Measurement and Analysis of Transverse Beam Transfer Functions in the Fermilab Main Ring

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Abstract

Beam transfer function measurements provide accelerator impedance, beam frequency spread and effective feedback gain information. An analysis technique is used to derive these quantities from beam measurements. Transverse bunched beam measurements in the Fermilab Main Ring are reported.

I. INTRODUCTION

It has been known for many years that the Main Ring suffers from a vertical coupled-bunch instability at injection [1, 2]. An extensive study project is underway to find the offensive modes and the impedance source which causes this instability. Beam transfer function measurements are a useful method for determining the beam impedance, effective damper gain and frequency space distribution. Beam transfer function(BTF) for coasting beam and bunched beam cases are measured. The development of a theory for the bunched beam transfer function is also underway.

II. COASTING BEAM THEORY

A. Closed-Loop BTF Measurements

The beam is a collection of particles, each having its own oscillation frequency $q_i\omega_0$ driven by a common external force amplitude $F(\omega)$ where q_i is the fractional tune and ω_0 is the angular revolution frequency. The equation for each individual particle is

$$\ddot{x}_i + (q_i\omega_0)^2 x_i = F(\omega)\cdot\exp(j\omega t) \qquad (1)$$

The BTF is defined as [3]

$$B(\omega) \equiv \frac{j\omega\langle x_i\rangle}{F(\omega)} = \frac{1}{2}[\pi\rho(\omega) + j\cdot pv\int\frac{\rho(q_i\omega_0)d(q_i\omega_0)}{q_i\omega_0-\omega}] \qquad (2)$$

where

$$\rho(\omega) = \frac{2}{\pi}\mathrm{Re}[B(\omega)] \qquad (3)$$

$j=-i$, $\rho(q_i\omega_0)$ is the normalized distribution of the betatron frequency among the particles.

When there is a self-coupling force due to the transverse impedance[4], eq.(1) is changed to

$$\ddot{y}_i + (q_i\omega_0)^2 y_i = F(\omega)\cdot\exp(j\omega t) - j\frac{eI_0Z_\perp}{mL}\langle y_i\rangle \qquad (4)$$

where e and m are the charge and mass of each individual particle respectively, I_0=total beam intensity, L=accelerator circumference and $Z_\perp$ is the transverse impedance. The coupled BTF $B'(\omega)$ is given by $j\omega\langle y\rangle/F(\omega)$ and has the following relation

$$\frac{1}{B'(\omega)} = \frac{1}{B(\omega)} + H(\omega) \qquad (5)$$

with $H(\omega) = eI_0Z_\perp/m\omega L$. The above equation represents a feedback loop as depicted in Fig.1 .

If a damper system is used to cure an instability, this will add a damping term to the right hand side of eq.(3), say $-jI_0G\langle y_i\rangle$,.where G is complex. Then the equation for the new feedback loop becomes

$$\frac{1}{B'(\omega)} = \frac{1}{B(\omega)} + H(\omega) + D(\omega) \qquad (6)$$

with $D(\omega) = GI_0/\omega$. The new feedback loop is depicted in Fig.2 . A plot of imag(1/B') vs. real(1/B') gives us the well known stability diagram. The effect of impedance and damper system is to shift the unperturbed stability diagram (1/B) by

$$\frac{eI_0Z_\perp}{m\omega L} + \frac{GI_0}{\omega} \qquad (7)$$

At lower intensity, an open loop BTF through a damper system can generate the betatron frequency distribution. We can determine the contour shift by comparing with the unperturbed stability diagram from calculation. A plot of contour shift vs. beam intensity provides us the information about the coupling impedance and damper gain.

B. Open-Loop BTF Measurements

The experimental setup is shown in Fig.3 and the corresponding loop diagram depicted in Fig.4. The output signal is

$$V_{out} = B(-DV_{in} - HV_{out}) \qquad (8)$$

The measured BTF is

$$B'(\omega) = \frac{-D(\omega)B(\omega)}{1+H(\omega)B(\omega)} \qquad (9)$$

Furthermore, the open-loop stability diagram is described by

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$$\frac{1}{B'(\omega)} = \frac{-1}{D(\omega)B(\omega)} - \frac{H(\omega)}{D(\omega)} \qquad (10)$$

When the self-coupling effect is very small, the BTF is approximately equal to $-D(\omega)B(\omega)$.

