Waveguide Side-Wall Coupling in RF Guns

Leon C.-L. Lin, S. C. Chen, J. Gonichon, S. Trotz, and J. S. Wurtele
Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, MA02141

Abstract

Waveguide side-wall coupling for RF guns is investigated both theoretically and experimentally. We model this aperture-coupling problem by an integral equation which is solved by the method of moments. The analysis yields an equivalent circuit representation of the system. Of the two normal modes of cavity resonance, the π-mode and 0-mode, we show that only the π-mode is excited. Experimental results show good agreement with theory.

I. Introduction

Photocathode RF guns are promising high brightness electron beam sources for free electron lasers and next generation linear colliders. Among existing systems, the $1\frac{1}{2}$-cell RF cavity design with a waveguide-side wall coupling scheme is most widely used[1,2]. This coupling scheme has been shown experimentally to successfully excite the desired π-mode resonance. However, until now there has been no solid theoretical understanding of the coupling. In this paper, we present a theoretical study of the waveguide side-wall coupling. First, we represent this problem by an integral equation based upon the equivalence principle. Next, we solve this integral equation by the method of moments. Then we construct an equivalent circuit representation for this problem: a transmission line shunt with a coupled GLC circuit. This circuit can be solved readily. The experimental data[3] are in good agreement with theory.

II. Theory and Cold Test Results

Consider a $1\frac{1}{2}$-cell RF cavity coupled to a waveguide via side-wall apertures, as shown in Fig. 1. The aim is to excite the π-mode resonance for the RF cavity with power

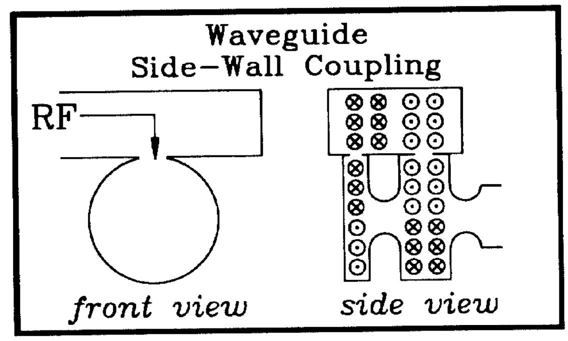

Figure 1: A $1\frac{1}{2}$-cell RF cavity coupled to a waveguide via side-wall apertures.

fed through the coupler. It is sufficient, from the equivalence principle, to solve for the unknown tangential electric field on the aperture (denoted as $\mathbf{E}^a$)[4]. The fields elsewhere can be uniquely determined by $\mathbf{E}^a$. We introduce an equivalent surface magnetic current $\mathbf{M}_s = \mathbf{E}^a \times \hat{n}$, where $\hat{n}$ is the unit vector normal to the aperture. $\mathbf{M}_s$ can be expanded by a linear combination of basis functions $\mathbf{M}_j$ ($j = 1, 2, ..., N$): $\mathbf{M}_s = \sum_j V_j \mathbf{M}_j$. From the continuity of the $\mathbf{H}$ field on the aperture, one obtains

$$[\mathbf{Y}^{wg} + \mathbf{Y}^{cv}]\mathbf{V} = \mathbf{I}, \tag{1}$$

with

$$Y_{ij}^{wg} = \iint_S ds \mathbf{M}_i \cdot \mathbf{H}^{wg}(\mathbf{M}_j), \tag{2}$$

$$Y_{ij}^{cv} = \iint_S ds \mathbf{M}_i \cdot \mathbf{H}^{cv}(\mathbf{M}_j), \tag{3}$$

$$I_i = -\iint_S ds \mathbf{M}_i \cdot \mathbf{H}^{inc}, \tag{4}$$

where $\mathbf{H}^x(\mathbf{M}_j)$ denotes the $\mathbf{H}$ field generated by $\mathbf{M}_j$ (for $x = wg, cv$), $\mathbf{H}^{inc}$ denotes the incident $\mathbf{H}$ field, and S denotes the aperture region. The formulation is greatly simplified if, on either side of the aperture, one has a canonical geometry for which a Green's function in close form exists. In our waveguide side-wall coupling problem, the waveguide is a canonical geometry, but the $1\frac{1}{2}$-cell RF cavity is not. However, it is obvious that the two cells are coupled

*This research is supported by DOE under grant DE-FG02-91-ER40648.

