ANALYTICAL STUDY OF RFQ CHANNEL BY MEANS OF THE EQUIVALENT CHARGES MODEL

Vitaly M. Pirozhenko and Oleg V. Plink

Moscow Radiotechnical Institute
113519 Moscow, Russia

Abstract

A simple model of RFQ channel is proposed. The model represents the periodic chain of twin point charges with a screw symmetry axis and alternating sign of the charges. For this approach the expressions both for longitudinal and azimuthal potential harmonics are determined in closed forms. The relationship of model parameters with RFQ geometry is defined. The results of calculations for different vane shapes are given including idealized electrodes and electrodes with extremely large modulation.

1. INTRODUCTION

In most cases the well-known concept of idealized electrodes or two terms potential is used when calculating the main parameters of RFQ accelerator [1]. Further more accurate definition is accomplished by means of computer simulations [2]. The use of electrodes with large modulation was supposed to increase both acceleration rate and output energy of RFQ accelerators [3,4]. However the difficulties of analytical and numerical studies of the RFQ with large modulation restrict the application of this system.

In this paper the equivalent charges model is proposed to calculate the RFQ parameters. The method of the equivalent charges was earlier used to study axisymmetric channel with alternating sign accelerating field [5]. It allowed to define harmonic contents of the field in explicit form, tha channel capacitance per unit length, the transit time factor and the field enhancement factor. Optimization of the axisymmetric channel was also carried out by means of the equivalent charges description of the channel [5].

The equivalent charges model of the RFQ channel is described and studied below. On base of this, RFQ geometry is defined for the various vane shapes. A number of expressions was earlier obtained in the paper [6].

2. MODEL DESCRIPTION

To model the RFQ channel an endless periodic chain of twin point charges is used (fig.1). The system has a screw symmetry axis. The sign of charges alternates along the channel axis. The charges of similar polarity are located periodically in pairs. The period of the system is $2h = \beta\lambda$. The distance from the charges to the channel axis b and the value of the point charge Q are the fitting model parameters. For this approach various vane configurations can be generated by means of metallizing different isopotential surfaces for different values of b. The value of Q is arbitrary as a matter of fact.

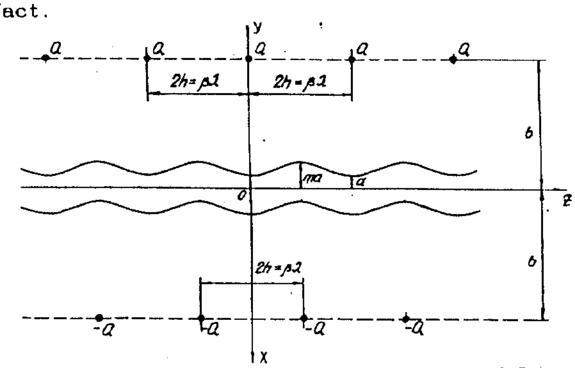

Figure 1. RFQ equivalent charges model

The minimal distance from the longitudinal profile of a chosen equipotential to the channel axis gives the channel bore radius a. the maximum distance defines the modulation factor m.

3. LONGITUDINAL HARMONICS OF POTENTIAL

The potential for the system in cylindrical coordinates can be expressed in the following form:

$$\varphi(r,\theta,z) = \frac{Q}{4\pi\varepsilon_0}\sum_{n=-\infty}^{\infty}\sum_{i=0}^{3}(-1)^i\left((\nu_{in}h - z)^2 + R_i^2\right)^{-1/2}$$

where $\varepsilon_0 = 8.85$ pF/m,

$$R_i^2 = r^2 + b^2 - 2rb\sin(\theta + i\pi/2),$$

$$\nu_{in} = \begin{cases} 2n & \text{for } i = 0,2 \\ 2n+1 & \text{for } i = 1,3 \end{cases}$$

The longitudinal Fourier series expansion of the potential is

$$\varphi(r,\theta,z) = \tau\sum_{l=0}^{\infty} A_l(r,\theta)\cos lkz.$$

where $\tau = Q/(2\pi\varepsilon_0 h)$, $k = \pi/h$.

Taking into account the relations

$$\int_0^{\infty}\left((c_1^2 + x^2)^{-1/2} - (c_2^2 + x^2)^{-1/2}\right)dx = \ln c_2/c_1,$$

$$\int_0^{\infty}(c^2 + x^2)^{-1/2}\cos\lambda x\,dx = K_0(\lambda c).$$

where $K_0(x)$ is the modified Bessel function, we have the expressions for longitudinal harmonics of the potential:

$$A_0(r,\theta) = h^{-1}\int_0^h\varphi(r,\theta,z)dz = 0.5\sum_0^3(-1)^{i+1}\ln R_i,$$

$$A_l(r,\theta) = 2h^{-1}\int_0^h\varphi(r,\theta,z)\cos lkz\,dz = \sum_0^3(-1)^{i(l+1)}K_0(lkR_i).$$

The axis potential distribution coincides with the axisymmetric case [5]:

$$\varphi(z) = 4\cdot\tau\sum_0^{\infty}A_{2l+1}(0)\cos((2l+1)kz),$$

with $A_{2l+1}(0) = K_0((2l+1)kb)$.

