Simulation of the Transverse Dipole Mode Multibunch Instability for the SSC Collider

S. Chen and G. López
Superconducting Super Collider Laboratory*
2550 Beckleymeade Ave., Dallas, TX 75237 USA

Abstract

We investigate a simulation code to study the transverse dipole mode multibunch instability due to a single RF cavity and the resistive wall for the Superconducting Super Collider. The growth time calculated from our code agrees well with analytical calculations (ZAP code). The code can be used for studying the interplay of wake field, noise and feedback system, i.e. for design the feedback system.

I. INTRODUCTION

The Superconducting Super Collider (SSC) Collider will be the first machine operating with 17424 bunches of protons separated by a relatively short distance [1], 5 m. The bunches are injected from the High Energy Booster (HEB) into the Collider, which requires eight HEB batches to fill almost symmetrically (there is an abort gap) one of its two rings. The batches are injected into the upper and lower ring one at a time. To inject one batch in the same ring requires approximately 515 s, and the eight batches are injected in approximately 1.14 h. Therefore, there is a great deal of concern about possible multibunch instabilities that the beam might suffer during this time. To study the transverse dipole mode (rigid motion) multibunch instability, a computer code was developed to calculate the instability growth time due to Positron-Electron Project (PEP) RF cavity and resistive wall impedance. The results are compared with the analytical approach, and the feedback system design is incorporated to control this instability.

II. TADIMMI CODE AND RF-INSTABILITY

Ignoring the possible coupling of different directions in the motion , the transverse amplitude of motion, Y, of a point-like bunch suffering the electromagnetic wakefield interaction, ww, can be described by the differential equation

$$d^2Y/dt^2 + \omega_\beta^2 Y = ww \; , \qquad (1)$$

where ω_β is the free angular betatron oscillation frequency. For simplicity, let us assume the following: there is a single cavity in the ring where there is a point-like interaction, the ring is perfectly linear, and the ring has M equally pair-spaced bunches. With these assumptions the short-range wakefield can be omitted, and the long-range wakefield is given by the Higher Order Modes (HOM) of the RF cavity. Since the length of the RF cavity is usually much smaller than the betatron wave length, the betatron phase advance within the RF cavity can be ignored. When the k-bunch passes through the RF cavity for $n+1$ turn, the transverse position does not change, $Y(k,n+1) = Y(k,n)$, but its momentum is changed by the wakefield of the RF cavity. For dipole mode wakefield, the change in the transverse momentum is

$$\acute{Y}(k,n+1) = \acute{Y}(k,n) + \frac{Ne^2}{E}\sum_{m=1}^{n}\sum_{J=1}^{M} D(j)Y(J,m)W(s/c) \; , \qquad (2)$$

where the summation is carried out over the wakefield left behind for all the previous bunches and turns. The variable s can be written in terms of the space between bunches, S_B, as $s = (k-J)S_B + (n-m)MS_B$, where $(k-J)$ and $(n-m)M$ represent the relative bunch and turn numbers. N is the total number of protons in the bunch, e is the proton charge, $E = \gamma m_o c^2$, c is the speed of light, m_o is the rest mass of the particle, and $W(s/c)$ is the wake function defined as [2]

$$W(s/c) = \sum_\lambda A_\lambda \exp(-\alpha_\lambda s/c)\sin(\Omega_\lambda s/c) \; , \qquad (3)$$

if $s > 0$, and it is equal to zero otherwise. The variables A_λ, α_λ, and Ω_λ are defined as $A_\lambda = \omega_\lambda^2 R_\lambda/\Omega_\lambda Q_\lambda$, $\alpha_\lambda = \omega_\lambda/2Q_\lambda$, and $\Omega_\lambda = \omega_\lambda\sqrt{1-1/4Q_\lambda^2}$, respectively. The summation is carried out over all the transverse HOM of the cavity. Each cavity-mode is characterized by the resonant angular frequency ω_λ, the shunt impedance R_λ, and the resonant quality factor Q_λ. The quantity $c/\alpha_\lambda S_B = 2Q_\lambda c/\omega_\lambda S_B$ gives us the number of bunches that the exited transverse HOM can affect before it decays by a factor of $1/e$. The factor $D(j)$, $0 \leq D(j) = N(j)/N \leq 1$, defines the distribution of bunches and the number of protons per bunch. The bunch is transported from the output to input of the RF cavity using the Courant-Snyder [3] map and together with the Eqs. (2) and (3) comprise the model for simulating transverse multibunch instabilities in the computer program TADIMMI.

A 358.9 MHz PEP cavity will be taken as the test cavity. At the RF location, the dispersion function is zero, and the Courant-Snyder parameters have the following values: $\beta = 112472.0$ mm, $\alpha = 0$, and $\mu = 2\pi\text{frac}(\nu) = 1.759292$. The characteristics of the bunches are $N = 7.5 \times 10^9$, $M =$

*Operated by the Universities Research Association, Inc., for the U.S. Department of Energy under Contract No. DE-AC35-89ER40486.