0-7803-1203-1/93$03.00 © 1993 IEEE

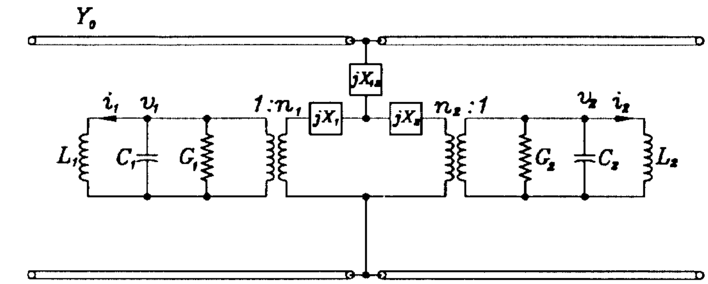

Figure 2: An equivalent circuit for the waveguide side-wall coupling problem.

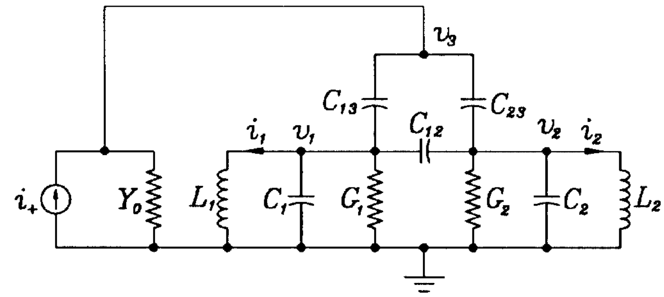

Figure 3: An equivalent circuit for the case when the waveguide is shorted at quarter-wavelength away from the aperture.

to each other primarily via the waveguide rather than via the circular iris (note that the circular iris is below cutoff, while the waveguide is not). Therefore, one can adopt the following steps to solve this problem: (1)Simplify by ignoring the iris and by representing each cell as a pillbox cavity — a canonical geometry. (2)Solve the simplified problem by using the moment method and construct an equivalent circuit representation. (3)Include the effects of the iris by introducing a reactive junction in the equivalent circuit.

The details of deriving Y_{ij}^{wg} and Y_{ij}^{cv} using Green's functions are given in [5]. Once $\mathbf{Y}^{wg}$ and $\mathbf{Y}^{cv}$ are calculated, the coefficients V_j can be obtained using Eq. 1. Then one can obtain the reflection and transmission coefficients in the presence of the surface magnetic current $\mathbf{M}_s = \sum_j V_j \mathbf{M}_j$ using the Green's function for the waveguide. Then one can construct an equivalent circuit, as shown in Fig. 2, which gives the same reflection and transmission coefficient as those in the original problem. The equivalent circuit elements are

$$X_1 = \left(\frac{Im\{Y_{11}^{wg}\}}{Re\{Y_{11}^{wg}\}} - \frac{Im\{Y_{12}^{wg}\}}{Re\{Y_{12}^{wg}\}}\right)/2Y_0, \qquad (5)$$

$$X_2 = \left(\frac{Im\{Y_{22}^{wg}\}}{Re\{Y_{22}^{wg}\}} - \frac{Im\{Y_{12}^{wg}\}}{Re\{Y_{12}^{wg}\}}\right)/2Y_0, \qquad (6)$$

$$X_{12} = \frac{Im\{Y_{12}^{wg}\}}{2Y_0 Re\{Y_{12}^{wg}\}}, \qquad (7)$$

$$C_i = \frac{\pi\epsilon_0 R_i^2 |J_0'(\chi_{01})|^2}{d_i}, \qquad (8)$$

$$L_i = 1/\omega_i^2 C_i, \qquad (9)$$

$$G_i = \omega_i C_i / Q_i, \qquad (10)$$

$$n_i^2 = \frac{-2\Delta x_i^2 C_i}{\mu_0 \pi^3 R_i^2 d_i Y_0 Re\{Y_{ii}^{wg}\}}, \qquad (11)$$

where R_i, d_i, ω_i, and Q_i are the radius, length, resonant frequency, and quality factor of the i-th pillbox cavity, respectively, Δx_i is the aperture width for the i-th pillbox cavity, and Y_0 is the characteristic admittance of the waveguide.

From the equivalent circuit, it is obvious that the waveguide must be shorted a quarter wavelength away from the aperture so as to obtain maximum coupling. In that case, by using a $Y-\Delta$ transformation and transforming the *GLC* circuits to the other side of the transformer, one obtains the circuit shown in Fig. 3.