4. AZIMUTHAL HARMONICS OF POTENTIAL

For further transformations the following addition theorems are used:

$$\ln R/b = -\sum_1^{\infty}(r/b)^s s^{-1}\cos s\theta$$

$$K_0(\lambda R) = K_0(\lambda b)I_0(\lambda r) + 2\sum_1^{\infty}K_s(\lambda b)I_s(\lambda r)\cos s\theta.$$

where $R^2 = r^2 + b^2 - 2rb\cos\theta$, $r < b$,
$I_s(x)$, $K_s(x)$ are modified Bessel functions.

Then the longitudinal Fourier coefficients are connected with azimuthal ones

$$A_0 = -\sum_0^{\infty} A_{0,2s+1}(\alpha)\cos 2(2s+1)\theta.$$

$$A_{2l} = -8\sum_0^{\infty} A_{2l,2(2s+1)}(\beta.\gamma)\cos 2(2s+1)\theta.$$

$$A_{2l+1} = 4A_{2l+1,0}(\beta.\gamma) + 2\sum_1^{\infty} A_{2l+1,4s}(\beta.\gamma)\cos 4s\theta.$$

where $A_{0,s}(\alpha) = \alpha^{2s}/s. \quad A_{l,s} = K_s(l\beta)I_s(l\gamma).$

$\alpha = r/b. \quad \beta = kb. \quad \gamma = kr \quad for \quad r < b.$

$\alpha = b/r. \quad \beta = kr. \quad \gamma = kb \quad for \quad r > b.$

5. RFQ EQUIVALENT PARAMETERS

For proper choice RFQ geometry it is desirable to define the following characteristics of the channel.

The effectiveness of acceleration in RFQ is characterized by

$$\xi = 2\cdot\varphi(0)T/V.$$

where V is the intervane voltage.

$T = \pi\cdot A_1(0)/(4\sum_{l=0}^{\infty} A_{2l+1}(0))$ is the transit time factor.

The effectiveness of focusing in RFQ is characterised by

$$x = 2\cdot\tau A_{0,1}(a/b)/V$$

where a is the bore radius of the channel.

When choicing RFQ geometry, it is necessary to know the coefficients of unlinearity and asymmetry of focusing field:

$$\delta = |1 + E_r(a.\pi/2.0)a/(xV)|.$$

$$\eta = |1 + E_r(a.\pi/2.0)/E_r(a.0.0)|.$$

The above obtained results leads to

$$\xi = 2\cdot\pi\tau K_0(kb)/V. \qquad x = 2\cdot\tau(a/b)^2/V.$$

$$T = \pi\cdot K_0(kb)/(4\sum_0^{\infty} K_0((2l+1)kb)).$$

One can consider $T = \pi/4$ for $kb \geq 2$. This result corresponds to the one term potential distribution along the channel axis.

6. ANALYSIS AND RESULTS

The different electrodes shapes can be synthetized by means of changing of the generating parameter kb. As examples the results of calculations of isopotential surfaces (electrodes shapes) are given at Fig.2 - 5 for $kb = 5$. $kb = \pi$ and $kb = \pi/2$.

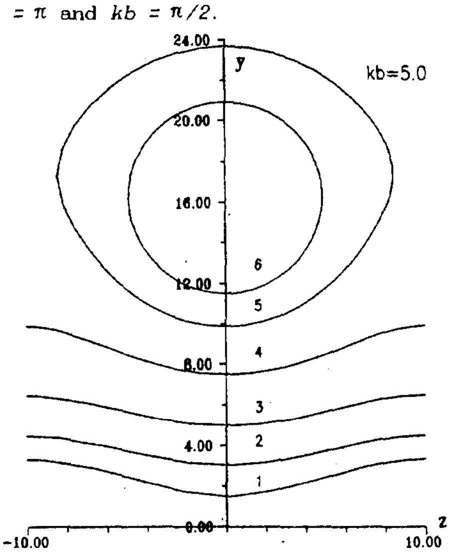

Figure 2. Equipotentials for $kb = 5$

The equipotentials of the system can be either enclosed configurations or surfaces which are continuous along the channel axis. For the systems with large value of kb the part of equipotentials is enclosed. the others are the unenclosed surfaces modulated periodically in the longitudinal direction. These types of equipotentials are separated from each to other by the boundary surface - the space separatrix (the curve number 3 at Fig.3). The cross sections of the electrodes in $X - Z$ plane are shown at Fig.4. For $y < ma$ the $X - Z$ equipotentials are closed. for $y > ma$ they are unclosed.