0-7803-1203-1/93$03.00 © 1993 IEEE

17424, $E = 2$ TeV, and $S_B = 5$ m.

The calculated growth time was 9 s per cavity. This simulation result for uniform distribution ($D(j) = 1$) is in very good agreement with the analytical approach (ZAP [4]). Studies show that damping several dangerous rf-HOMs of the cavity is not enough to avoid the instability. One way to control the multibunch instabilities rising from the HOM cavities is DE-Q these HOM. For the growth time to be higher than the injection time, the following De-Q quality factor, Q_d, for N_c cavities is deduced: $Q_d \leq 4.8 \times 10^6\ \Omega\ \mathrm{m}^{-1} / < R_\perp/Q > N_c$, where $< R_\perp/Q >= 125\ \Omega/\mathrm{m}$, $N_c = 32$, and $Q_d \leq 1200$ for the PEP cavity model.

A single cell normal cavity with HOM-couplers or a single cell superconducting (sc) cavity with couplers [5] can satisfy this restriction to a large extent. Another way to control multibunch instabilities is to use an active system such as a feedback system [6]. The normal operation of the transverse feedback system is to have a beam position monitor (BPM) that measures the transverse displacement of the bunch. The signal is amplified and transmitted to the kicker (K), located downstream at a phase advance of $\pi/2$, which produces an angular deviation to the bunch given by [7]

$$\Delta \acute{Y} = g\ (Y_{BPM} + \delta Y)/\sqrt{\beta_{BPM}\beta_K} \qquad (4)$$

where g is the gain of the system, β_{BPM} and β_K are the beta function at the location of the BPM and the kicker, Y_{BPM} and δY are the displacement measured by the BPM and the error transmitted of this displacement (random variable with a Gaussian distribution). This error is called the resolution of the BPM and is due to the electronic noise in the system (white noise). Using the following values of the Collider west utility region $\beta_{BPM} = \beta_K \approx 420$ m, $g = 0.1$, $\delta Y = 10\ \mu$m, and with the bandwidth (BW) of the feedback system assumed to be $BW \geq 30$ MHz, the damping of the dipole mode multibunch instability was verified with the simulations for the same-turn and one-turn delay correction schemes.

III. RESISTIVE WALL INSTABILITY

The resistive wall instability is important for angular frequencies, ω_k, close to the angular revolution frequency, ω_o; i.e., $\omega_k = \omega_o(\Delta_\beta + k)$, where Δ_β is the fractional part of the tune of the machine such that $-1/2 < \Delta_\beta < 1/2$. Each k corresponds to one possible multibunch mode of oscillation. For lower modes and at the frequency ω_k, the real part of the transverse resistive wall impedance can be approximated by a resonant impedance, where the resonant frequency is ω_k, the shunt impedance is just $Re[Z_\perp(\omega_k)]$, and the quality factor Q_o can be selected to approximate the shape of the resistive wall impedance. Therefore, the problem is reduced to the previous multibunch instability due to an RF cavity, and the above program can be applied using this new resonator impedance. This is important since the explicit expression of the copper coated resistive wall wakefield is too complicated, and computations would require huge amounts of interactions among the bunches per each turn. Figure 1 shows the phase space generated by the bunches due to this instability. Figure 2 shows the comparison of the simulations with ZAP analytical code.

Table 1. Performance for $-1 + \Delta_\beta = -0.75$.

Feedback System	One BPM-One K	TBK
BPM Resolution $\delta Y(\mu\mathrm{m})$	≤ 100	≤ 500
Gain (g) g	≥ 0.30	≥ 0.03

For the nominal Collider beam tube parameters, the growth time of the instability is 56 ms. To control this instability, a conventional one BPM-one K and a novel two BPM-two K (TBK), configured as B1-K1-B2-K2, feedback systems were used. The kick is given by Eq. (4), but instead of using Y_{BPM}, the p-turns average displacement of the bunches at the BPM,

$$< Y >_p = \sum_{i=1}^{pM} Y_i^{BPM}/pM \qquad (5)$$

, was used. That is, each bunch receives p-times the same correction. This implies that the kicker flat-top time, ft, will be

$$ft = p(C/c) = p \times 2.904 \times 10^{-4}\ \mathrm{s}\ . \qquad (6)$$

The bandwidth of the feedback system is assumed to equal that defined by the batch-to-batch separation ($\sim 1.7\ \mu$s), $BW \geq 0.3$ MHz. Table 1 summarizes the calculations for the two damper systems.

The copper and stainless steel thicknesses were taken as 0.1 mm and 1.0 mm. The kickers' flat-top was that with $p = 2$. Figure 3 shows the damping effect on average displacement of the bunches, as recorded by the BPM1, using the TBK damping system. The behavior of one particular bunch (number 1000) as seen by the BPM1 in the TBK damping system is shown is Figure 4.

IV. COMMENTS AND CONCLUSIONS

A computer program, TADIMMI, was developed [8] to study the dipole mode multibunch instability in the SSC Collider due to RF cavities. A resonant impedance approximation to the resistive wall impedance at low frequencies allows us to use the same code to study this instability. The design of the feedback sytem to control these instabilities was incorporated in the code. A two BPM- two K feedback system may be required to control the resistive wall instability since it has much better control of this instability than the conventional one BPM-one K. Study of the feedback system noise effect on the emittance growth is needed.

