Nonlinear evolution of longitudinal bunched-beam instabilities.

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Abstract

Numerical results of the nolinear evolution of longitudunal instabilities of bunched beams are presented. Saturation effects due to the decoherence (tune spread) are categorized according to the magnitude and type of impedances. New phenomenon of non-saturating instability (beam splitting) is described.

I. INTRODUCTION

Nonlinear stages of development of coherent instabilities have been studied by a few authors /1-3/ in relation to the longitudinal instability of the coasting beam. Some numerical simulation studies were carried out for the bunched beam, with the emphasis on the thresholds of instabilities /4/. Numerical simulation results /1/ indicated that the longitudinal instability of the coasting beam always saturates and eventually decays due to the effect of decoherence. In the present paper, we undertake a numerical simulation study of the nonlinear development of the longitudinal instability of bunched beams. The saturation effects due to the decoherence from the tune spread are the primary objects of interest. More detailed presentation of this study is available in Ref./8/

II. THE MODEL.

The simulation is carried out for the model that consists of a single bunch interacting with a localized single-mode longitudinal wakefield under the assumption of the short bunch length relative to the wavelength of the wakefield (Long wavelength/ low frequency approximation in the analysis of Ref./5/):

$$\ddot{x}_i + \omega_s^2 x_i - \lambda x_i^3 = \sqrt{\frac{\epsilon}{N}} q \delta_{2\pi}(t) \qquad (1)$$
$$\ddot{q} + \alpha \dot{q} + \omega_c^2 q = \sqrt{\epsilon N} \bar{x} \delta_{2\pi}(t)$$

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where time is normalized to make the revolution frequency $\omega_0 = 1$, x_i $(i = 1, N)$ is the coordinate of the i-th particle, $\bar{x}$ is the coordinate of the centre of gravity of the beam $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$, and q is the amplitude of the wakefield. The quantity ϵ measures the strength of interaction in the continuous limit $N \to \infty$, while $\delta_{2\pi}$ is 2π-periodic δ-function. Frequencies ω_s and ω_c are respectively the synchrotron and resonant impedance frequencies. The constant $\lambda > 0$ measures the nonlinearity of the RF potential well and is always small in our study.

We will assume the interaction strength ϵ together with the tune spread ω_s to be small, $\delta\omega_s \ll \alpha$ (where $\delta\omega_s$ is the tune spread $\delta\omega_s = \frac{3}{8\omega_s}\lambda\langle x^2\rangle$), $\epsilon \ll \alpha\omega_s^2\omega_c$. In the zero tune spread limit, the complex coherent frequency shift $\Delta\omega_s = \omega - \omega_s$ can be found (see, e.g., /5/)to be:

$$\Delta\omega_s = \frac{i\epsilon\tilde{Z}}{2\omega_s} \qquad (2)$$

where the complex effective impedance $\tilde{Z} = \tilde{Z}_r + i\tilde{Z}_i$ is defined as $\tilde{Z} = Z(\omega_s)$, with the regular frequency-dependent impedance:

$$Z(\omega) = -\frac{i}{(2\pi)^2}\sum_n \frac{1}{\omega_c^2 + i\alpha(\omega+n) - (\omega+n)^2} \qquad (3)$$

For the finite value of the tune spread, the relevant (real) parameters that define the nonlinear evolution are $\delta\omega_s$, $Re(\Delta\omega_s)$ and $Im(\Delta\omega_s)$. One of these parameters defines the units of time, so that the evolution depends essentially on two dimensionless parameters:

$$C_r = -\frac{Re(\Delta\omega_s)}{\delta\omega_s}$$
$$C_i = -\frac{Im(\Delta\omega_s)}{\delta\omega_s} \qquad (4)$$

III. SCENARIOS OF EVOLUTION.

The evolution in model (1) is directly simulated by using many particles and implementing the single-turn mapping. In that mapping, the nonlinearity of oscillations λ is treated perturbatively, i.e. the mapping for $x_i, \dot{x}_i$ between the δ-functional "kicks" is that of a linear oscillator with

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the shifted frequency. The number of particles N was taken to be large enough to reproduce the continuous limit. We present in Figs.1-4 four characteristic cases of instability evolution. These cases are representative of four different scenarios that we loosely define by the relative strength of instability (distance from the threshold):

I) Strong instability $|C_r| \gg C_{rcr}$, $|C_i| \gg C_{icr}$

II) Weak instability $|C_r| \sim C_{rcr}$, $|C_i| \sim C_{icr}$

and by the type of impedance:

a) *Reactive impedance* $|C_r| > |C_i|$ (or $|Z_i| > |Z_r|$)

b) *Active impedance* $|C_r| < |C_i|$ (or $(|Z_i| < |Z_r|)$

The quantities C_{rcr}, C_{icr} are the critical (i.e.corresponding to the stability border) values defined by the ratio C_r/C_i. Stability border in C_r, C_i plane is shown in Fig.5.

