to $\Delta t = 0.1$. Three variants were run with the following parameters:

No. of case	1	2	3
r_{max}	2.	7.	2.
Δ_b	0.5	7.	0.5
n_{0b}/n_{0e}	0.5	0.5	0.05
$S = (n_b/n_e)^{1/3}\gamma$	7.94	7.94	3.68
$(n_b/(n_e\gamma))^{1/3}$	0.37	0.37	0.17
$\mu = ((n_b\gamma)/n_e)^{1/3}$	1.71	1.71	0.79

The results of the computer simulation are presented in figures 1,2,3 according to the number of variants. In the figures it is shown the longitudinal section of the cylindrical system at the line $r = 1.75$ i.e. along the beam line. It is clearly seen that the beam front excites the plasma and the disturbances are high i.e. the plasma is nonlinear both in the cases of the narrow beams and in the case of the wide beam. The development of the beam instability occurs only for the wide beam (see fig.2) as the Čerenkov resonance condition $\gamma < \omega_e/(ck_\perp)$ is not valid but the beam current is greater the threshold value. The fact is important that the instability development takes place without the charge and current compensations and this essentially influences on both the excitations dynamics and the electromagnetic fields spectrum. The almost entire charge compensation occurs for the narrow beams ($\Delta_b < c/\omega_e$) as the potential distributions show in fig.1a,3a at $t = 160$. No current compensation are available both in the case of the high density beam (fig. 1b) and in the case of low density beam (fig. 3b).

Thus the self-consistent electromagnetic fields generated by a beam at the initial state radically alter both the linear and the nonlinear instability stages.