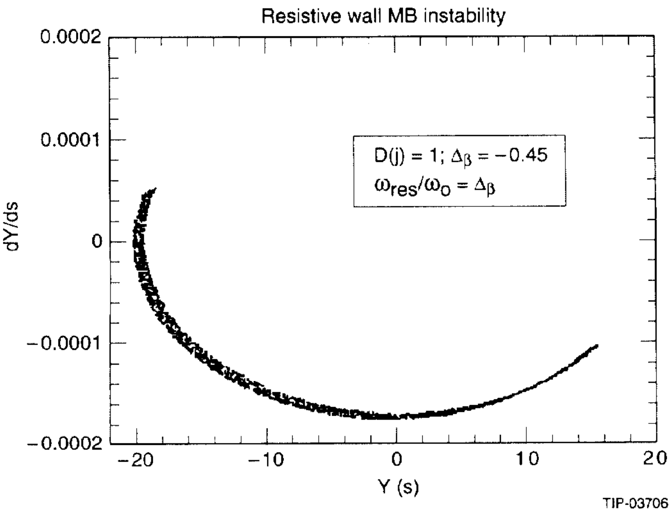

Figure 1. Bunch-generated phase space due to resistive-wall instability.

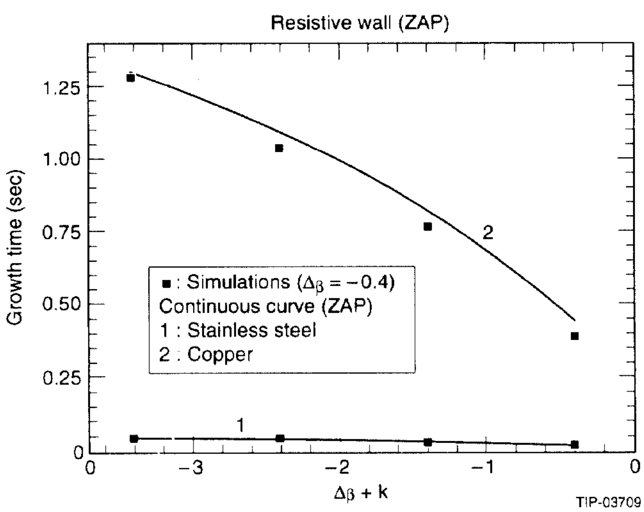

Figure 2. Comparison of simulations and analytical code.

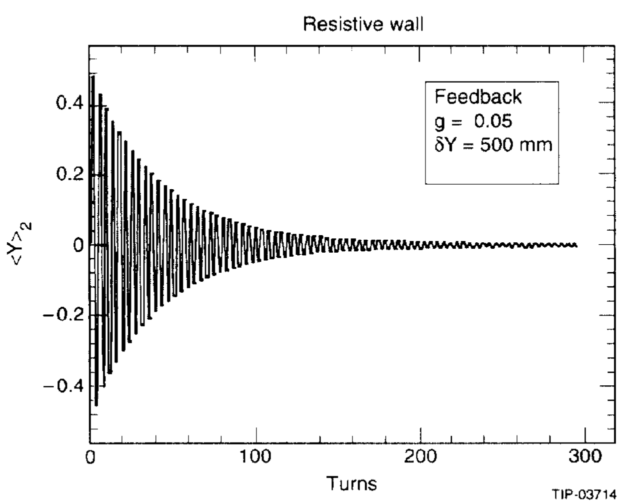

Figure 3. Average displacement in TBK damping system.

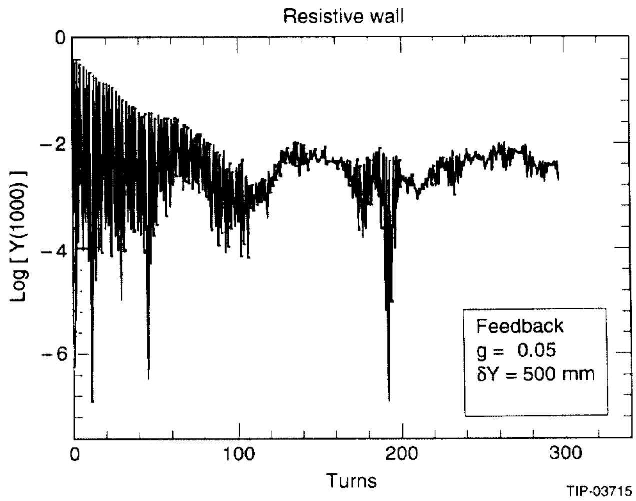

Figure 4. Bunch 1000 in TBK damping system.

V. ACKNOWLEDGEMENTS

We wish to thank Dr. A. Chao for many valuable discussions and suggestions, and Dr. R. Meinke, Dr. R. Gerig, and Dr. R. Schwitters for their support at the SSC.

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