Examples of the time dependencies of centroid position $\bar{x}(t)$ and emittance $\sigma(t) = \langle\langle x^2\rangle\rangle$ (averaged over the particles and the fast synchrotron oscillations) for four different scenarios are shown in Figs.1-4. Time scale is given in turns, and the scaling factor is the instability rise time in the absence of tune spread $\tau_{gr} = 1/Im(\Delta\omega_s)$.

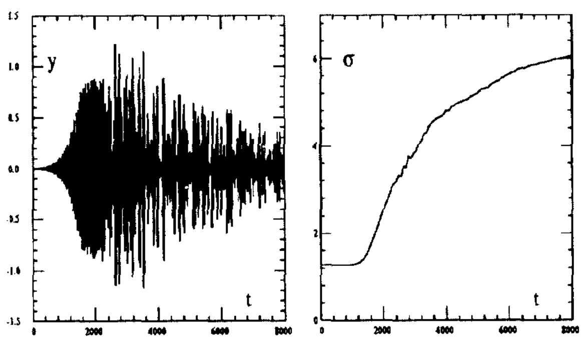

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Fig.1. (a) Centroid oscillations $y = \bar{x}(t)$ and (b) Emittance growth $\sigma(t)$ for the case of (Strong instability, Reactive impedance) with $C_r = 4.16$, $C_i = 1.65$, and $\tau_{gr} = 188$..

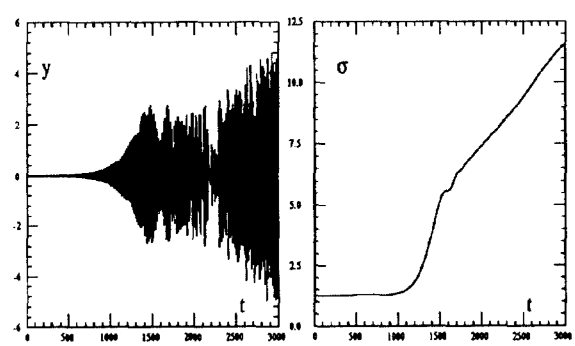

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Fig.2. (a) Centroid oscillations $y = \bar{x}(t)$ and (b) Emittance growth $\sigma(t)$ for the case of (Strong instability, Active impedance) with $C_r = 1.18$, $C_i = 4.40$, and $\tau_{gr} = 142$.

The centroid oscillations in Figs.1-4 present itself a fast-oscillating sinusoidal signal (with the synchrotron frequency) with a slow-changing envelope. The initial growth demonstrates the saturation at some level. After that, one observes some apparently random "turbulent" oscillations. In the *Strong instability* regime the envelope of oscillations grows monotonically until the saturation. For the *Weak instability*, the envelope of oscillations is not a monotonically growing function of time even before the saturation. The first maximum in the envelope of oscillations occurs early before the saturation and is quite small. It is followed by several more maxima of increasing amplitude before the saturation. The maximally attainable amplitudes of centroid oscillations are much smaller than in the case of a strong instability.

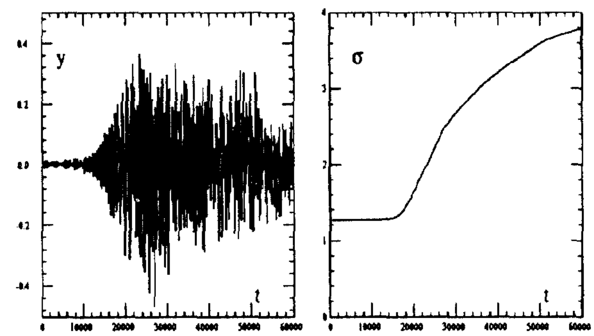

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Fig.3. (a) Centroid oscillations $y = \bar{x}(t)$ and (b) Emittance growth $\sigma(t)$ for the case of (Weak instability, Reactive impedance) with $C_r = 1.25$, $C_i = .49$, and $\tau_{gr} = 315$..

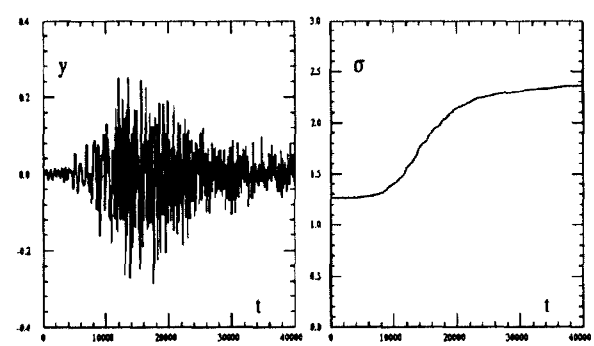

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Fig.4. (a) Centroid oscillations $y = \bar{x}(t)$ and (b) Emittance growth $\sigma(t)$ for the case of (Weak instability, Active impedance) with $C_r = .24$, $C_i = .89$ and $\tau_{gr} = 175$..