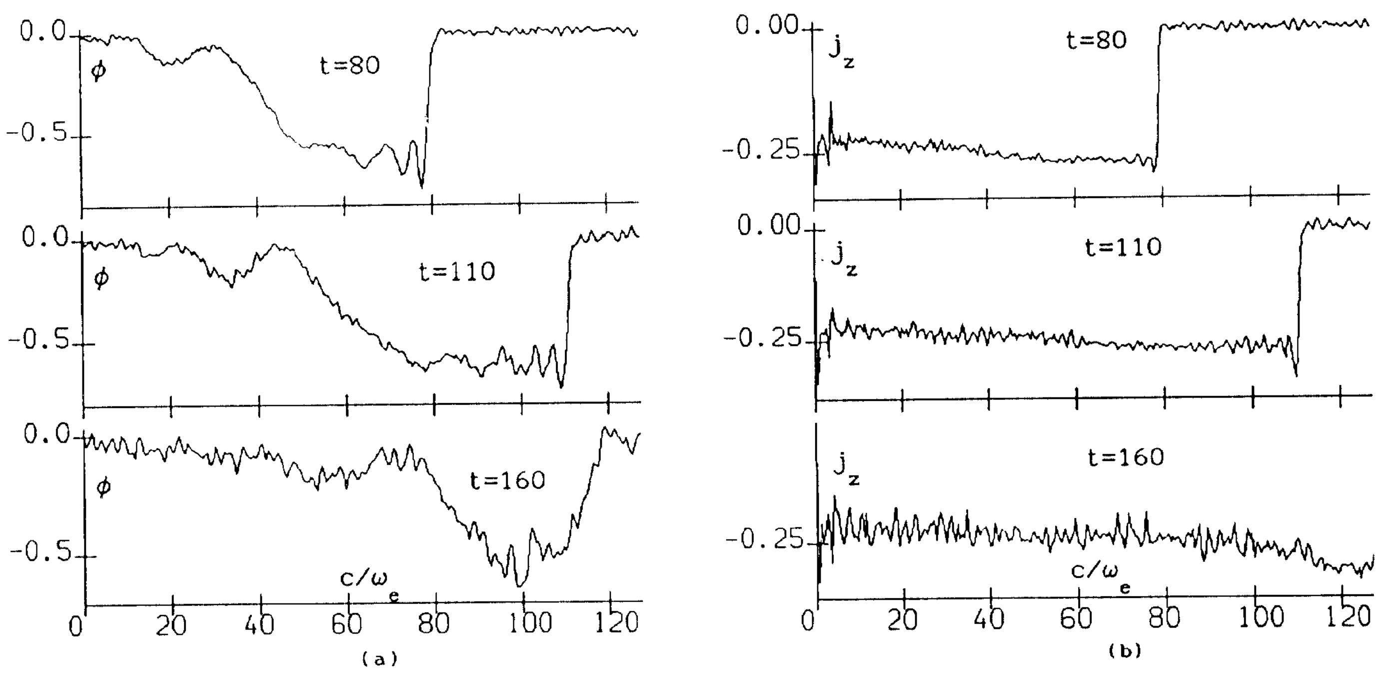

Figure 1: The longitudinal section ($r_b = 1.75$) of (a) the scalar potentials $\phi(r_b, z)$ and (b) the axial current density $j_z(r_b, z)$ for the narrow high-current beam along the beam line at $t = 80$, $t = 110$, $t = 160$.

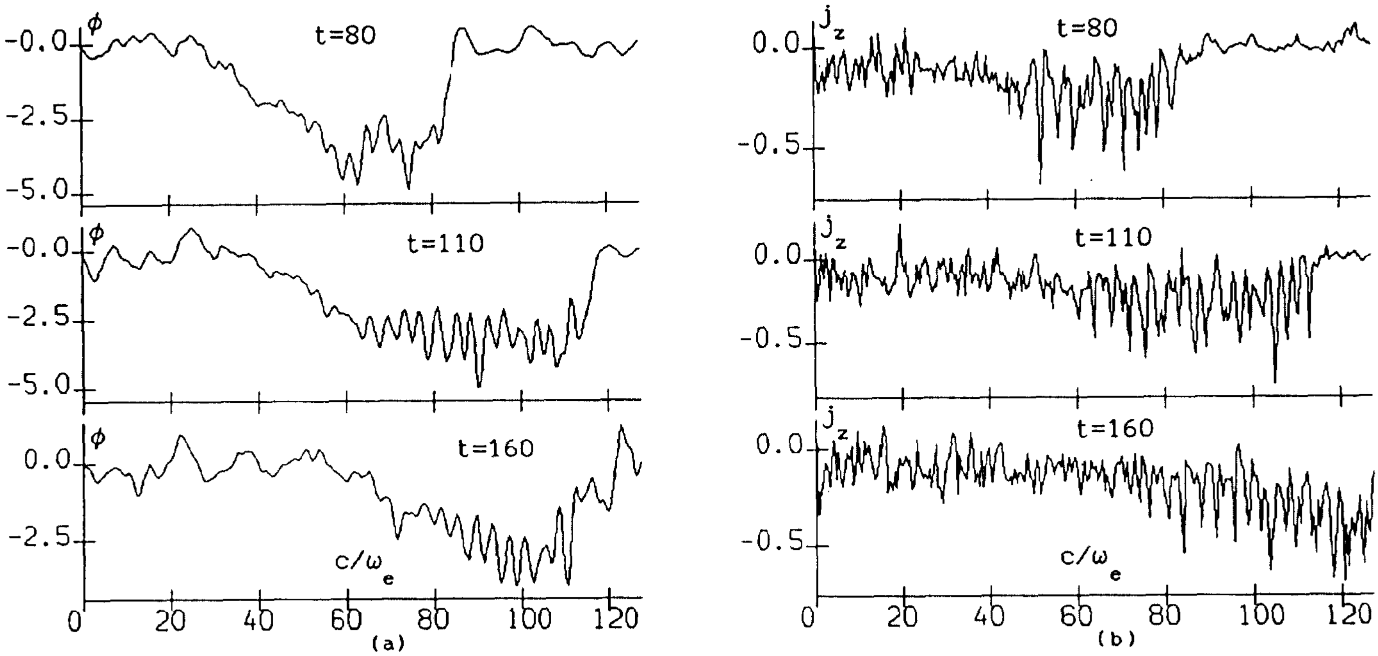

Figure 2: The longitudinal section ($r_b = 1.75$) of (a) the scalar potentials $\phi(r_b, z)$ and (b) the axial current density $j_z(r_b, z)$ for the wide high-current beam along the beam line at $t = 80$, $t = 110$, $t = 160$.