III. MEASUREMENTS

Beam transfer functions are measured with coasting and bunched beam in the Main Ring at 8 GeV flat top. Fig.5, Fig.6 and Fig.7 are results of coasting beam case for n=1053+q betatron sideband. The revolution frequency is 47.4 kHz and vertical tune is 0.4 for the Main Ring. Fig.5 and Fig.6 give the amplitude and phase of BTF respectively. Fig.7 is the distribution of betatron frequency spread derived from eq.(3). Due to limited beam study time, we did not average data from multiple cycles, a process which significantly improves the signal to noise ratio of the results. The presented signals are too noisy to provide a smooth enough stability diagram. We can see a substantial shift in the coherent frequency for different beam intensity from Fig.5 to Fig.7. This frequency change is proportional to the product of beam intensity and transverse impedance. A measurement of coherent frequency shift vs. beam intensity will give the transverse impedance. Fortunately we also do have averaged data for bunched BTFs. The results are depicted in Fig.8, Fig.9 and Fig.10. It is necessary to calibrate the experimental apparatus in order to derive the impedance and damper gain from the stability diagram, see eq.(7). Further work is underway to attain quantitative results from BTF measurements.

IV. REFERENCES

[1] R. Stiening and E.J.N. Wilson, Nucl. Instr. and Methods 121 (1974) 283-285.
[2] G. Jackson, Proc. IEEE Part. Acc. Conf. , (1991) 1755.
[3] D. Boussard, CERN SPS/86-11(ARF).
[4] G. Nassibian and F. Sacherer, Nucl. Instr. and Methods 159 (1979) 21-27.

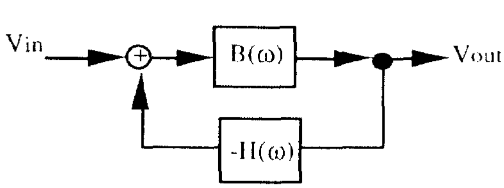

Fig.1: feedback loop with the coupling impedance.

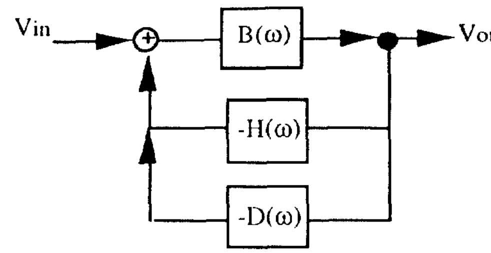

Fig.2: feedback loop with the coupling impedance and damper system.

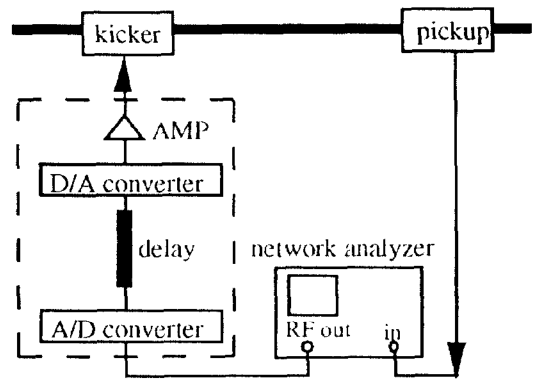

Fig.3: experimental setup for BTF measurements.

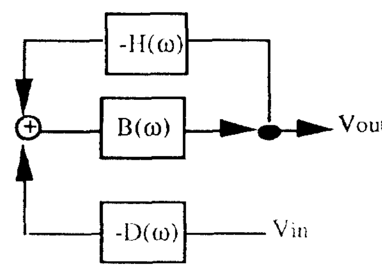

Fig.4: loop diagram corresponding to the BTF measurements.

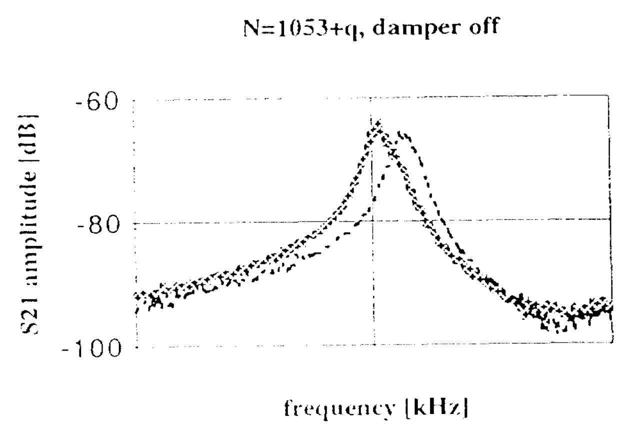

Fig.5: BTF measurement for coasting beam -- amplitude, dashed line corresponds to low beam intensity.

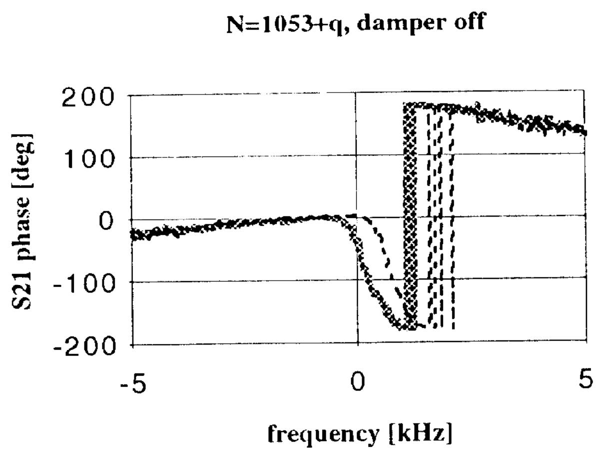

Fig.6: BTF measurement for coasting beam -- phase, dashed line corresponds to low beam intensity.