We define the variables

$$a_i = \sqrt{\frac{C_i}{2}} v_i + j\sqrt{\frac{L_i}{2}} i_i \qquad (12)$$

for $i = 1,2$ such that $|a_i|^2$ = stored energy in the i-th cell. Because one is interested only in the vicinity of resonant frequencies of both cells, the following approximations can be used: (1)$1-(\frac{w}{w_i})^2 = (1+\frac{w}{w_i})(1-\frac{w}{w_i}) \cong 2(1-\frac{w}{w_i})$, and (2)$a_i \cong \sqrt{\frac{C_i}{2}} V_i e^{j\omega t}$. The circuit is readily solved given the above approximations:

$$\begin{bmatrix} j(\omega-\omega_1') & -\kappa \\ -\kappa & j(\omega-\omega_2') \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \qquad (13)$$

where

$$\omega_1' \cong \omega_1\sqrt{\frac{C_1}{C_1'}} + j\frac{G_1}{2C_1'}, \qquad (14)$$

$$\omega_2' \cong \omega_2\sqrt{\frac{C_2}{C_2'}} + j\frac{G_2}{2C_2'}, \qquad (15)$$

$$\kappa \cong \frac{j\omega C_{12}}{2\sqrt{C_1' C_2'}}, \qquad (16)$$

$$b_1 = \frac{-j\omega C_{13} I_+}{\sqrt{8C_1'}(Y_0 + j\omega C_3')}, \qquad (17)$$

$$b_2 = \frac{-j\omega C_{23} I_+}{\sqrt{8C_2'}(Y_0 + j\omega C_3')}, \qquad (18)$$

$$C_1' = C_1 + C_{12} + C_{13}, \qquad (19)$$

$$C_2' = C_2 + C_{12} + C_{23}, \qquad (20)$$

$$C_3' = C_{13} + C_{23}. \qquad (21)$$

It is convenient to normalize frequencies with respect to the coupling coefficient κ: $\Omega \equiv \frac{w}{-j\kappa}$. Then the eigenvalues and eigenvectors of this system are

$$\Omega_\pm = \frac{\Omega_1' + \Omega_2'}{2} \pm \sqrt{(\frac{\Omega_1' - \Omega_2'}{2})^2 + 1}, \qquad (22)$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{a}_+ & \mathbf{a}_- \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \qquad (23)$$

$$\theta = \tan^{-1}\left\{-\frac{\Omega'_1-\Omega'_2}{2}+\sqrt{\left(\frac{\Omega'_1-\Omega'_2}{2}\right)^2+1}\right\} \quad (24)$$

The eigenmodes $\mathbf{a}_+$ and $\mathbf{a}_-$ are known as the symmetric (π-) and antisymmetric (0-) mode, respectively. The response $\mathbf{a}$ is a linear combination of $\mathbf{a}_+$ and $\mathbf{a}_-$: $\mathbf{a}=\mathbf{S}\mathbf{c}=c_+\mathbf{a}_++c_-\mathbf{a}_-$ with $\mathbf{c}=\mathbf{S}^{-1}\mathbf{A}^{-1}\mathbf{b}$. Consider three special cases: (1)$|\frac{\Omega'_1-\Omega'_2}{2}|\gg 1$ and $\Omega'_1>\Omega'_2$. Here, the two cells are nondegenerate and the first cell has higher resonant frequency. (2)$|\frac{\Omega'_1-\Omega'_2}{2}|\gg 1$ and $\Omega'_1<\Omega'_2$. Here, the two cells are nondegenerate and the second cell has higher resonant frequency. (3)$|\frac{\Omega'_1-\Omega'_2}{2}|\ll 1$. Here, two cells are degenerate. The resonant frequencies $\Omega_\pm$, normal modes $\mathbf{a}_\pm$, and coefficients $c_\pm$ for these three cases are summarized in Table 1. In general, it is very likely that the cavity will be in either Case (1) or Case (2) before tuning; in either case both the π-mode and 0-mode can be excited. To excite a pure π-mode, one must tune the cavity until Case (3) is reached. Then, if the condition $c_-\propto(b_2-b_1)\sim 0$ holds, one can efficiently suppress the 0-mode.

