Some Aspects of the Long Range Beam-Beam Interaction in Storage Rings *

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Introduction

In this paper we develop a linear model of the long range beam-beam tune shift for cases when the electron and positron orbits are separated at all crossing points [4]. Theoretical considerations show that the tunes of coherent beam-beam modes depend crucially on the betatron phase advance between crossing points. Under certain conditions the tune split may be reduced, or even made equal to zero in spite of a strong beam-beam interaction. Measurements of coherent beam-beam modes were made at CESR. In most cases, the experimental results are in good agreement with the linear model.

The behavior of coherent beam-beam modes was examined theoretically, taking into account the coherent tune shift caused by impedance effects. As in [2] we form the single turn matrix describing the bunch centroid motion and evaluate the eigenvalues. In the simplest case, this motion includes the linear transport through the magnetic lattice, for which betatron phase advance does not depend on a bunch current, followed by a long range beam-beam 'kick' in the linear approximation. The measurements and comparisons with model predictions are discussed in the final section.

This work was motivated by the CESR upgrade program [1]. A central part of this program is to increase the average stored beam current by forming multiple trains of bunches. However, it is not possible to get large enough separation at all crossing points to be able to ignore the long range beam-beam interaction because the length of each train is comparable with a half betatron wavelength.

Long range beam-beam interaction in the linear approximation

We assume the separation distances at the crossing points are large enough compared with the beam sizes that the linear approximation of the beam-beam kick angle does not contain terms significant terms coupling vertical and horizontal motion. Likewise, we assume the linear transport between crossing points does not include coupling and therefore develop only a one dimensional model.

Consider the simplest case of one bunch per beam and refer to figure 1. The electron and positron bunches should interact with each other only at two points **A** and **B** located at the opposite sides of the storage ring. If these points are not on a symmetry axis of the lattice, the betatron phase advance from **A** to **B** on side **c** would not in general be equal to the phase advance on side **d**. This kind of asymmetry leads to some unexpected behavior of coherent modes.

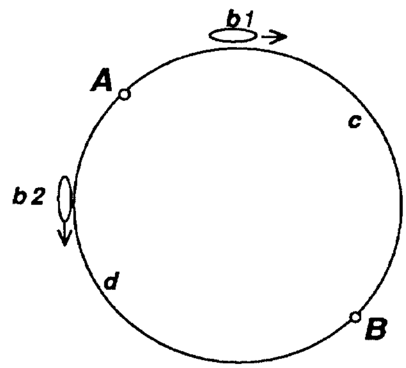

Figure 1: Electrons in bunch b_1 interact with positrons in bunch b_2 at positions **A** and **B**. The phase advance along arc **c** may be different from the phase advance along arc **d**.

Lets form the vector (X_1, X_1', X_2, X_2'), where $X_{1,(2)}$ and $X'_{1,(2)}$ are is the horizontal coordinates and associated angles of bunch $b_{1,(2)}$, appropriately normalized by the horizontal beta function. The matrix that transports both bunches simultaneously (in opposite directions) through the magnetic structure from **A** to **B** is,

$$M_{A,B} = \begin{pmatrix} \cos\mu_c & \sin\mu_c & 0 & 0 \\ -\sin\mu_c & \cos\mu_c & 0 & 0 \\ 0 & 0 & \cos\mu_d & \sin\mu_d \\ 0 & 0 & -\sin\mu_d & \cos\mu_d \end{pmatrix} \qquad (1)$$

where μ_c and μ_d are the absolute values of phase advance from **A** to **B** along side **c** and **d** accordingly. For simplicity we have assumed that magnitude of horizontal beta function is equal to one and its derivative is equal to zero at both points **A** and **B**.

To get the matrix describing the long range beam-beam interaction consider the kick angle produced by the electromagnetic field of b_2 on b_1. If distance between centers of bunches is much larger than the bunch size, then the change of angle will be $\delta X_1' = 2N_2 r_0/\gamma d$, where N_2 is

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the number of particles in bunch 2, r_0 the classical electron radius, γ, the Lorentz factor, and d is the distance between bunch centers. Note that d is composed of a closed orbit separation, d_0, and $X_{1,2}$ which are the displacements of bunches $b_{1,2}$ relative to the closed orbit, i.e., $d = d_0 + X_1 - X_2$ Assuming $|X_{1,2}| \ll d_0$ we can rewrite formula for angle change as:

$$\delta X_1' = \frac{2N_2 r_0}{\gamma d_0} - \frac{2N_2 r_0}{\gamma d_0^2}(X_1 - X_2) \qquad (2)$$

Here the first term is a dipole kick, which gives a very small orbit distortion that does not depend on X_1 or X_2. The second term is proportional to bunch displacements. It is like a gradient error and couples the motion of the two beams to produce coherent motion. In what follows, we will ignore the first term and only take into account the gradient term. Conceptually this means we must include the effects of the dipole error as a distortion of the closed orbit. In practice, the distortion of the closed orbit is too small to matter.