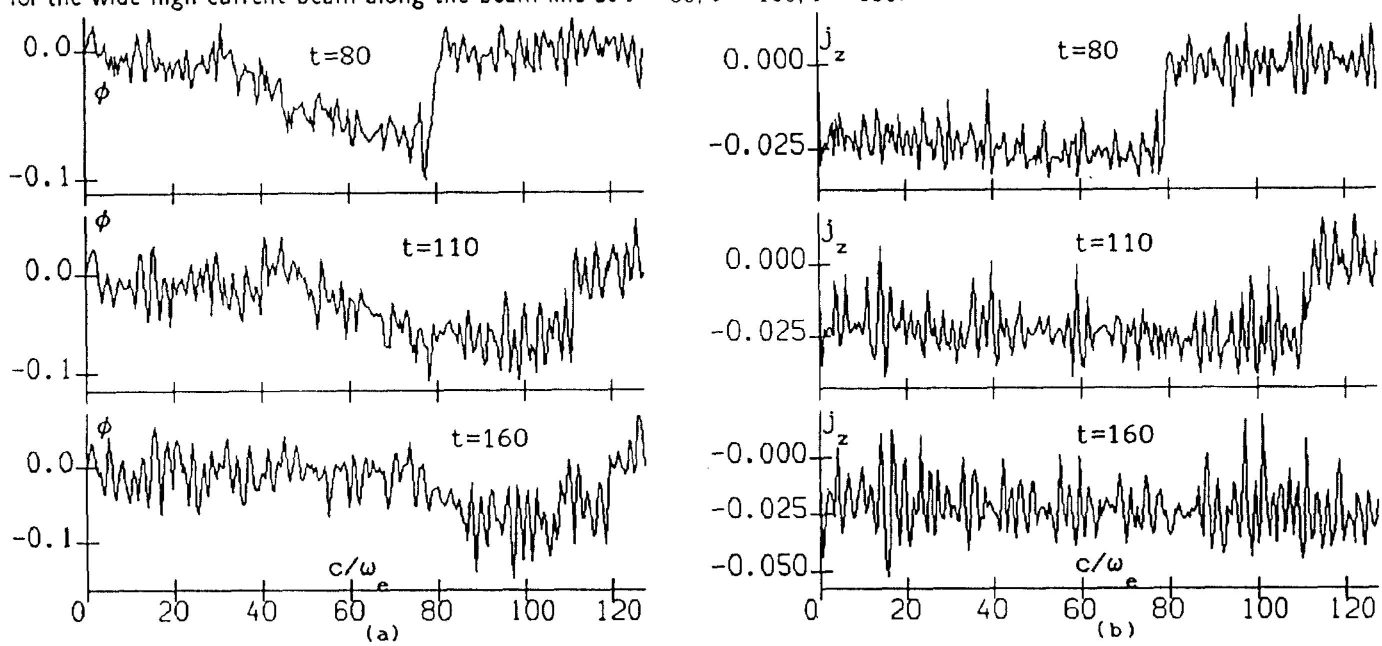

Figure 3: The longitudinal section ($r_b = 1.75$) of (a) the scalar potentials $\phi(r_b, z)$ and (b) the axial current density $j_z(r_b, z)$ for the narrow low-current beam along the beam line at $t = 80$, $t = 110$, $t = 160$.

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