	Case (1)	Case (2)	Case (3)
Ω_+	Ω'_1	Ω'_2	$\frac{\Omega'_1+\Omega'_2}{2}+1$
Ω_-	Ω'_2	Ω'_1	$\frac{\Omega'_1+\Omega'_2}{2}-1$
S	$\begin{bmatrix}1&0\\0&1\end{bmatrix}$	$\begin{bmatrix}0&-1\\1&0\end{bmatrix}$	$\frac{1}{\sqrt{2}}\begin{bmatrix}1&-1\\1&1\end{bmatrix}$
c_+	$\frac{b_1}{\kappa(\Omega-\Omega'_+)}$	$\frac{b_2}{\kappa(\Omega-\Omega'_+)}$	$\frac{b_2+b_1}{\sqrt{2}\kappa(\Omega-\Omega'_+)}$
c_-	$\frac{b_2}{\kappa(\Omega-\Omega'_-)}$	$\frac{-b_1}{\kappa(\Omega-\Omega'_-)}$	$\frac{b_2-b_1}{\sqrt{2}\kappa(\Omega-\Omega'_-)}$

Table 1: Eigenmodes and their coefficients for three special cases.

We use a network analyzer to measure the reflection coefficient (S_{11}) for the 17GHz RF gun cavity. In our experiment, the untuned cavity belongs to Case (2). Also, the two apertures are commensurate in size and, consequently, $b_1\sim b_2$. Therefore, one observes two distinct resonances with comparable magnitude, as shown in Fig. 4. After we tune the first cell to Case (3), only the symmetric (π-) mode is seen, as shown in Fig. 5. Note that the resonance of the symmetric mode is upshifted by an amount κ, which is about $16MHz$ in our experiment.

III. Conclusion

We have constructed an equivalent circuit representation for side-wall coupling in RF guns. We solved the equivalent circuit and found that, in our coupling scheme, only the π-mode of the $1\frac{1}{2}$-cell cavity is excited. Cold tests of our 17GHz gun confirm these predictions.

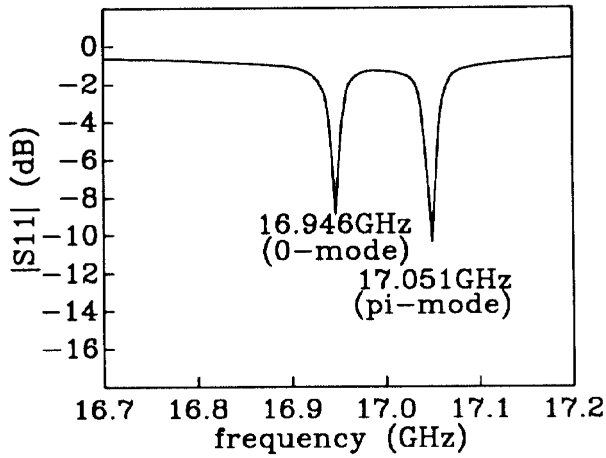

Figure 4: The reflection coefficient for the untuned cavity.

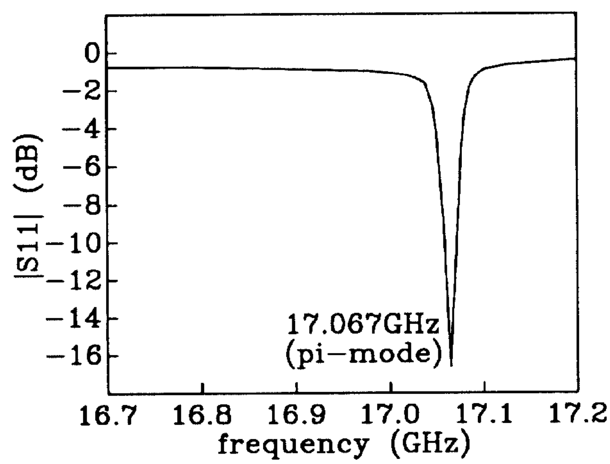

Figure 5: The reflection coefficient for the tuned cavity.

References

[1] K. Batchlor *et al.*, "Design and Modelling of a 5MeV Radio Frequency Electron Gun," BNL-41766 (1988).

[2] F. Aghamir *et al.*, "SATURNUS: The UCLA High Gain Infrared FEL project," UCLA-CAA0068-9/90 (1990).

[3] S. C. Chen *et al.*, "High Gradient Acceleration in a 17 GHz Photocathode RF Gun," These Proceedings (1993).

[4] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*, New York: McGraw Hill, 1961.

[5] Leon C.-L. Lin *et al.*, "An Equivalent Circuit Formulation for Waveguide Side-Wall Coupling in RF Guns," to be submitted to *IEEE Trans. Microwave Theory and Technology*.