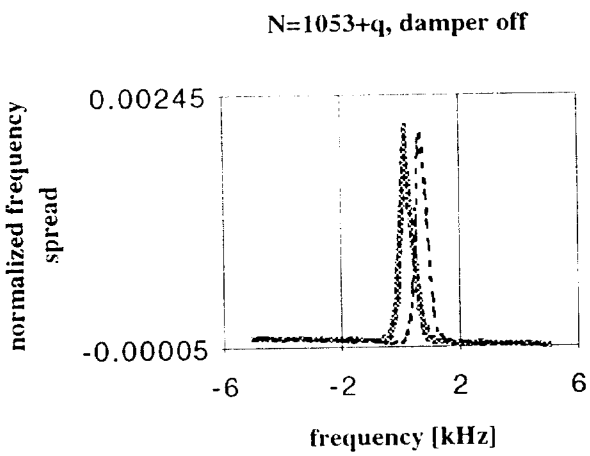

Fig.7: distribution of frequency spread derived from eq.(3), dashed line corresponds to low beam intensity.

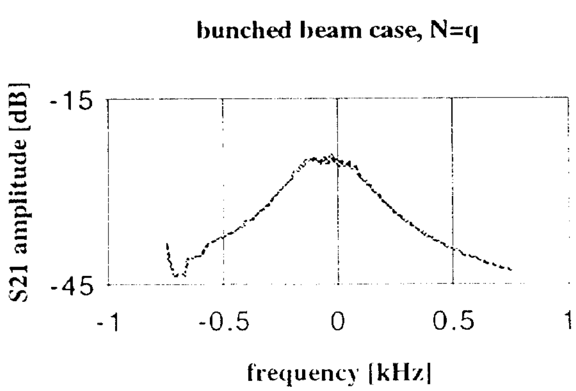

Fig.8: bunched BTF measurement -- amplitude.

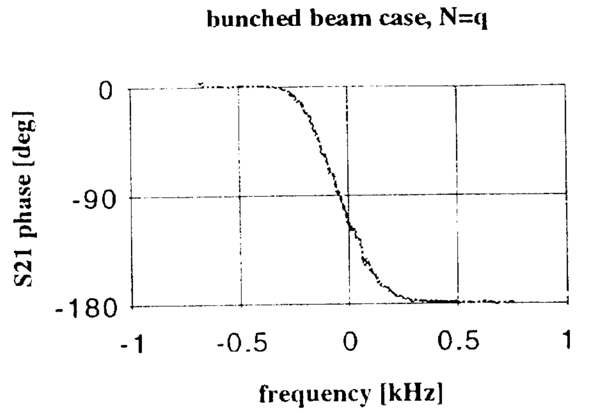

Fig.9: bunched BTf measurement -- phase.

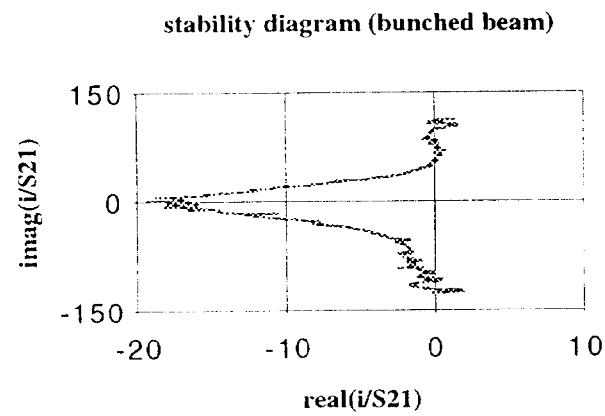

Fig.10: bunched BTF measurement -- stability diagram.

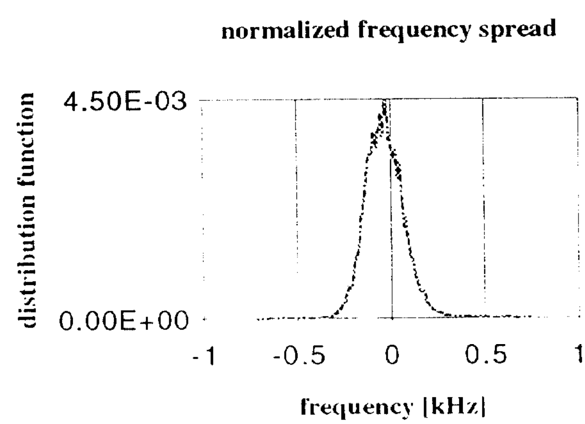

Fig.11: bunched BTF measurement -- distribution of frequency spread derived from eq.(3).