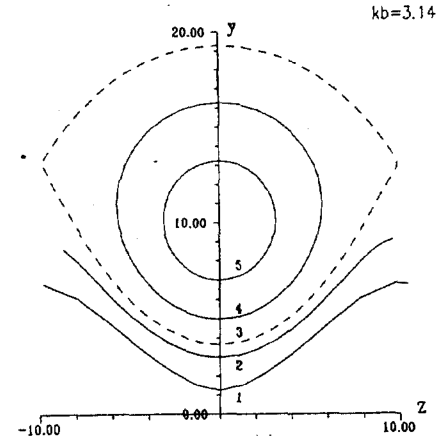

Figure 3. Equipotentials for $kb = \pi$

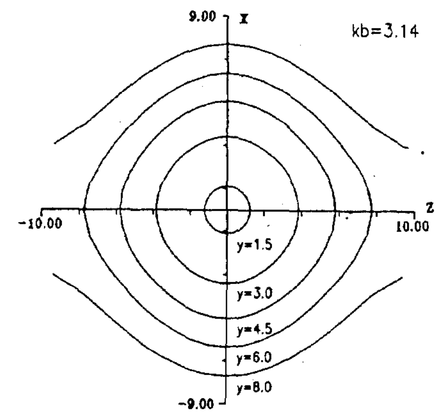

Figure 4. Equipotentials for kb = π

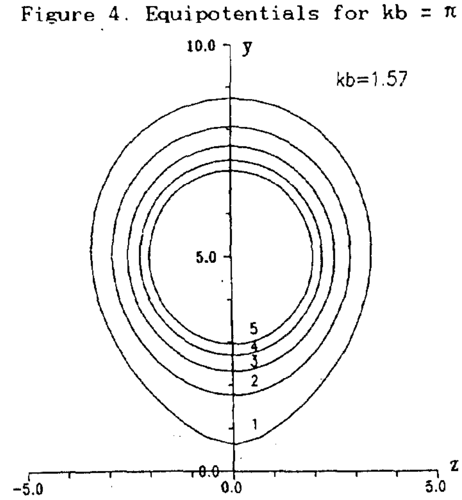

Figure 5. Equipotentials for $kb = \pi/2$

For small values of generating parameters kb (Fig.5) equipotentials have the closed configuration. This case corresponds so called stem electrodes or the electrodes with extremely large modulation.

The results of calculations are summarized in Tables 1 - 3.

Table 1
Results of calculations for kb = 5

Curve number	1	2	3	4	5	6
ka	0.468	0.958	1.57	2.36	3.10	3.61
m	2.23	1.47	1.30	1.32		
x_i	0.360	0.661	0.773	0.794		
$4\xi_i/\pi$	0.606	0.272	0.132	0.070		
x	0.359	0.661	0.772	0.776	0.692	0.587
$4\xi/\pi$	0.603	0.266	0.116	0.051	0.027	0.017

Table 2
Results of calculations for kb = π

Curve number	1	2	3	4	5
ka	0.390	0.933	1.15	1.57	2.20
m	5.50	3.07	3.62		
x_i	0.105	0.350	0.466		
$4\xi_i/\pi$	0.862	0.528	0.393		
x	0.111	0.361	0.425	0.474	0.403
$4\xi/\pi$	0.860	0.488	0.379	0.225	0.098

Table 3
Results of calculations for kb = π/2

Curve number	1	2	3	4	5
ka	0.193	0.560	0.732	0.848	0.930
x	0.018	0.114	0.156	0.174	0.182
$4\xi/\pi$	0.912	0.685	0.548	0.457	0.395

In Tables 1 - 2 the effectivenesses of acceleration and focusing for the idialized electrodes are also given. They were calculated in accordance with [1]:

$$x_i = (I_0(ka) + I_0(mka))/(m^2 I_0(ka) + I_0(mka)).$$
$$4\xi_i/\pi = (1 - x_i)/I_0(ka).$$

These relations can be used for the unenclosed configurations with $ma < b$. For such systems the relationship of the effectiveness of focusing with the model parameters has the simple form:

$$x = (1 + 4(b/a)^2 K_0(kb) I_0(ka))^{-1}.$$
$$\delta = \eta/2 = (kb)^2 \cdot K_0(kb).$$

7. CONCLUSION

The equivalent charges model allows to describe various RFQ electrodes configurations beginning with the idealized electrodes up to the electrodes with extremely large modulation - the stem electrodes. The use of electrodes with large modulation may allow to increase an acceleration rate in RFQ. The additional advantage of such a system is small value of capacitance per unit length. But the increase of the modulation leads to rise of unlinearity of focusing field. A final choice of the RFQ channel parameters requires analysis of influence of this effect on beam dynamics.

8. ACKNOWLEDGMENTS

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