Equivalent Equations and Incoherent Lifetime Calculated from e^+e^- Beam-Beam Simulation

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Abstract

For given tunes (ν_x, ν_y, ν_s), the influence of different terms $x^k y^l s^m$ in the nonlinear beam-beam forces is different, so only some of them are important as sources of regular (nonstochastic) effects. As for irregular effects, it seems that in e^+e^- colliders the simultaneous action of both quantum fluctuations and three-dimensional nonlinearities creates effectively an additional stochastic force. The developed program of beam-beam simulation including Fourier analysis of different moments $M^{klm}(t) = x^k(t)y^l(t)s^m(t)$ permits us to identify both regular and stochastic effective forces in the equivalent equations of the particles' motion. These differential equations can be used, for example, for the calculation of the particles' lifetime.

I. INTRODUCTION

The idea of extracting equivalent differential equations for particle coordinates x, y and s from incoherent spectrum densities calculated in the course of beam-beam simulations has been described in detail earlier [1,2]. The scheme of calculating the equivalent equations is the following.

The individual spectrum of a single particle $\#j$ is:

$$m_j^{k\ell m}(\nu) = \frac{1}{n_{\max}} \sum_{n}^{n_{\max}} x_j^k(n) y_j^\ell(n) s_j^m(n) e^{-2\pi i\nu n}$$

$\Delta\nu = 1/n_{\max}$; $n =$ the number of revolutions.

Incoherent "Schottky noise":

$$\mid F_{\text{incoh}}^{k\ell m}(\nu) \mid^2 = \frac{1}{N} \sum_{j}^{N} \mid m_j^{k\ell m}(\nu) \mid^2$$

$N =$ the number of particles. x, y, s are taken in the IP (interaction point).

An example of the effective equation for the particles' vertical movement (as explained in [2]):

$$\begin{aligned} \ddot{y} + \gamma\dot{y} + \omega^2(x,y)y &= \\ &= asy + b(x^2 - \langle x^2 \rangle)y + \\ &+ (c_o + c_B)\textstyle\sum_{t_k<t}\delta(t-t_k) \end{aligned}$$

(we can use the term $a_1\dot{s}y$ instead of asy). Analogous equations can be written for x and s oscillations.

II. DIFFUSION

c_o corresponds to the radiation fluctuations without BB interactions.

c_B describes the result of the combined effect of the radiation fluctuations and stochastic nonlinear diffusion.

For $\sigma^2 = \beta\epsilon$ we get

$$\frac{d\sigma^2}{dt} = -2\gamma\sigma^2 + \text{const}\cdot(c_o^2 + c_B^2) \approx 0$$

$$\sigma^2 = \sigma_o^2 + \sigma_B^2 = \beta(\epsilon_o + \Delta\epsilon_B)$$

$$\frac{\sigma_B^2}{\sigma_o^2} = \frac{c_B^2}{c_o^2} \approx \frac{\Delta\epsilon_B}{\epsilon_o};\ \boxed{c_B \cong c_o\sqrt{\frac{\Delta\epsilon_B}{\epsilon_o}}}$$

$$\Delta\epsilon_B = \text{const}\cdot\int \Delta d\nu$$

(see Fig. 1).

$$\epsilon_o = \text{const}\int \mid F_o^v(\nu) \mid^2 d\nu$$

(Using *real*, not logarithmic scales!)

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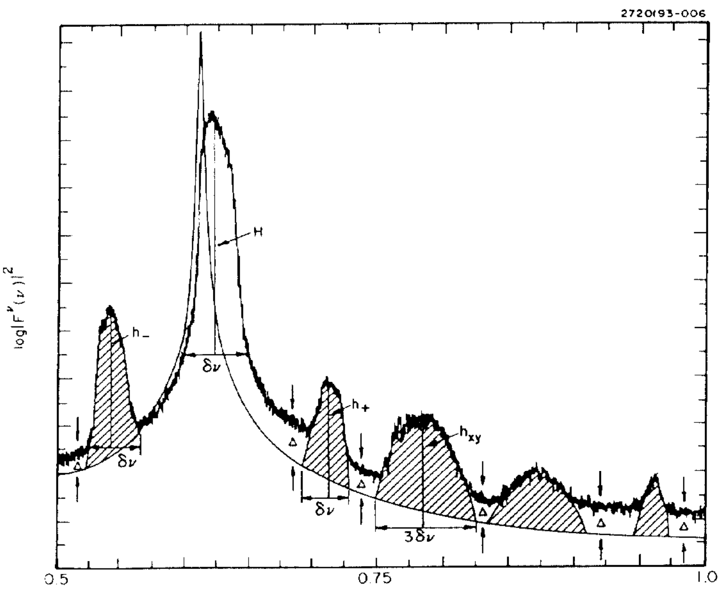

Fig. 1: Vertical spectrum density with and without b-b interaction. $\nu_{x0} = 0.568$, $\nu_{y0} = 0.610$.