The long range beam-beam interaction matrix from just before the kick to just after kick will be

$$M_{int} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 4\pi\delta\nu_1 & 1 & -4\pi\delta\nu_1 & 0 \\ 0 & 0 & 1 & 0 \\ -4\pi\delta\nu_2 & 0 & 4\pi\delta\nu_2 & 1 \end{pmatrix} \qquad (3)$$

where $\delta\nu_{1,2} = N_{2,1} r_0 \beta / 2\pi\gamma d^2$ is the tune shift for a single crossing point and $N_{2,1}$ is the number of particles in bunch 2,1. To get a single turn matrix M_{tot} we must make a matrix multiplication:

$$M_{tot} = M_{int} M_{B,A} M_{int} M_{A,B} \qquad (4)$$

where $M_{B,A}$ describes the bunch motion from **B** to **A**. Eigenvalues and eigenvectors of M_{tot} characterize the coherent modes.

In figure 2 we present results of numerical calculation of beam-beam coherent modes as described above. We can see that at low total current, where one bunch is very weak, the tune of the higher frequency mode doesn't depend on bunch intensity, but the lower tune goes down with increasing bunch current. The resulting tune split is proportional to bunch intensity. This picture has a simple interpretation. The unaffected tune belongs to the strong bunch, while the decreasing tune is associated with the motion of the weak bunch. The fact that the tune is decreasing means there is a defocussing effect by large bunch. The tune shift or tune split, both are the same in this case, is the sum of the tune shifts calculated for each of the single crossing points, i.e., $2\delta\nu_1$. An asymmetry in phase advance between **c** and **d** does not matter.

A different situation arises in the case where both bunches have significant intensity. Here the asymmetry in phase advance $\mu_{as} = |\mu_c - \mu_d|$ plays an important role. In figure 2 we see that if the phase advance is zero, i.e., ($\mu_{as} = 0$), the dependence of mode tunes resembles that

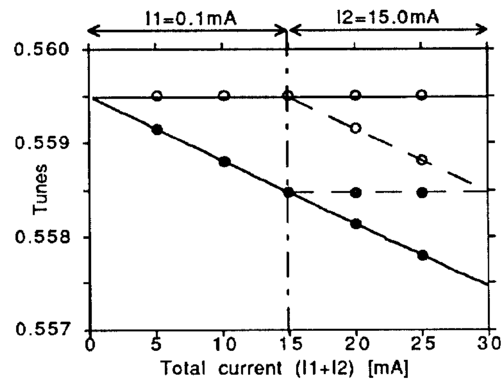

Figure 2: Calculated mode tunes. Open and solid circles refer to the two different modes. Here the effects of asymmetries in the phase advance are evident. Dotted and continuous lines are for the cases of $|\mu_c - \mu_d| = \pi$ or 0. The calculation was done for $\gamma = 10^4$, $d = 29$ mm, $\beta = 40$ m, $N_b = 1.6 \times 10^{11} \times I\ [mA]$.

of the strong-weak case. The tune shift of the lower mode is proportional to sum of bunch intensities and tune split equals to sum of tune shifts at both crossing points. However, if $\mu_{as} = \pi/2$, the higher tune goes down with increasing bunch intensities, while the lower tune remains constant. At the point where the bunch intensities become equal one to another, both coherent beam-beam modes have the same tune shift, which is equal one half the tune split in the $\mu_{as} = 0$ case. Moreover *the tune split equals zero in spite of beam-beam interaction.*

To get a more realistic model which can be compared with experimental data, the beam-wall coherent tune shift and multiple bunches must be taken into account. The beam-wall coherent tune shift should be introduced as an extra phase advance $\delta\mu_{b-w}$ for each bunch. The magnitude of $\delta\mu_{b-w}$ is proportional to bunch intensity and is taken from single beam measurements. To describe a configuration with k bunches per beam, the eigenvalues of matrices of order $2k$ must be evaluated.