IV. Beam splitting.

The *II*b *(Strong instability, active impedance)* example of Fig.2 shows peculiar non-decaying and even slightly growing oscillations after the saturation. More insight into this behavior is provided by a few phase space snapshots in

Fig.2c. Time labels are in the same scale as in Fig.2a.

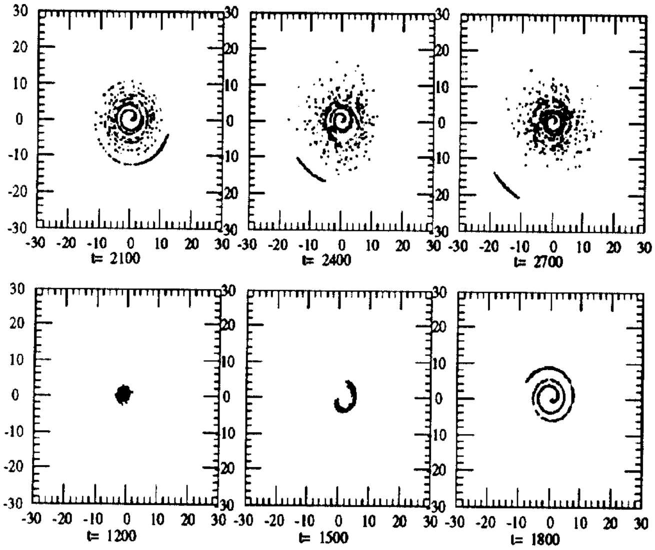

Fig.2c. Phase space snapshots for the parameters of Fig.2a and b.

One can see that the particles that happen to be at the larger radia (amplitude of oscillations) at the moment of saturation start moving toward increasing radia and finally form a distinct beamlet that is separate from the core of the beam. This beamlet does not decohere but oscillates with increasing amplitude as a rigid entity .

The results of a special-purpose series of simulations to explore the onset of beam splitting are presented in Fig.5. The beam splitting was diagnosed by observing a non-decohering beamlet in the phase space.

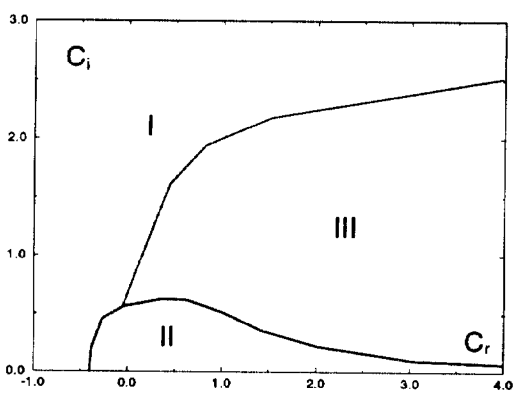

Fig.5 State diagram of instability. I is the beam splitting region, II is stable region and III is unstable, no beam splitting region.

One interesting feature of Fig.5 is the overlap of stability border and splitting border for negative C_r. For large negative C_r one can observe that the beam does not actually split, but rather oscillates as a whole with increasing amplitude without decohering.

The border of the beam splitting is a "soft" one, i.e. the percentage of particles trapped in the "beamlets" is approaching zero when approaching the border. In some cases one can also see several "beamlets" succesively splitting from the core of the distribution.

V. Discussion and Conclusions.

The most interesting nonlinear effect observed is the beam splitting phenomenon. We suggest to explain, or rather interpret it as the trapped-particles nonlinear modes, in extension of a similar concept of persistent nonlinear (BGK) waves in plasma physics (see, e.g. /6/). We expect by that analogy that in our system a group of particles can be trapped, under certain conditions, near the center of self-driven nonlinear resonance. The elongated shape of the "beamlet" in Fig.2c corraborates that interpretation. The resonant frequency will change in time under the influence of (anti)dissipative impedance component Z_r, carrying the trapped particles towards larger radia. The difference with conventional BGK modes is in this (anti)dissipation in the system causing the frequency sliding. We suggest thus the term "sliding trapped (BGK) modes".

Beam turbulence is another important class of essentially nonlinear phenomena. Even when after the saturation of instability the emittance becomes large enough to make a beam stable for any smooth bell-shaped distribution, the small-scale "microstructure" of the density can persist in the beam for a long time, causing random-like low-level centroid oscillations and slow emittance growth.

In our case of fast synchrotron oscillations $\delta\omega_s \ll \omega_s$ emittance growth can be related to the amplitude of centroid oscillations $\bar{x} = a\sin(\omega_s t)$ through the convinient scaling law:

$$\frac{d\sigma}{dt} = \frac{A^2}{\tau_{gr}} \tag{5}$$

where τ_{gr} is the instability rise time in the absence of the tune spread $\tau_{gr} = \frac{1}{Im(\Delta\omega_s)}$. A theoretical quasilinear "overshoot" approach /7/ for predicting $\sigma(t)$ and $A(t)$ was developed in Ref./7/ along the same lines as in the coasting beam theory /2-3 /.

VI. Acknowledgements

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