III. LONGITUDINAL-VERTICAL COUPLING

Let us consider the equation (*after* the beginning of the BB interaction):

$$\ddot{y} + \omega^2 y = asy$$

$$y = y_0 + y_a \;;\;\; y_0 = Ae^{i\omega t} + A^* e^{-i\omega t} \;;$$

$$s = Be^{i\Omega t} + B^* e^{-i\Omega t}$$

$$y_a = \frac{-a}{\Omega(2\omega + \Omega)} \left(ABe^{i(\omega+\Omega)t} + A^* B^* e^{-i(\omega+\Omega)t} \right) +$$

$$+(\text{the analogous term for} - \Omega) \; .$$

$$\frac{< y_a^2 >}{< y_o^2 >} = \frac{h_\pm}{H} \approx \frac{a^2 \sigma_s^2}{8\omega_o^4 \nu_y^2 \nu_s^2}$$

(ω_o = revolution frequency)

$$\boxed{a \cong \frac{2\sqrt{2}\omega_o^2 \nu_y \nu_s}{\sigma_s} \sqrt{\frac{<h_\pm>}{H}}} \; (\sigma_s \equiv \sigma_L)$$

IV. HORIZONTAL-VERTICAL COUPLING

$$\boxed{b \cong \frac{8\omega_o^2 \nu_x (\nu_x + \nu_y)}{\sigma_x^2} \sqrt{\frac{h_{xy}}{H}}}$$

V. FAST LOSS OF PARTICLES IN THE BEGINNING OF BB INTERACTION

The distribution before BB interaction:

$$\phi_o = e^{-\frac{A^2}{2\sigma_o^2}} \;\;, \;\; A^2 = y^2 + \dot{y}^2/\omega_y^2$$

In the beginning of the BB, after $\Delta t \sim \tau_{\rm rad} \sim \frac{1}{\gamma}$

$$\phi_B = e^{-A^2/2\sigma_B^2}$$

The technique previously used in [3] is used here. The number of particles n =const $\int \phi dA^2$. ϕ_B is the *new* distribution of the particles that survive after the beginning of BB plus Δt. After that the slow loss begins.

The fast loss of particles (during $\Delta t \sim \tau_{\rm rad}$)

$$\delta_y = \int_o^\infty \frac{dA^2}{2\sigma_o^2} \left(e^{-A^2/2\sigma_o^2} - e^{-A^2/2\sigma_B^2} \right) = 1 - \frac{\sigma_o^2}{\sigma_B^2}$$

The full fast loss is

$$\boxed{\delta = 1 - \frac{\sigma_{ox}^2 \sigma_{oy}^2 \sigma_{os}^2}{\sigma_{Bx}^2 \sigma_{By}^2 \sigma_{Bs}^2}}$$

VI. LIFETIME OF THE BEAM CENTER DENSITY, $n(o)$

Here we give only an approximate estimation. We take into account only the diffusion mentioned in II. In this approximation, the lifetime τ_D depends mainly (and very strongly) on ξ , $\tau_D = f(\xi)$, [3]

$$\xi = \frac{A^2_{(\rm permit.)}}{2\sigma^2} \;\; .$$

$A_{(\rm permit.)}$ is the permitted amplitude (the distance to the physical border, or to the dynamic aperture).

σ is rather uncertain. For big *vertical* amplitudes, when $A_y \sim A_{\text{permit}}$, $x - y$ coupling is essential; so, $\sigma \sim \sigma_x$ (and not σ_y).

According to [3]

$$\boxed{\frac{n(o,t)}{n(o,o)} \approx \xi^{(\xi e^{-\xi})} \cdot e^{-2\gamma t \xi e^{-\xi}}}$$

Here $t = o$ means just after the fast loss. For $t = \tau_D = 4$ hours, $\gamma = 1/400T$, $\frac{n(o,\tau)}{n(o,o)} = 1/e$ we get $\xi \sim 24$; $A_{(\text{permit.})}/\sigma \sim 7$.

VII. REFERENCES

1. [1] Y.F. Orlov, C.M. O'Neill, A. Soffer. Fourier Analysis of $x^k y^\ell$ Moments in Beam-Beam Simulations. XV International Conference on High Energy Accelerators. Int. J. Mod. Phys. A (Proc. Suppl.) 2B (1993).
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3. [3] Yu. F. Orlov and S. A. Khejfets. Losses of Particles in Ring Accelerators Taking Damping Into Account. Proceedings of the International Conference on High-Energy Accelerators and Instrumentation, CERN, 1959.