Measurements

Three machine studies were carried out at CESR to measure long range tune shifts under different conditions as a function of beam current. The choice of bunches and the pretzel configuration insured that there were no head-on collisions at the normal interaction points. Betatron tune shifts were measured on a spectrum analyzer connected to beam pickup electrodes. For an accurate measurement of the frequency it was necessary to artificially spread the tunes of the two beam enough that the peaks would not overlap. This was done by varying sextupole strengths in the region of separated orbits. The betatron resonance widths were about 1 kHz wide which is larger than many of the tune shifts. Signal averaging and careful attention to the frequency measurement were paid – not always successfully – resulting in a variance of the frequency shift measurements of order 0.1 kHz. The optics used were the same as those used during normal operation of CESR.

The first measurement consisted of one bunch of positrons circulating against one bunch of electrons. The electron bunch was used as a 'probe' beam. Its betatron tunes were measured and its current held constant at 2 mA/bunch while the positron bunch current was reduced from 12 mA to 2 mA. One of the nicer features of this technique is that there is no confusion introduced by frequency shifts due to impedance as the measured beam is held at constant current. The results are summarized in the table below:

Crossing points	Bunch # e^+	Bunch # e^-	measured ν_y $\left[\frac{Hz}{mA}\right]$	theory ν_y $\left[\frac{Hz}{mA}\right]$
2,7	1	7	-15 ± 7	-0.5
3,6	1	6	-8.6 ± 5	-8
4,5	1	5	56.6 ± 6	53.8

The theoretical values of β_v and the betatron phase ϕ_h at the parasitic crossing points are probably only good to about 15% and constitute the main uncertainty in the theoretical values. The variance in the experimental values is dominated by the variance in the frequency measurements. Within these uncertainties the measurements are consistent with the theoretical predictions.

The second set of measurements was quite similar to the first except that 'improved' techniques for frequency measurement and data taking were used, and data was taken at two different pretzel amplitudes. The results are given below:

Bunch e^+	Bunch e^-	ν_x $\left[\frac{Hz}{mA}\right]$ data	ν_x theory	ν_y $\left[\frac{Hz}{mA}\right]$ data	ν_y theory	pretzel
1	5	-6 ± 2	-16.4	60 ± 2	66.3	1200
1	5	-42 ± 5	-36.9	183 ± 3	149	800
1	6	-10 ± 1	-20	14 ± 2	9.3	1200
1	6	-28 ± 2	-44	35 ± 3	21	800

Here there is substantial disagreement between the theory and experiment. The experimental horizontal tune changes are substantially less than the predicted values, while the experimental vertical tune changes are generally somewhat greater than the predictions. If this is to be explained by errors in the assumed beta function values, the horizontal beta functions must be in error by about a factor of 2. More likely is the possibility that the "improved technique" was in fact worse.

In the third machine studies we measured the tuneshifts generated by two trains of 7 bunches each. In this configuration, each bunch undergoes 14 crossings with opposing bunches, each at a different separation distance and beta function [3]. The total tune shift received may be different for different bunches. We measured the only the tune shifts for the highest and lowest frequency modes. The total current in the positron beam was held constant while the current in the electron beam was varied. An attempt was made to keep the individual bunch currents more or less equal for each beam. The results are also summarized the following table.

Mode	ν_x $\left[\frac{Hz}{mA}\right]$ measured	ν_x theory	ν_y $\left[\frac{Hz}{mA}\right]$ measured	ν_y theory
highest	-3.2 ± 1	-12	$21.1 \pm .4$	21
lowest	-25 ± 2	-22	$-32.9 \pm .8$	-33

The tune shift due to impedance was measured during earlier machine studies opportunities and added to the theoretical prediction. In most cases the theoretical and measured slopes of frequency versus total electron current are in good agreement. However, for the data with the highest horizontal frequency, the measured slope is less negative than the theoretical indicating less long range tune shift than expected. One possible reason for the discrepancy is betatron phase errors. Another possible reason for the discrepancy may arise from the implicit assumption that the change of position of the peak in the betatron spectrum is exactly representative of the tune change. This would not be the case when two peaks are overlapping since the peak of the sum of the signals would not follow the tune of each mode independently. This is particularly relevant to the horizontal tune because the frequency spread of the modes is not large and does not increase with higher currents as much as for the vertical modes.

Conclusions

The use of tune splits of coherent beam-beam modes to test parasitic interaction points, analogous to the use of $\pi - mode$ and $\sigma - mode$ for head-on collisions, may lead to confusing results. Under certain conditions the tune split may be reduced, moreover it may be zero in spite of a strong beam-beam interaction. The best way to study the tune shifts due to the long range beam-beam interaction is to use one bunch per beam and measure the dependence of the coherent tunes on the intensity of one of the bunches keeping the intensity of the other bunch fixed and quite small. Only in this case can you be sure that the tune shift of smallest bunch will be equal to sum of tune shifts for the single interaction points.

References

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